

# Modeling Optimization Study of Two-Dose Vaccine Distribution Considering Timeliness

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**Keywords:** Epidemic Control, Vaccine Distribution, Timeliness.

**Abstract:** Vaccination is one of the most effective measures for epidemic prevention and control. In this paper, we firstly construct a vaccine timeliness function based on vaccine effectiveness and completion time. Then we propose a dynamic two-dose vaccine distribution optimization model base on the age-structure compartment model, to reduce infection and speed up infection clearance. Comparing with the pro rata strategy and the not considering timeliness strategy, the results showed that the strategy designed in this paper not only advanced the clearing time but also reduced the number of infections.

## 1 INTRODUCTION

Vaccination is one of the most efficient ways to halt the spread of the COVID-19 outbreak. Most countries advise 2 rounds of vaccination to avoid COVID-19 pneumonia. But optimizing the distribution of two doses of the vaccine in the absence of adequate vaccine production presents an objective and practical difficulty.

Vaccine distribution needs to be based on the dynamics of epidemic transmission, and the compartment model is an effective means to characterize epidemic transmission (Mukandavire et al. 2007, Althaus et al. 2014, Glasser et al. 2016, He et al. 2020).

A compilation of the literature related to vaccine distribution reveals that most vaccine distribution studies consider only one-time distribution of one-dose vaccines, such as Enayati et al. reduced the effective regeneration number of influenza epidemics to less than or equal to 1 by a one-time vaccine distribution (Enayati et al. 2020). Matrajt et al. proposed an optimal allocation strategy for the COVID-19 vaccine, and they assumed that people had been vaccinated according to the optimal allocation before the experiment (Matrajt et al. 2021). A small number of studies have also considered two-

dose vaccine allocation, such as Matrajt et al. who, based on a previous article, proposed an optimal allocation strategy with a mixture of one- and two-dose vaccines (Matrajt et al. 2021). Few other studies conducted dynamic distribution studies of vaccines in the spread of the epidemic, such as Han et al. conducted an optimal distribution study of the new crown vaccine to obtain a time-varying vaccine distribution strategy (Han et al. 2021). Chen et al. studied the COVID-19 vaccine allocation strategy in New York City and found that the dynamic distribution strategy outperformed the static distribution strategy (Chen et al. 2018).

Specifically, Parino et al. conduct a dynamic distribution study of two-dose vaccine in an Italian research context (Parino et al. 2021). Further, they proposed a stochastic optimal vaccine allocation model to explore the problem of optimal allocation of two-dose vaccine (Calafiore et al. 2022). Based on their study, this paper further considers the age heterogeneity of virus transmission and vaccine efficacy, and innovatively defines vaccine timeliness and uses it as the objective function for optimal two-dose vaccine distribution.

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## 2 MODEL FORMULATION

### 2.1 Parameter and Variable Definitions

The necessary notation for this paper is first defined as follows to make it easier to articulate the model in the following section:

$K$ : the set of age groups,  $k, \bar{k} \in K$

$\bar{k}$ : any age group that encounters the age group  $k$ , the age group  $k$  will encounter this age group and other age groups

$N_k$ : Number of people in the age group  $k$

$\varsigma_k$ : Susceptibility of the age group  $k$

$\tau_{1k}$ : Effectiveness of the first dose of vaccine for the age group  $k$

$\tau_{2k}$ : Effectiveness of two full doses of vaccine for the age group  $k$

$\beta$ : Transmission rate

$C_{k,\bar{k}}$ : Contact rate between age group  $k$  and age group  $\bar{k}$

$\delta$ : Probability of conversion from latent to infected

$\gamma$ : Probability of conversion of an infected person to a recovered person

$\omega$ : Interval between the first and second doses

$T$ : Duration of the experiment,  $t \in T$

$C$ : Number of vaccines distributed per day

$S_k(t)$ : Number of susceptible persons in the age group  $k$  at the time  $t$

$V_{1k}(t)$ : Number of people in the group  $k$  who had received the first dose of vaccine and not the second dose at the time  $t$

$V_{2k}(t)$ : Number of people in the age group  $k$  who had received the full two doses of vaccine at the time  $t$

$E_k(t)$ : Number of latent cases in the age group  $k$  at the time  $t$

$I_k(t)$ : Number of infected persons in the age group  $k$  at the time  $t$

$R_k(t)$ : Number of recovered persons in the age group  $k$  at the time  $t$

$u_{1k}(t)$ : Number of first vaccine doses assigned to the age group  $k$  at the time  $t$

$u_{2k}(t)$ : Number of second vaccine doses assigned to the age group  $k$  at the time  $t$

### 2.2 Model Explanation

#### 2.2.1 Timeliness Definition

This study evaluates the vaccination's timeliness in terms of both vaccine efficiency and completion time. The total number of infections devotes the efficiency of the vaccination. The clearing time of infected persons indicates completion time of the vaccination. The vaccine effect and completion time functions are created using the sigmoid function ( $f(x) = \frac{1}{1 + e^{-ax}}$ ). The

precise steps are as follows.

$$i = \sum_{k=1}^K \sum_{t=1}^T I_k(t) \quad (1)$$

$$f(i) = \frac{1}{1 + e^{-\frac{\ln(\frac{1}{\epsilon_1} - 1)^2}{I_{max}}(i - \frac{I_{max}}{2})}} \quad (2)$$

$$t = \begin{cases} t | \sum_{k=1}^K I_k(t) = 0, \sum_{k=1}^K I_k(T) = 0 \\ T, \sum_{k=1}^K I_k(T) > 0 \end{cases} \quad (3)$$

$$g(t) = \frac{1}{1 + e^{-\frac{\ln(\frac{1}{\epsilon_2} - 1)^2}{t - \omega}(t - \frac{T + \omega}{2})}} \quad (4)$$

$$Max(mf(i))(ng(t)) \quad (5)$$

#### 2.2.2 Vaccine Distribution Model

In this study, the vaccine distribution operation is added to the age-structured seir model with two new compartments included to create the SVEIR model, as illustrated in Figure 1 below.

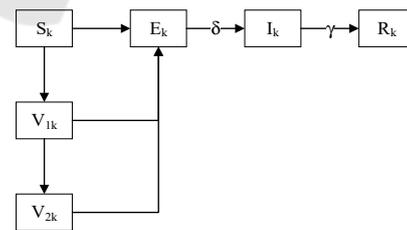


Figure 1: The SVEIR model.

In the SVEIR model, we divide the population into six categories, namely susceptible (S), first dose vaccine recipient ( $V_1$ ), full dose vaccine recipient ( $V_2$ ), exposed (E), infected (I) and recovered (R). Susceptible persons ( $S_k$ ) are transformed into exposed persons ( $E_k$ ) in part due to contact with infected persons ( $I_k$ ) and into first dose vaccine recipients ( $V_{1k}$ ) in part due to first dose vaccination. After at

least  $\omega$  days, the first dose vaccinated persons ( $V_{1k}$ ) can receive the second dose vaccination and become full dose vaccinated persons ( $V_{2k}$ ). The total number of first and second doses of vaccine allocated to each age group each day will not exceed the supply for that day ( $o$ ). The first dose vaccine recipients ( $V_{1k}$ ) and the full dose vaccine recipients ( $V_{2k}$ ) are also partially transformed into exposed persons ( $E_k$ ) due to contact with infected persons ( $I_k$ ) in each age group.

Exposed persons ( $E_k$ ) become infected ( $I_k$ ) after an incubation period of  $\frac{1}{\delta}$ . Infected persons ( $I_k$ ) are transformed into recovered persons ( $R_k$ ) after a period of infection.

According to the above description, the dynamic conversion process between different populations during the outbreak is as follows:

$$\frac{dS_k(t)}{dt} = -u_{1k}(t) - S_k(t)\beta \sum_{\bar{k}=1}^K (c_{k,\bar{k}} \frac{I_{\bar{k}}(t)}{N_{\bar{k}}}) \quad (6)$$

$$\frac{dV_{1k}(t)}{dt} = u_{1k}(t) - u_{2k}(t) - V_{1k}(t)(1 - \tau_{1k})\beta \sum_{\bar{k}=1}^K (c_{k,\bar{k}} \frac{I_{\bar{k}}(t)}{N_{\bar{k}}}) \quad (7)$$

$$\frac{dV_{2k}(t)}{dt} = u_{2k}(t) - V_{2k}(t)(1 - \tau_{2k})\beta \sum_{\bar{k}=1}^K (c_{k,\bar{k}} \frac{I_{\bar{k}}(t)}{N_{\bar{k}}}) \quad (8)$$

$$\frac{dE_k(t)}{dt} = (S_k(t) + V_{1k}(t)(1 - \tau_{1k}) + V_{2k}(t)(1 - \tau_{2k}))\beta \sum_{\bar{k}=1}^K (c_{k,\bar{k}} \frac{I_{\bar{k}}(t)}{N_{\bar{k}}}) - \delta E_k(t) \quad (9)$$

$$\frac{dI_k(t)}{dt} = \delta E_k(t) - \gamma I_k(t) \quad (10)$$

$$\frac{dR_k(t)}{dt} = \gamma I_k(t) \quad (11)$$

$$\sum_{k=1}^K (u_{1k}(t) + u_{2k}(t)) \leq o \quad (12)$$

$$0 \leq u_{1k}(t) \leq S_k(t) \quad (13)$$

$$0 \leq u_{2k}(t + \omega) \leq V_{1k}(t) \quad (14)$$

$$S_k(t), V_{1k}(t), V_{2k}(t), E_k(t), I_k(t), R_k(t) \geq 0 \quad (15)$$

### 3 SOLUTION PROCEDURE

The model described above is a nonlinear optimization model and solved by the particle swarm algorithm. The solution process is outlined as follows.

Step 1: Randomly initialize the position vector so that the sum of elements is equal to the daily vaccine supply ( $o$ ).

Step 2: The positions are brought into the SVEIR model to calculate the infection situation, and the objective function value is obtained as the fitness.

Step 3: Compare the fitness values, get the individual optimal solution and population optimal solution, and update the position and velocity of the particles.

Step 4: The algorithm terminates when the maximum number of iterations or the upper limit of running time is reached.

## 4 CASE STUDY

### 4.1 Parameter Setting

The test population of 100,000 people was divided into four groups: 0-14, 15-39, 40-64, and 65+. The parameter settings considering age heterogeneity are referred to the literature (Al Kaabi et al. 2021, China 2021, Hu et al. 2021), as shown in the following table.

Table 1: Parameter settings considering age heterogeneity.

Age group	No.	Population	Susceptibility	Effectiveness of first dose	Effectiveness of full dose
0-14	1	17973	0.58	0.2	0.6
15-39	2	32816	1	0.3	0.8
40-64	3	35689	1	0.3	0.8
65+	4	13522	1.65	0.2	0.6

Assuming an effective reproduction number of 1.5 at the beginning of the outbreak, the resulting transmission rate was 0.0610 (Diekmann et al. 1990). The contact rate between age groups was calculated from literature (Zhang et al. 2019). The transition rate from exposed to infected was 0.1562 (incubation period 6.4 days). The probability of recovery was 0.1754 (mean infection period 5.7 days) (Hu et al. 2021). The minimum interval between the first and second dose was 21 days. The total duration of the experiment was set at 300 days, with 1500 doses of vaccine distributed daily and 5 infected individuals in each age group at the beginning of the experiment.

### 4.2 Test Result

The algorithm yielded a locally optimal solution after 349 iterations with a cumulative number of infections of 3338 and 178 days of clearing under a 1-hour runtime limit.

The distribution of the vaccine is shown in Fig. 2-3 below. The 65+ age group, which is sensitive to the virus and has low vaccine efficacy, needs to be vaccinated as fast as possible until fully covered. When the outbreak appears to be under control in the 65+ age group, the 65+ age group can postpone receiving the first dose, which already has high coverage, and continue receiving vaccinations once the disease has been contained in other age groups.

The 15-39 and 40-64 age groups, which have the most contact with other age groups, also need to be vaccinated quickly until complete coverage is achieved. The 0-14 age group basically does not receive vaccination in the early stage due to its own low susceptibility to the virus, and a small amount of vaccination is slowly administered after the epidemic is controlled in other age groups.

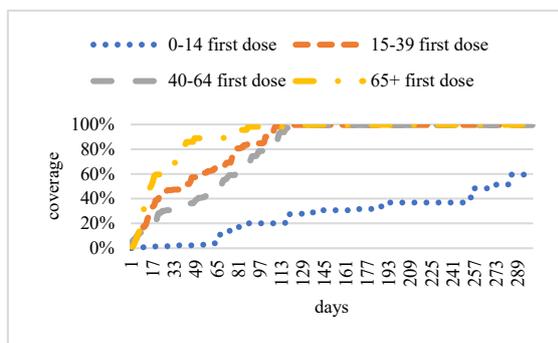


Figure 2: First dose distribution.

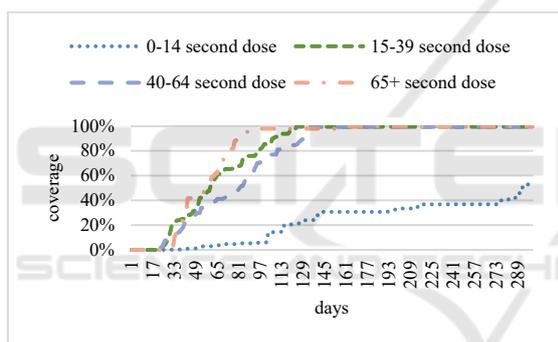


Figure 3: Second dose distribution.

## 5 CONCLUSION

In this paper, a time-efficient objective function is innovatively defined in the vaccine distribution problem to reduce infections and achieve early zero community transmission. We introduce vaccination operations into an age-structured compartment model and constructs a dynamic vaccine distribution model that can be extended to other age heterogeneous infectious diseases for resource distribution decisions. Afterwards, we combine particle swarm algorithm with ideal point method to solve the location model. Based on data experiments, we suggest that vaccination of the 65+ age group should be performed as fast as possible until complete coverage is achieved. The 15-39 and 40-64 age groups should be followed by timely and complete

coverage; the 0-14 age group can be vaccinated in small amounts in the early stages.

This paper also has some shortcomings that could be a direction for future research. We do not consider asymptomatic infected people and the effect of vaccine on reducing the symptoms of infection. We used the particle swarm algorithm to solve the model, but the search for a faster and more accurate solution can be continued.

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