Game Analysis of Manufacturers and Gray Market Speculators Under Different Government Policies

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Abstract: This article aims to explore the impact of different government policies on the sales of licensed products by manufacturers and the sales of parallel products by gray market speculators. A market structure with manufacturers as dominant players and gray market speculators as followers is established. Based on the Stackelberg game approach, three scenarios are considered in which the government does not implement a policy, the government implements a subsidy policy for manufacturers, and the government implements a regulatory policy for gray market speculators. In the three cases, the influence of government subsidy amount and supervision intensity on the equilibrium solution of manufacturers and gray market speculators is solved and compared. Research shows that when the government implements subsidy policies, both manufacturers and gray market speculators will reduce the unit sales price of their products. When the government implements regulatory policies, manufacturers will increase the unit sales price of licensed products, and gray market speculators will reduce the sales price of parallel products. Regardless of whether the government implements subsidies or regulatory policies, it will increase the after-sales service level, sales volume and profits of manufacturers, and reduce the sales and profits of gray market speculators.

1 INTRODUCTION[1]

Gray markets, also known as parallel imports, are market channels that sell branded goods without the authorization of the trademark holder (Liu et al., 2020). Unauthorized sellers are called "gray market speculators" and goods sold through gray market speculators are called "gray market products" or "parallel products". Factors such as volume discounts implemented by manufacturers and large fluctuations in exchange rates that cause price differences in different markets for the same product are the main reasons why gray markets occur (Cavusgil et al., 1988). In recent years, with the rapid development of e-commerce, Internet technology and globalization of logistics, the gray market phenomenon has become increasingly prominent. For example, according to the Financial Times, in the European Union, the gray market accounts for billions of dollars in pharmaceutical sales each year. Kanavos et al. (2004) find that from 1997 to 2002, the share of the gray market in the overall pharmaceutical industry increased from less than 2% to 10.1% in Sweden, from 1.7% to about 7% in Germany, and from less than 1% to 21.6% in Greece during the same period. A report released by a company called iSuppli shows that 145 million cell phones were shipped in the gray market in 2009, with a nearly 13% market share of the global cell phone market, a significant increase of 43.6% compared to 2008. Gray markets are also found in other sectors around the world, with the airline industry, the automotive industry, watches and jewelry, and beauty and health products all involving gray market speculators (Wang, 2014). The increasingly large size of the gray market can cause a decline in profits from the perspective of companies. It also reduces consumer satisfaction and damages brand image in the long run as the quality of after-sales service for consumers cannot be guaranteed. From the stakeholder's point of view, it will make gray market speculators and unscrupulous black marketers gain higher profits, leading to chaotic market channels and social harmony in turmoil. Therefore, it is of some practical importance to motivate companies to solve the gray market problem from the government's perspective.

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The increasingly prominent issue of gray market has attracted domestic and international scholars to study it from different perspectives. For example, Su et al. (2012) find that volume discount contracts and revenue sharing contracts can reduce retailer participation in gray market transactions. Hong et al. (2018) establish compensation contracts to achieve overall supply chain coordination and incentives in a gray market environment. Since parallel imports are a common form of gray market, scholars have also studied parallel imports and manufacturer channel choice. For example, Su et al. (2017) based on the perspective of parallel imports find that manufacturers' choice of channel depends on the effects of parallel import costs, the elasticity of market demand in two countries and production costs. Hong et al. (2021) construct four channel structures based on manufacturers selling directly or distributing in two markets to investigate the conditions for the existence of gray markets under different channel structures and the optimal channel choice for manufacturers. Yeung et al. (2013) examine how official distribution channels of multinational companies, asset specificity, limited rationalization of franchised dealers, and parallel traders lead to the sustainability of parallel imported cars. Other scholars have studied gray markets in terms of pricing and service strategies. For example, Ahmadi et al. (2000) give manufacturers' pricing strategies under different conditions by developing a three-level supply chain model with manufacturers, gray market speculators, and consumers. The findings suggest that the presence of gray market speculators may help manufacturers to expand the global scope of their products and even increase their global profits. Iravani et al. (2016) find that the emergence of gray markets leads manufacturers to increase service levels in both high- and low-price markets and that service decisions can be used as a non-price mechanism to manage gray markets. Rong et al. (2020) consider a multinational manufacturer and a local manufacturer selling products to two independent markets and compare and analyze the effects of different power structures on supply chain members' pricing and profits with and without gray markets. In addition, it has been found that RFID technology (Ding et al., 2022), manufacturers' after-sales service quality decisions (Hu et al., 2021), and brands' remanufacturing decisions (Huang et al., 2020) can be used to manage and control gray markets.

Throughout the literature, manufacturers' after-sales service quality decisions (Iravani et al., 2016; Hu et al., 2021) can be an effective tool for managing gray markets, but few scholars have incentivized firms to improve after-sales service quality from the government's perspective to address the gray market problem. Most of the scholars are combining government policies with green supply chain and closed-loop supply chain. For example, Shang et al. (2020) construct a green supply chain model in which the government subsidizes R&D costs and production costs, respectively, and find that both government subsidy strategies positively affect product greenness, sales volume, and supply chain members' profits. Cao et al. (2020) establish three scenarios based on secondary supply chains with no government subsidies, government subsidies for manufacturers, and government subsidies for retailers. The results show that government subsidies can stimulate the green effort behavior of manufacturers and retailers, which is always beneficial to the green development of the supply chain. Xia et al. (2017) analyze the impact of government adoption of subsidy policy, adoption of regulatory policy, and no policy on the reverse recycling of end-of-life vehicles in formal and informal channels in three cases. Although Wu (2017) studies the impact of government regulatory policies on manufacturers in the presence of gray markets, the relationship between government subsidy policies and gray markets is not studied.

In view of this, this paper considers the existence of a manufacturer and a gray market speculator in the market, constructs three different scenarios in which the government does not implement a policy, the government implements a subsidy policy for the manufacturer, and the government implements a regulatory policy for the gray market speculator, and uses a dominant-subordinate game approach to study the decision problem of different government policies for the manufacturer and the gray market speculator, respectively.

2 MODEL INTRODUCTION

2.1 Model Description

In this paper, we consider a market structure with a manufacturer (denoted as *M*) as the dominant player and a gray market speculator (denoted as *A*) as the follower. The manufacturer sells a licensed product to consumers through an authorized channel at price p_m, and the gray market speculator sells a parallel product to consumers through an unauthorized channel at price p_a. In contrast to gray market

speculators, manufacturers need to provide after-sales services to consumers. The government, in order to motivate manufacturers to provide after-sales services and to reduce arbitrage by gray market speculators, has considered three scenarios: no government policy, a government policy of subsidies to manufacturers, and a government policy of regulation of gray market speculators.

2.2 Model Assumptions

(1) Assume that the manufacturer's cost of providing after-sales service is $\frac{ks^2}{2}$, where k indicates the manufacturer's after-sales service cost factor.

(2) Although both licensed and parallel products are genuine in the market, parallel products do not have access to after-sales service, quality assurance, etc., and their perceived product quality is lower than that of licensed products. Therefore, it is assumed that in the market, the perceived quality of the licensed product is 1 and the perceived quality of the parallel product is θ ($0<\theta<1$).

(3) Assume that the consumer can only purchase a maximum of one product.

(4) Denote consumers' marginal willingness to pay for the perceived quality of a product by x. Let x obey a uniform distribution of $[0,1]$, which yields the consumer's perceived value of the licensed product as x and the perceived value of the parallel product as θx.

(5) Assume that transportation costs, production costs, etc. are all zero.

2.3 Description of Symbols

Table 1: Symbol settings and their descriptions.

Symbol	Symbol description
n	Government does not implement policies
v	Government imposes subsidy policy on manufacturers, unit subsidy amount
τ	Government imposes regulatory policy on gray market speculators, unit regulation
s_m^o	The after-sales service level of a manufacturer when the government implements policy o, where $o\in\{n,v,\tau\}$
p_m^o	The unit selling price of a manufacturer selling a licensed product when the government implements policy o, where $o\in\{n,v,\tau\}$
p_a^o	The unit selling price of a gray market speculator selling a parallel product when the government implements policy o, where $o\in\{n,v,\tau\}$
q_m^o	The number of sales of licensed products sold by manufacturers when the government implements policy o, where $o\in\{n,v,\tau\}$
q_a^o	The number of sales of parallel products sold by speculators in the gray market when the government implements policy o, where $o\in\{n,v,\tau\}$
π_m^o	Manufacturer's profit on the sale of licensed products when the government implements policy o, where $o\in\{n,v,\tau\}$
π_a^o	The profits of gray market speculators from the sale of parallel products when the government implements policy o, where $o\in\{n,v,\tau\}$
u_m^o	The utility function of a consumer purchasing a licensed product when the government implements policy o, where $o\in\{n,v,\tau\}$
u_a^o	The utility function of a consumer's purchase of a parallel product when the government implements policy o, where $o\in\{n,v,\tau\}$

2.4 Demand Function Creation

2.4.1 Government Non-Implementation Policy and Government Implementation of Subsidy Policy

According to the assumptions, the utility function of a consumer purchasing a licensed product is expressed as $u_m^o = x - p_m^o + s_m^o$ and the utility function of a consumer purchasing a parallel product is expressed as $u_a^o = \theta x - p_a^o$, respectively, where $o\in\{n,v\}$. When the utility function $u_m^o \ge u_a^o, u_m^o \ge 0$, the consumer chooses to buy the licensed product; when $u_a^o \ge u_m^o, u_a^o \ge 0$, the consumer chooses to buy the parallel product. At this point the demand function is

$$q_m^o = 1 - \frac{p_m^o - p_a^o - s_m^o}{1-\theta} \quad (1)$$

$$q_a^o = \frac{p_m^o - p_a^o - s_m^o}{1-\theta} - \frac{p_a^o}{\theta} \quad (2)$$

2.4.2 Government Implementation of Regulatory Policies

According to the assumptions, the utility function of consumers purchasing licensed products is $u_m^\tau = x - p_m^\tau + s_m^\tau$, and the utility function of consumers purchasing parallel products is $u_a^\tau = \theta x - p_a^\tau - \tau$. When the utility function $u_m^\tau \geq u_a^\tau, u_m^\tau \geq 0$, the consumer chooses to buy the licensed product; when $u_a^\tau \geq u_m^o, u_a^\tau \geq 0$, the consumer chooses to buy the parallel product. At this point the demand function is

$$q_m^\tau = 1 - \frac{p_m^\tau - p_a^\tau - s_m^\tau - \tau}{1-\theta} \quad (3)$$

$$q_a^\tau = \frac{p_m^\tau - p_a^\tau - s_m^\tau - \tau}{1-\theta} - \frac{p_a^\tau + \tau}{\theta} \quad (4)$$

3 MODEL SOLVING

3.1 Government Non-Implementation Policy

The government has neither a subsidy policy for manufacturers nor a regulatory policy for gray market speculators. At this point, the profit functions for the manufacturer and the gray market speculator are

$$\pi_m^n(p_m^n, s_m^n) = p_m^n q_m^n - \frac{k(s_m^n)^2}{2} \quad (5)$$

$$\pi_a^n(p_a^n) = p_a^n q_a^n \quad (6)$$

Theorem 1: In the absence of government intervention, the optimal solution for manufacturers and gray market speculators is

$$p_m^{n^*} = \frac{4k(1-\theta)^2}{(2-\theta)(4k+\theta-4k\theta-2)}$$

$$s_m^{n^*} = \frac{2(1-\theta)}{4k+\theta-4k\theta-2}$$

$$p_a^{n^*} = \frac{\theta(1-\theta)(2k+\theta-2k\theta-2)}{(2-\theta)(4k+\theta-4k\theta-2)}$$

Substituting $p_m^{n^*}$, $s_m^{n^*}$ and $p_a^{n^*}$ into equations (1) (2) (5) (6), we get

$$q_m^{n^*} = \frac{2k(1-\theta)}{4k+\theta-4k\theta-2}$$

$$q_a^{n^*} = \frac{2k+\theta-2k\theta-2}{(2-\theta)(4k+\theta-4k\theta-2)}$$

$$\pi_m^{n^*} = \frac{k(2\theta-2)^2}{2(2-\theta)(4k+\theta-4k\theta-2)}$$

$$\pi_a^{n^*} = \frac{\theta(1-\theta)(2k+\theta-2k\theta-2)^2}{(\theta-2)^2(4k+\theta-4k\theta-2)^2}$$

Proof 1: According to the reverse solution method, p_a^n is solved first. Since $\frac{\partial^2 \pi_a^n}{\partial (p_a^n)^2} = \frac{2}{\theta(\theta-1)} < 0$, it is known that π_a^n has a great value about p_a^n. Let $\frac{\partial \pi_a^n}{\partial p_a^n} = 0$ and solve for $p_a^n = \frac{\theta(p_m^n - s_m^n)}{2}$. Substituting p_a^n into π_m^n, we find the Hessian matrix of π_m^n with respect to p_m^n and s_m^n as

$$H = \begin{bmatrix} \frac{2-\theta}{\theta-1} & \frac{\theta-2}{2(\theta-1)} \\ \frac{\theta-2}{2(\theta-1)} & -k \end{bmatrix}.$$

When $4k+\theta-4k\theta-2>0$, we have $|H_1| = \frac{2-\theta}{\theta-1} < 0$ and $|H_2| = \frac{-(\theta-2)(4k+\theta-4k\theta-2)}{4(\theta-1)^2} > 0$. It is known that the matrix H is negative definite and π_m^n has great values with respect to p_m^n and s_m^n. We combine $\frac{\partial \pi_m^n}{\partial p_m^n} = 0$ and $\frac{\partial \pi_m^n}{\partial s_m^n} = 0$ to obtain the values of p_m^n and s_m^n. Substitute them back into $p_a^n = \frac{\theta(p_m^n - s_m^n)}{2}$ to obtain the optimal solution for p_a^n. The proof is complete.

3.2 Government Subsidy Policies for Manufacturers

The profit functions for manufacturers and gray market speculators when the government implements a subsidy policy for manufacturers are

$$\pi_m^v(p_m^v, s_m^v) = (p_m^v + v)q_m^v - \frac{k(s_m^v)^2}{2} \quad (7)$$

$$\pi_a^v(p_a^v) = p_a^v q_a^v \quad (8)$$

Theorem 2: In the case of a government subsidy policy for manufacturers, the optimal solution for manufacturers and gray market speculators is

$$p_m^{v^*} = \frac{4k(1-\theta)^2 + v[(2-\theta)^2 + 2k(\theta-1)(2-\theta)]}{(2-\theta)(4\mathrm{k}+\theta-4\mathrm{k}\theta-2)}$$

$$s_m^{v^*} = \frac{2(1-\theta)+\mathrm{v}(2-\theta)}{4k+\theta-4k\theta-2}$$

$$p_a^{v^*} = \frac{\theta(1-\theta)(2\mathrm{k}+\theta-2k\theta-2kv+k\theta v-2)}{(2-\theta)(4\mathrm{k}+\theta-4\mathrm{k}\theta-2)}$$

Substituting $p_m^{v^*}$, $s_m^{v^*}$ and $p_a^{v^*}$ into equations (1) (2) (7) (8), we obtain

$$q_m^{v^*} = \frac{k(2-2\theta+2v-\theta v)}{4k+\theta-4k\theta-2}$$

$$q_a^{v^*} = \frac{2k+\theta-2k\theta-2kv+k\theta v-2}{(2-\theta)(4\mathrm{k}+\theta-4\mathrm{k}\theta-2)}$$

$$\pi_m^{v^*} = \frac{k(2\theta-2v+\theta v-2)^2}{2(2-\theta)(4\mathrm{k}+\theta-4\mathrm{k}\theta-2)}$$

$$\pi_a^{v^*} = \frac{\theta(1-\theta)(2k+\theta-2k\theta-2kv+k\theta v-2)^2}{(\theta-2)^2(4k+\theta-4k\theta-2)^2}$$

Proof 2: The proof procedure is the same as in Proof 1 and is therefore omitted. It should be noted that the condition for the existence of the optimal solution is $v < \frac{2k-2k\theta+\theta-2}{k(2-\theta)}$.

3.3 Government Regulatory Policies for Gray Market Speculators

The profit function for manufacturers and gray market speculators when the government imposes a regulatory policy on gray market speculators is

$$\pi_m^\tau(p_m^\tau, s_m^\tau) = p_m^\tau q_m^\tau - \frac{k(s_m^\tau)^2}{2} \quad (9)$$

$$\pi_a^\tau(p_a^\tau) = p_a^\tau q_a^\tau \quad (10)$$

Theorem 3: The optimal solution for manufacturers and gray market speculators in the case of a government policy of regulation of gray market speculators is

$$p_m^{\tau^*} = \frac{2(\theta-1)(2k+k\tau-2k\theta)}{(\theta-2)(4k+\theta-4k\theta-2)}$$

$$s_m^{\tau^*} = \frac{\tau-2\theta+2}{4k+\theta-4k\theta-2}$$

$$p_a^{\tau^*} = \frac{(\theta-1)m}{(\theta-2)(4k+\theta-4k\theta-2)}$$

Where $m = 2\tau-2\theta-4k\tau+2k\theta-\tau\theta-2k\theta^2+\theta^2+3k\tau\theta$

Substituting $p_m^{\tau^*}$, $s_m^{\tau^*}$ and $p_a^{\tau^*}$ into equations (1) (2) (9) (10), we obtain

$$q_m^{\tau^*} = \frac{k(\tau-2\theta+2)}{4k+\theta-4k\theta-2}$$

$$q_a^{\tau^*} = \frac{m}{\theta(2-\theta)(4k+\theta-4k\theta-2)}$$

$$\pi_m^{\tau^*} = \frac{-k(\tau-2\theta+2)^2}{2(\theta-2)(4k+\theta-4k\theta-2)}$$

$$\pi_a^{\tau^*} = \frac{(1-\theta)m^2}{\theta(\theta-2)^2(4k+\theta-4k\theta-2)^2}$$

Proof 3 The proof procedure is the same as in Proof 1 and is therefore omitted. It should be noted that the condition for the existence of the optimal solution is $\tau < \frac{\theta(2k-2k\theta+\theta-2)}{4k-3k\theta+\theta-2}$.

4 MODEL ANALYSIS

Proposition 1: $\frac{\partial p_m^{v^*}}{\partial v} < 0$, $\frac{\partial s_m^{v^*}}{\partial v} > 0$, $\frac{\partial q_m^{v^*}}{\partial v} > 0$, $\frac{\partial \pi_m^{v^*}}{\partial v} > 0$; $\frac{\partial p_a^{v^*}}{\partial v} < 0$, $\frac{\partial q_a^{v^*}}{\partial v} < 0$, $\frac{\partial \pi_a^{v^*}}{\partial v} < 0$.

Proposition 1 suggests that the government's subsidy policy for manufacturers will result in a decrease in the manufacturer's unit sales price for selling licensed products and an increase in the level of after-sales service, product sales and profits for manufacturers selling licensed products. At the same time, it will lead to a decrease in the unit selling price, sales volume and profits of gray market speculators selling parallel products. The main reason for this is that manufacturers increase the number of licensed products sold by lowering the unit sales price of licensed products in order to receive more government subsidies, at which point the manufacturer's profit increases by selling more at a lower price. As a result of this increase in profits, manufacturers are more motivated to provide better quality after-sales service to their customers. Gray market speculators will reduce the selling price of parallel products in order to maintain their original

competitive advantage, but the sales volume will be partially reduced due to the impact of government subsidies, which ultimately leads to a reduction in profits for gray market speculators.

Proposition 2: $\frac{\partial p_m^{\tau^*}}{\partial \tau}>0$, $\frac{\partial s_m^{\tau^*}}{\partial \tau}>0$, $\frac{\partial q_m^{\tau^*}}{\partial \tau}>0$, $\frac{\partial \pi_m^{\tau^*}}{\partial \tau}>0$, $\frac{\partial s_m^{\tau^*}/\partial \tau}{\partial p_m^{\tau^*}/\partial \tau}<1$; $\frac{\partial p_a^{\tau^*}}{\partial \tau}<0$, $\frac{\partial q_a^{\tau^*}}{\partial \tau}<0$, $\frac{\partial \pi_a^{\tau^*}}{\partial \tau}<0$.

Proposition 2 suggests that the government's implementation of regulatory policies on gray market speculators will increase the unit sales price, after-sales services level, sales volume and profit of licensed products sold by manufacturers, and the increase in service level is smaller than the increase in price. For gray market speculators, government regulation will simultaneously reduce the unit sales price, sales volume and profit of the parallel product. The main reason for this is that when the government imposes regulatory policies on gray market speculators, it reduces the competition between manufacturers and gray market speculators, allowing manufacturers to restore the prices of licensed products in the market and sell them in the market at high prices. Although manufacturers will increase costs by raising after-sales service levels as regulation increases, manufacturers can increase their profits with government protection as the increase in after-sales service levels is less than the increase in price. When the government implements regulatory policies, it causes some consumers who buy parallel products to switch to buying licensed products, and gray market speculators sell fewer parallel products. To prevent a reduction in demand for their products, gray market speculators will continue to reduce the unit selling price of their parallel-imported products, ultimately leading to a reduction in their profits.

Proposition 3: When $v \geq \bar{v}$ ($\bar{v}=\frac{2k-2k\theta+\theta-2}{k(2-\theta)}$), then $q_a^{v^*} \leq 0$. When $\tau \geq \bar{\tau}$ ($\bar{\tau}=\frac{\theta(2k-2k\theta+\theta-2)}{4k-3k\theta+\theta-2}$), then $q_a^{\tau^*} \leq 0$ and $\bar{\tau}-\bar{v}<0$.

Proof 4: By $\frac{\partial q_a^{v^*}}{\partial v}<0$, we know that the quantity of parallel products sold by gray market speculators will fall to a demand of 0 as the amount of government subsidy increases, i.e. $q_a^{v^*}=0$, and we get $\bar{v}=\frac{2k-2k\theta+\theta-2}{k(2-\theta)}$. Similarly, by $\frac{\partial q_a^{\tau^*}}{\partial \tau}<0$, we know that the number of gray market speculators selling parallel products will fall to a demand of 0 as government regulation increases. i.e. $q_a^{\tau^*}=0$, we get $\bar{\tau}=\frac{\theta(2k-2k\theta+\theta-2)}{4k-3k\theta+\theta-2}$. In addition, we can conclude that $\bar{\tau}-\bar{v}=\frac{(2k+\theta-2k\theta-2)(4k+\theta-5k\theta+k\theta^2-2)}{k(\theta-2)(4k+\theta-3k\theta-2)}<0$.

Proposition 3 suggests that whether the government implements a regulatory or a subsidy policy, gray market speculators will choose to exit the market when either the level of regulation or the amount of subsidy reaches a certain level. For the government, it takes less effort to implement a regulatory policy than a subsidy policy to fully combat and regulate the gray market.

Proposition 4:

$$\frac{\tau}{v}<\frac{2k\theta-k\theta^2}{4k-3k\theta+\theta-2}$$

$$\Rightarrow\begin{cases} s_m^{v^*}>s_m^{\tau^*}>s_m^{n^*} \\ p_m^{\tau^*}>p_m^{n^*}>p_m^{v^*}, p_a^{n^*}>p_a^{\tau^*}>p_a^{v^*} \\ q_m^{v^*}>q_m^{\tau^*}>q_m^{n^*}, q_a^{n^*}>q_a^{\tau^*}>q_a^{v^*} \\ \pi_m^{v^*}>\pi_m^{\tau^*}>\pi_m^{n^*}, \pi_a^{n^*}>\pi_a^{\tau^*}>\pi_a^{v^*} \end{cases};$$

$$\frac{2k\theta-k\theta^2}{4k-3k\theta+\theta-2}\leq\frac{\tau}{v}\leq 2-\theta$$

$$\Rightarrow\begin{cases} s_m^{v^*}\geq s_m^{\tau^*}>s_m^{n^*} \\ p_m^{\tau^*}>p_m^{n^*}>p_m^{v^*}, p_a^{n^*}>p_a^{v^*}\geq p_a^{\tau^*} \\ q_m^{v^*}\geq q_m^{\tau^*}>q_m^{n^*}, q_a^{n^*}>q_a^{v^*}\geq q_a^{\tau^*} \\ \pi_m^{v^*}\geq \pi_m^{\tau^*}>\pi_m^{n^*}, \pi_a^{n^*}>\pi_a^{v^*}\geq \pi_a^{\tau^*} \end{cases};$$

$$\frac{\tau}{v}>2-\theta$$

$$\Rightarrow\begin{cases} s_m^{\tau^*}>s_m^{v^*}>s_m^{n^*} \\ p_m^{\tau^*}>p_m^{n^*}>p_m^{v^*}, p_a^{n^*}>p_a^{v^*}>p_a^{\tau^*} \\ q_m^{\tau^*}>q_m^{v^*}>q_m^{n^*}, q_a^{n^*}>q_a^{v^*}>q_a^{\tau^*} \\ \pi_m^{\tau^*}>\pi_m^{v^*}>\pi_m^{n^*}, \pi_a^{n^*}>\pi_a^{v^*}>\pi_a^{\tau^*} \end{cases}.$$

Proposition 4 suggests that, for the manufacturer, we obtain the following conclusion:

(1) From the manufacturer's after-sales service level, when the ratio of government regulation to the amount of government subsidy satisfies $\tau/v<2-\theta$, the manufacturer provides the highest

level of after-sales service when the government implements a subsidy policy, followed by the implementation of a regulatory policy and the lowest when no policy is implemented. This is because government-imposed subsidy policies can cover the after-sales service costs of manufacturers, resulting in the highest level of after-sales service. Compared with the government does not implement the policy, the manufacturer's after-sales services level is higher when the government implements the regulatory policy, in order to attract consumers to change from unauthorized channels to authorized channels, thereby supporting licensed products. Whichever policy the government implements, it will help to improve the level of after-sales service for manufacturers.

(2) In terms of manufacturer prices for the sale of licensed products, manufacturer unit sales prices are highest when the government implements a regulatory policy, followed by no policy and lowest when a subsidy policy is implemented. The main reason is that when the government implements a subsidy policy, manufacturers will increase their sales volume by reducing their prices in order to obtain more government subsidies. When the government imposes a regulatory policy, competition between manufacturers and gray market speculators is reduced. This will eliminate the need for manufacturers to reduce their unit selling prices, which are greater than they would be if the government did not implement the policy.

(3) In terms of sales of licensed products by manufacturers, when the ratio of government regulation to the amount of government subsidy satisfies $\tau / v < 2 - \theta$, the number of sales by manufacturers is highest when the government implements the subsidy policy, followed by the implementation of the regulation policy and lowest when the policy is not implemented. The reason is that when the government implements the subsidy policy, the manufacturer has the lowest unit sales price and the highest level of after-sales service, resulting in the highest number of sales. When the government imposes a regulatory policy, the manufacturer has the highest unit sales price and the second highest level of after-sales service. Although the manufacturer's unit sales price is higher than it would have been in the absence of the government's policy, the government's regulation of gray market speculators has resulted in a shift of consumers to the authorised channel, ultimately resulting in higher sales than would have been the case in the absence of the government's policy. Manufacturer profits are not discussed here as they change in line with changes in their sales volumes.

For gray market speculators, the conclusions we can draw are as follows:

(1) From the perspective of the price of parallel products sold by gray market speculators, when the ratio of government supervision to government subsidy amount meets $\tau / v < \frac{2k\theta - k\theta^2}{4k - 3k\theta + \theta - 2}$, the sales price of gray market speculators is the highest when the government does not implement policies, followed by the implementation of regulatory policies, and the lowest is the implementation of subsidy policies. The main reason for this is that both government regulation and subsidy policies are unfavourable to gray market speculators, who will then lower their unit sales prices in order to maintain their competitive advantage. Compared with government supervision, as the amount of subsidies increases, gray market speculators will sell parallel products at lower prices.

(2) From the perspective of the sales volume of parallel products sold by gray market speculators, when the ratio of government supervision to government subsidies meets $\tau / v < \frac{2k\theta - k\theta^2}{4k - 3k\theta + \theta - 2}$, the sales volume of gray market speculators is the highest when the government does not implement policies, followed by the implementation of regulatory policies, and the least is the implementation of subsidy policies. The main reason is that compared to government regulation, when the subsidy is greater than a certain value, the government subsidy is most beneficial to the manufacturer, causing more consumers to turn to authorized channels, and ultimately leading to the lowest sales volume of gray market speculators.

Since the relationship between the quantity sold and the profit of a gray market speculator satisfies $\pi_a^i = \theta(1-\theta)(q_a^i)^2$, $i \in \{n, v, \tau\}$, the change in the profit of a gray market speculator is consistent with the change in the quantity it sells.

5 NUMERICAL ANALYSIS

The above model mainly analyses the optimal strategies of manufacturers selling licensed products and gray market speculators selling parallel products under different policies implemented by the government. In the following, the relevant parameters of the model are taken to further analyse and verify the relevant findings of this paper. According to Wu (2017), we assume that the parameter $\theta = 0.25$ and take $k = 1.5$ in order to ensure that the hypothesis $k > \frac{2-\theta}{2(1-\theta)}$ holds. At this point, we can get $\frac{2k\theta - k\theta^2}{4k - 3k\theta + \theta - 2} = 0.21$ and $2-\theta = 1.75$. Since the change in sales of manufacturers and gray market speculators coincides with the change in profits, the portrayal and analysis of profits is omitted below.

5.1 The Impact of Government Subsidies and Regulatory Policies on the Unit Sales Prices of Manufacturers and Gray Market Speculators

As seen in Figure 1, as the amount of government subsidies increases, the unit sales price of licensed and parallel products will continue to decrease. As government regulation increases, the unit sales price of licensed products continues to increase and the unit sales price of parallel products continues to decrease. Neither regulation nor subsidy policies imposed by the government are favourable to gray market speculators and will result in gray market speculators reducing their unit sales prices.

For the unit sales price of licensed products, the price is from highest to lowest: government-imposed regulatory policy, government-imposed no policy, government-imposed subsidised policy. For the unit sales price of a parallel product, when $\tau/v > 0.21$, the price is from highest to lowest: government-imposed no policy, government-imposed subsidised policy, government-imposed regulatory policy. When $\tau/v < 0.21$, the price goes from high to low: government-imposed no policy, government-imposed regulatory policy, government-imposed subsidised policy.

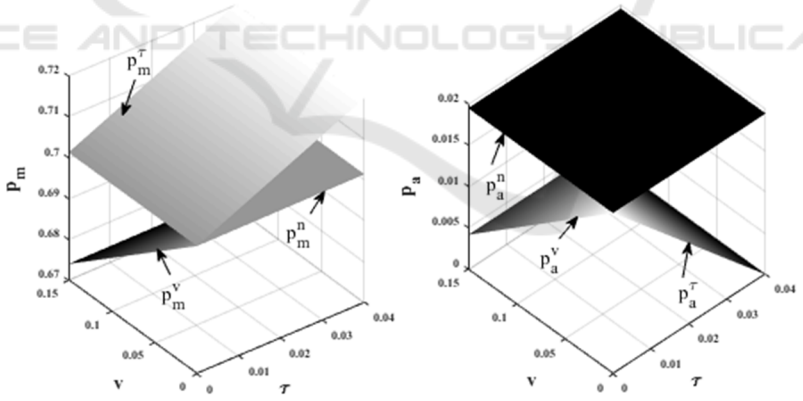

Figure 1: Impact of government subsidies and regulatory policies on unit sales prices.

5.2 Impact of Government Subsidies and Regulatory Policies on Sales Volumes of Manufacturers and Gray Market Speculators

As seen in Figure 2, sales of licensed products have been increasing and sales of parallel products have been decreasing as the amount of government subsidies or regulation has increased.

For the number of licensed products sold by manufacturers, when $\tau/v > 1.75$, the number of sales goes from high to low: government-imposed regulatory policy, government-imposed subsidy policy, and government-imposed no policy; when $\tau/v < 1.75$, the number of sales goes from high to low: government-imposed subsidy policy, government-imposed regulatory policy, and government-imposed no policy. For the number of

gray market speculators selling parallel products, when $\tau/v > 0.21$, the number of sales goes from high to low: no government policy, government policy of subsidies, government policy of regulation; when $\tau/v < 0.21$, the number of sales goes from high to low: no government policy, government policy of regulation, government policy of subsidies.

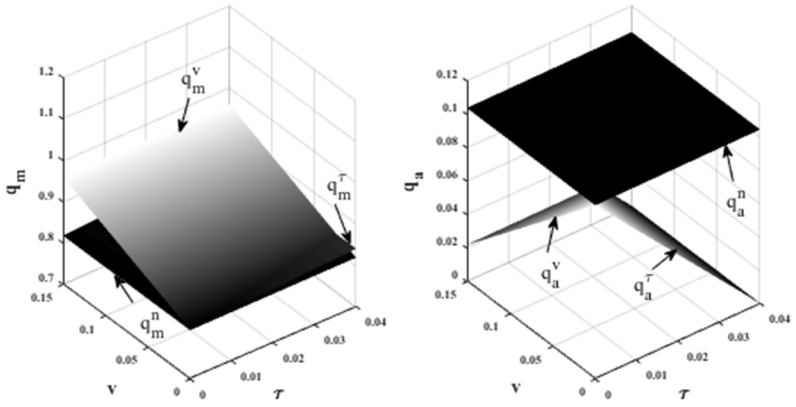

Figure 2: Impact of government subsidies and regulation on sales volumes.

5.3 The Impact of Government Subsidies and Regulatory Policies on Manufacturers' after-Sales Service Levels

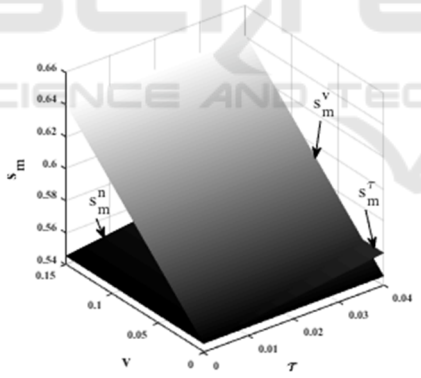

Figure 3: The impact of government subsidies and regulatory policies on manufacturers' after-sales service levels.

As seen in Figure 3, the level of manufacturer after-sales service increases as the amount of subsidy or regulation increases. When $\tau / v > 1.75$, the level of service goes from high to low: government-imposed regulatory policy, government-imposed subsidy policy, and government-imposed no policy; when $\tau / v < 1.75$, the level of service goes from high to low: government-imposed subsidy policy, government-imposed regulatory policy, and government-imposed no policy.

6 CONCLUSION

We consider a market in which manufacturers sell licensed products and gray market speculators sell parallel products. Based on the three scenarios of no government policy, government subsidy policy for manufacturers and government regulatory policy for gray market speculators, a game model with manufacturers as dominant players and gray market speculators as followers is developed to solve and analyse the impact of different government policies on each equilibrium solution for manufacturers and gray market speculators. The study shows the following.

(1) As the amount of government subsidies increases, the unit selling price of manufacturers selling licensed products will decrease and their sales volume and profits will increase. The unit sales price, quantity and profit of parallel products sold by gray market speculators will be reduced. With increased government regulation, the unit sales price, sales volume and profits of manufacturers selling licensed products have increased and the unit sales price, sales volume and profits of gray market speculators selling parallel products have decreased. Both government subsidies and regulatory policies can improve the level of after-sales service for manufacturers.

(2) For manufacturers, the unit sales price of licensed products is highest under government-imposed regulatory policies, followed

by no government-imposed policies, and lowest by government-imposed subsidy policies. When the ratio of unit regulation to unit subsidy is greater than a certain threshold value (i.e. $\tau / v > 2-\theta$), the highest level of sales, profits and after-sales service of licensed products is achieved when the government implements a regulation policy, the second highest level of sales, profits and after-sales service is achieved when the government implements a subsidy policy, and the lowest level of sales, profits and after-sales service is achieved when the government does not implement a policy. Conversely, sales, profits and service levels of licensed products are highest when the government implements a subsidy policy, second highest when the government implements a regulatory policy and lowest when the government does not implement a policy.

(3) For gray market speculators, when the ratio of government unit regulation to unit subsidy is greater than a certain threshold value (i.e. $\tau/v > \frac{2k\theta-k\theta^2}{4k-3k\theta+\theta-2}$), the unit sales price, sales volume and profit of the parallel product is highest when the government does not implement the policy, the unit sales price, sales volume and profit is second highest when the government implements the subsidy policy, and the unit sales price, sales volume and profit is lowest when the government implements the regulation policy. Conversely, the unit sales price, sales volume and profit of a parallel product are highest when the government does not implement a policy, second highest when the government implements a regulatory policy and lowest when the government implements a subsidy policy.

The above research has some implications for government and business control of gray market transactions, but there are still many shortcomings: for example, firstly, this paper only considers the case where the demand function is deterministic, and there are real-life cases where demand is uncertain. Secondly, the paper does not break down gray market speculators, and future research could consider the entry of retailers into the gray market separately.

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