Optimizing the Quality of Electric Lighting with the Use of Minkowski's Geometric Difference

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Abstract: In the paper, using the geometric difference of Minkowski, which are often used in the theory of differential games, the geometric data of the set of a certain lighting instrument are obtained. Found a way to build the set that needs to be installed in the lighting set to provide the lighting level corresponding to the requirement. In this work, conditions are obtained for the sufficiency and necessity of given triangles on the Euclidean plane, i.e. it is shown that if the place of illumination is a triangle of sufficiently large size and the illuminated place of the lighting set is also a triangle, then the place of installation of the set will have a triangular shape. Methods for finding the Minkowski difference of some groups of triangles by vectors corresponding to their sides are also shown and proved. At the end of the article is a theorem on the Minkowski difference of triangles. The theorem on the difference of Minkowski triangles is proved. The results obtained can be applied in the implementation of the installation of lighting devices for residential buildings, offices and enterprises.

1 INTRODUCTION

The effect of light and light pollution on nature, including humans, requires additional research. For example, in part when solving safety problems on highways, it is advisable to solve problems in an integrated manner, while simultaneously increasing the quality of lighting and the characteristics of the road surface. The last factor is essential for compliance with the requirements for standardizing brightness (Bowers, 1998).

Many works have been devoted to optimizing the qualities of electric lighting (Bommel', 2009). But these works do not consider the geometric data of the illuminated areas and the capabilities of the illuminating tool.

In the article, using the geometric difference of Minkowski (Bekker, Brink, 2004) - (Pontryagin, 1981), which are often used, the geometric data of the set of a certain lighting instrument are obtained (Mamatov, 2009) -(Tukhtasinov, 2009).

Definition 1. The sum of the two sets P_1 and P_2 given in the n-dimensional $\mathbb{R}^n$ space is defined as:

$$P \triangleq P_1 + P_2 \triangleq \{x \in \mathbb{R}^n \mid x = x_1 + x_2, x_1 \in P_1, x_2 \in P_2\}. \quad (1)$$

Equation (1) can also be expressed by the operation of union of sets

$$P_1 + P_2 \triangleq \bigcup_{x_1 \in P_1} (x_1 + P_2). \quad (2)$$

Definition 2. The Minkowski difference of two sets is defined as follows:

$$Q \triangleq P_1 \overset{*}{-} P_2 \triangleq \left\{x \in \mathbb{R}^n \mid x + P_2 \subset P_1\right\}; \quad (3)$$

If the set is P_1 the area that is being sanctified, P_2 is the possibility of the illuminating instrument, then Q is the set that must be set for the illuminating instrument. The purpose of the work is using the geometric Minkowski differences, to obtain geometric data for the location of a certain lighting set.

2 METHODS

It is necessary and sufficient for the condition $r_1 \geq r_2$ to exist for the Minkowski difference of closed circles $B_{r_1}(x_1)$, $B_{r_2}(y_1)$ with radius r_1, r_2 in the plane $\mathbb{R}^2$ (Satimov, 1973)

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In the first case, this means that if, for example, the length of the room is longer than the width, then the lighting fixtures must be installed along the segment, in the second case, along the circles, the radius of which is $r_1 - r_2$.

In (Pontryagin, 1981)-(Mamatov, 2009) geometric differences are calculated when X, Y it has a rather complex structure, that is, place of lighting (this X) and illuminated place of the lighting set (this Y) then you can find the place of installation of the set $X \overset{*}{-} Y$. In this work, the results obtained are more effective than previously known works. The proposed methods for calculating the geometric difference are new, they allow solving the problem when Y it has a complex structure, i.e. it can be an arbitrary set.

3 RESULTS AND DISCUSSIONS

Theorem 1. If sets P_1 and P_2 are triangles on the plane $\mathbb{R}^2$, and the radius of the incircle the triangle P_1 is not less than the radius of the circumcircle of the triangle P_2, then the difference $P_1 \overset{*}{-} P_2$ is not empty.

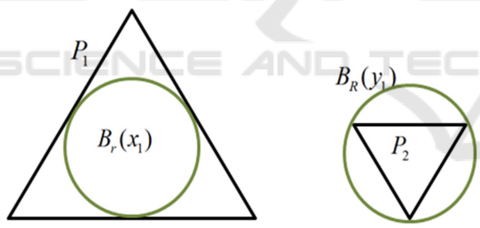

Figure 1.

Theorem 2. If the sets P_1 and P_2 are triangles on the plane, and the relation $P_1 \overset{*}{-} P_2$ is valid, then the radius of the incircle of the triangle P_1 is not smaller than the radius of the incircle of the triangle P_2.

Theorem 3. If the Minkowski difference of any triangle P_2 from any triangle P_1 in the plane $\mathbb{R}^2$ consists of more than one point, then the difference will be a triangle similar to triangle P_1.

Let circle $B_{r_1}(x_1) = \{x : \|x_1 - x\| \le r_1, x_1, x \in P_1,$ $r_1 \in \mathbb{R}\}$ be a incircle of the triangle P_1, and circle $B_{r_2}(y_1) = \{y : \|y_1 - y\| \le r_2,\ y_1, y \in \mathbb{R}^2, r_2 \in \mathbb{R}\}$ be a circumcircle of the triangle P_2. Then it is obvious that (1), (2)

$$B_r(x_1) \subset P_1 \text{ and } B_{r_2}(y_1) \supset P_2 \tag{4}$$

According to the condition of theorem $r_1 \ge r_2$ and by the

$$B_{r_1}(x_1) \overset{*}{-} B_{r_2}(y_1) \ne \varnothing \tag{5}$$

Since (4) and by the property in [6]

$$B_{r_1}(x_1) \overset{*}{-} B_{r_2}(y_1) \subset P_1 \overset{*}{-} P_2 \tag{6}$$

Considering expression (4), (5), it follows that $P_1 \overset{*}{-} P_2 \ne \varnothing$. The theorem has been proved.

Let circle $B_{r_1}(x_1) = \{x : \|x_1 - x\| \le r_1,$ $x_1, x \in P_1, r_1 \in \mathbb{R}\}$ be a incircle of the triangle P_1, and circle $B_{r_2}(y_1) = \{y : \|y_1 - y\| \le r_2,\ y_1, y \in P_2, r_2 \in \mathbb{R}\}$ be a circumcircle of the triangle P_2. Then it is obvious that

$$B_{r_1}(x_1) \subset P_1 \text{ and } B_{r_2}(y_1) \subset P_2 \tag{7}$$

According to the condition of theorem $P_1 \overset{*}{-} P_2 \ne \varnothing$. For any point $a \in P_1 \overset{*}{-} P_2$ we can write $a + P_2 \subset P_1$. Since (7)

$$a + B_{r_2}(y_1) \subset a + P_2 \subset P_1 \quad \text{and} \quad a \in P_1 \overset{*}{-} B_{r_2}(y_1).$$

This means that it is possible to place a circle $B_{r_2}(y_1)$ inside (6) a triangle P_1. Since circle $B_{r_1}(x_1)$ is a incircle of the triangle P_1 (circle with the largest radius lying inside triangle P_1), it follows that $r_1 \ge r_2$. The theorem has been proved.

The opposite of this theorem is not always valid, since $P_1 \overset{*}{-} P_2 \ne \varnothing$ does not mean that the radius of the incircle of the triangle P_1 is greater than or equal to the radius of circumcircle of the triangle B (Fig. 2).

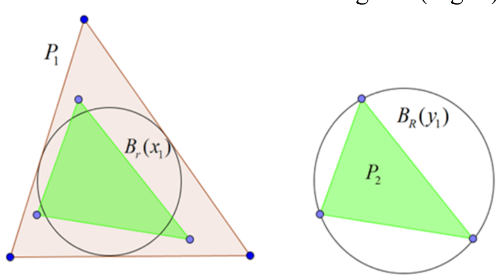

Figure 2.

Therefore, the given by above theorem only sufficient condition. Following theorem is a necessary condition.

1. Finding the Minkowski difference of triangles whose vectors on the corresponding sides are in the same direction.

In this case, the result of the difference is that the vectors on the sides are the same as the direction of the vectors on the corresponding sides of the given triangle, and their lengths are equal to the difference in the lengths of these vectors. Let the conditions $\vec{a}_1 \uparrow\uparrow \vec{b}_1, \vec{a}_2 \uparrow\uparrow \vec{b}_2,\ \vec{a}_3 \uparrow\uparrow \vec{b}_3$ and $|\vec{a}_1| \geq |\vec{b}_1|$, $|\vec{a}_2| \geq |\vec{b}_2|$, $|\vec{a}_3| \geq |\vec{b}_3|$ are satisfied for the vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3$ on the sides of triangle P_2 and the corresponding vectors $\vec{b}_1, \vec{b}_2, \vec{b}_3$ on the sides of triangle P_2 (Fig.3).

Then the difference $P_1 \overset{*}{-} P_2$ is also a triangle, and for the corresponding vectors $\vec{c}_1, \vec{c}_2, \vec{c}_3$ on its sides, the following relation holds:

$$\vec{c}_1 \uparrow\uparrow \vec{a}_1,\ \vec{c}_2 \uparrow\uparrow \vec{a}_2,\ \vec{c}_3 \uparrow\uparrow \vec{a}_3; \quad |\vec{c}_1| = |\vec{a}_1| - |\vec{b}_1|, |\vec{c}_2| = |\vec{a}_2| - |\vec{b}_2|, |\vec{c}_3| = |\vec{a}_3| - |\vec{b}_3|. \tag{8}$$

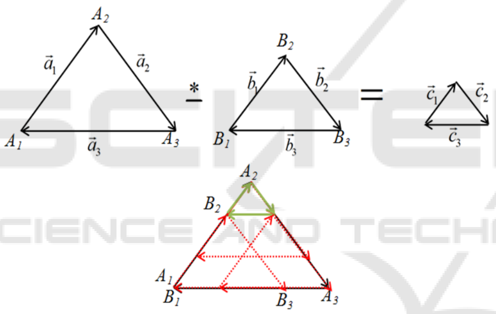

Figure 3.

2. Finding the Minkowski difference of triangles whose vectors on the two corresponding sides are in the same direction.

Let the conditions $\vec{a}_1 \uparrow\uparrow \vec{b}_1, \vec{a}_2 \uparrow\uparrow \vec{b}_2, \vec{a}_3 \not\uparrow\uparrow \vec{b}_3$ and $|\vec{a}_1| \geq |\vec{b}_1|, |\vec{a}_2| \geq |\vec{b}_2|$ are satisfied for the vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3$ on the sides of triangle P_1 and the corresponding vectors $\vec{b}_1, \vec{b}_2, \vec{b}_3$ on the sides of triangle P_2 (Fig.4).

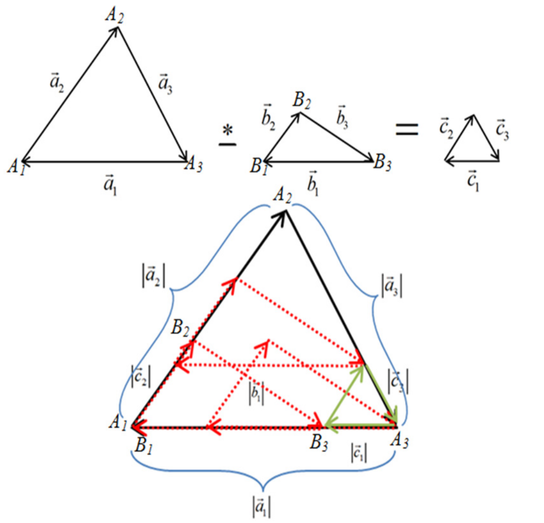

Figure 4.

Then the difference $P_1 \overset{*}{-} P_2$ is also a triangle, and for the corresponding vectors $\vec{c}_1, \vec{c}_2, \vec{c}_3$ on its sides, the following relation holds: $\vec{c}_1 \uparrow\uparrow \vec{a}_1,\ \vec{c}_2 \uparrow\uparrow \vec{a}_2,\ \vec{c}_3 \uparrow\uparrow \vec{a}_3$. If $\frac{|\vec{a}_1|}{|\vec{b}_1|} < \frac{|\vec{a}_2|}{|\vec{b}_2|}$, then the length of these vectors are

$$|\vec{c}_1| = |\vec{a}_1| - |\vec{b}_1|,\ |\vec{c}_2| = \frac{|\vec{a}_1| - |\vec{b}_1|}{|\vec{a}_1|}|\vec{a}_2|,\ |\vec{c}_3| = \frac{|\vec{a}_1| - |\vec{b}_1|}{|\vec{a}_1|}|\vec{a}_3|; \tag{9}$$

if $\frac{|\vec{a}_1|}{|\vec{b}_1|} > \frac{|\vec{a}_2|}{|\vec{b}_2|}$, then

$$|\vec{c}_1| = \frac{|\vec{a}_2| - |\vec{b}_2|}{|\vec{a}_2|}|\vec{a}_1|, |\vec{c}_2| = |\vec{a}_2| - |\vec{b}_2|, |\vec{c}_3| = \frac{|\vec{a}_2| - |\vec{b}_2|}{|\vec{a}_2|}|\vec{a}_3|. \tag{10}$$

3. Finding the Minkowski difference of triangles whose vectors on the only one corresponding sides are in the same direction.

Let the conditions $\vec{a}_1 \uparrow\uparrow \vec{b}_1, \vec{a}_2 \not\uparrow\uparrow \vec{b}_2, \vec{a}_3 \not\uparrow\uparrow \vec{b}_3$ and $|\vec{a}_1| \geq |\vec{b}_1|, |\vec{a}_2| \geq |\vec{b}_2|, |\vec{a}_3| \geq |\vec{b}_3|$ are satisfied for the vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3$ on the sides of triangle P_1 and the corresponding vectors $\vec{b}_1, \vec{b}_2, \vec{b}_3$ on the sides of triangle P_2. In this case the difference $P_1 \overset{*}{-} P_2$ may be empty (9). For example, Minkowski difference of triangles in fig.5 is not existence, because it is not possible to place a triangle $B_1B_2B_3$ inside a triangle $A_1A_2A_3$.

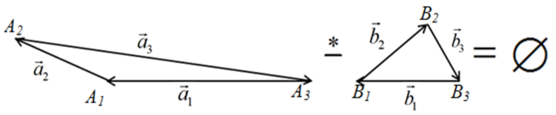

Figure 5.

For this reason, we also include the condition given in theorem 1. However, there are three possible cases of triangles in this group.

In first case, the angles adjacent to the $\vec{b}_1$ side of the triangle P_2 greater than the angles adjacent to the $\vec{a}_1$ side of the triangle P_1 as shown in fig. 6 that is, $\angle A_1 < \angle B_1$ and $\angle A_3 < \angle B_3$ are appropriate.

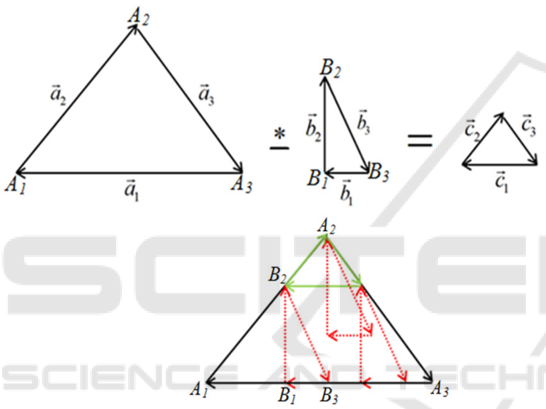

Figure 6.

Then the difference $P_1 \stackrel{*}{-} P_2$ is also a triangle, and for the corresponding vectors $\vec{c}_1, \vec{c}_2, \vec{c}_3$ on its sides, the relation $\vec{c}_1 \uparrow\uparrow \vec{a}_1$, $\vec{c}_2 \uparrow\uparrow \vec{a}_2$, $\vec{c}_3 \uparrow\uparrow \vec{a}_3$ is valid, that is, it is a triangle similar to triangle P_1, as in the second group. The lengths of the vectors $\vec{c}_1, \vec{c}_2, \vec{c}_3$ are as follows:

$$|\vec{c}_1| = \frac{h_{|\vec{a}_1|} - h_{|\vec{b}_1|}}{h_{|\vec{a}_1|}} |\vec{a}_1|, \ |\vec{c}_2| = \frac{h_{|\vec{a}_1|} - h_{|\vec{b}_1|}}{h_{|\vec{a}_1|}} |\vec{a}_2|, \qquad (11)$$

$$|\vec{c}_3| = \frac{h_{|\vec{a}_1|} - h_{|\vec{b}_1|}}{h_{|\vec{a}_1|}} |\vec{a}_3|.$$

Where $h_{|\vec{a}_1|}$ and $h_{|\vec{b}_1|}$ are the heights of side $|\vec{a}_1|$ of triangle P_1 and the heights of side $|\vec{b}_1|$ of triangle P_2, respectively. And they can be calculated as follows (10):

$$h_{|\vec{a}_1|} = \frac{2}{|\vec{a}_1|} \sqrt{p_a (p_a - |\vec{a}_1|)(p_a - |\vec{a}_2|)(p_a - |\vec{a}_3|)},$$

$$p_a = \frac{|\vec{a}_1| + |\vec{a}_2| + |\vec{a}_3|}{2};$$

$$h_{|\vec{b}_1|} = \frac{2}{|\vec{b}_1|} \sqrt{p_b (p_b - |\vec{b}_1|)(p_b - |\vec{b}_2|)(p_b - |\vec{b}_3|)},$$

$$p_b = \frac{|\vec{b}_1| + |\vec{b}_2| + |\vec{b}_3|}{2}.$$

In second case, the relation $\angle A_1 < \angle B_1$, $\angle A_3 > \angle B_3$ is satisfied for the angles of triangles P_1 and P_2, which are parallel to each other (Fig. 7).

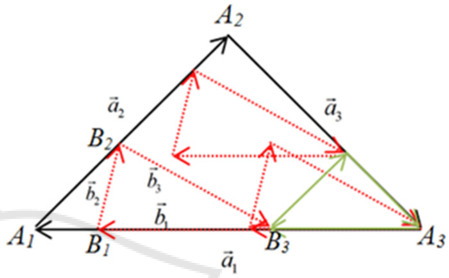

Figure 7.

The difference (3) $P_1 \stackrel{*}{-} P_2$ is a triangle similar to triangle P_1, and the lengths of the vectors on its sides are (11):

$$|\vec{c}_1| = \frac{h_{|\vec{a}_2|} - |\vec{b}_3 - proj_{\vec{a}_2} \vec{b}_3|}{h_{|\vec{a}_2|}} |\vec{a}_1|,$$

$$|\vec{c}_2| = \frac{h_{|\vec{a}_2|} - |\vec{b}_3 - proj_{\vec{a}_2} \vec{b}_3|}{h_{|\vec{a}_2|}} |\vec{a}_2|, \qquad (12)$$

$$|\vec{c}_3| = \frac{h_{|\vec{a}_2|} - |\vec{b}_3 - proj_{\vec{a}_2} \vec{b}_3|}{h_{|\vec{a}_2|}} |\vec{a}_3|.$$

Here $h_{|\vec{a}_2|}$ is the height of the triangle P_1 on the side $|\vec{a}_1|$, and $proj_{\vec{a}_2} \vec{b}_3$ is the orthogonal projection of the vector $\vec{b}_3$ on the vector $\vec{a}_2$, which are calculated as follows:

$$h_{|\vec{a}_2|} = \frac{2}{|\vec{a}_2|} \sqrt{p_a (p_a - |\vec{a}_1|)(p_a - |\vec{a}_2|)(p_a - |\vec{a}_3|)},$$

$$p_a = \frac{|\vec{a}_1| + |\vec{a}_2| + |\vec{a}_3|}{2};$$

$$\left|\vec{b}_3 - proj_{\vec{a}_2}\vec{b}_3\right| = \frac{\sqrt{|\vec{a}_2|^2\left|\vec{b}_3\right|^2 - (\vec{a}_2,\vec{b}_3)^2}}{|\vec{a}_2|},$$

In third case, the relation $\angle A_1 > \angle B_1, \angle A_3 > \angle B_3$ is satisfied for the angles of triangles P_1 and P_2, which are parallel to each other (Fig. 8).

The difference $P_1 \overset{*}{-} P_2$ is a triangle similar to triangle P_1, and the lengths of the vectors on its sides are(12):

$$|\vec{c}_1| = \frac{|\vec{a}_2| - \left|\vec{b}_2\right|}{|\vec{a}_2|}|\vec{a}_1|,\ |\vec{c}_2| = |\vec{a}_2| - \left|\vec{b}_2\right|, |\vec{c}_3| = \frac{|\vec{a}_2| - \left|\vec{b}_2\right|}{|\vec{a}_2|}|\vec{a}_3|.$$

4. Finding the difference Minkowski of triangles whose vectors on the corresponding side are not in the same direction.

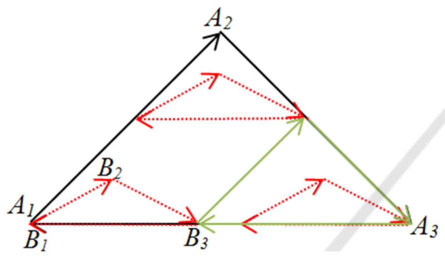

Figure 8.

Let the conditions $\vec{a}_1 \not\upuparrows \vec{b}_1, \vec{a}_2 \not\upuparrows \vec{b}_2, \vec{a}_3 \not\upuparrows \vec{b}_3$ and $|\vec{a}_1| \geq \left|\vec{b}_1\right|, |\vec{a}_2| \geq \left|\vec{b}_2\right|, |\vec{a}_3| \geq \left|\vec{b}_3\right|$ are satisfied for the vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3$ on the sides of triangle P_1 and the corresponding vectors $\vec{b}_1, \vec{b}_2, \vec{b}_3$ on the sides of triangle P_2. In this case, as in the third group, the difference $P_1 \overset{*}{-} P_2$ can be an empty set, so in this case, we assume that the condition of theorem 1 holds. It is not difficult to see that this difference also results in a triangle like P_1 (Fig. 9).

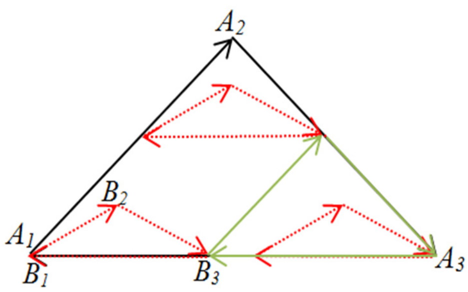

Figure 9.

The following equations are valid for the vectors $\vec{c}_1, \vec{c}_2, \vec{c}_3$ on the sides of this triangle:

$$|\vec{c}_1| = \left(1 - \sqrt{\frac{|\vec{a}_2|^2\left|\vec{b}_1\right|^2 - (\vec{a}_2,\vec{b}_1)^2}{|\vec{a}_1|^2|\vec{a}_2|^2 - (\vec{a}_1,\vec{a}_2)^2}} - \sqrt{\frac{|\vec{a}_3|^2\left|\vec{b}_3\right|^2 - (\vec{a}_3,\vec{b}_3)^2}{|\vec{a}_1|^2|\vec{a}_3|^2 - (\vec{a}_1,\vec{a}_3)^2}}\right)|\vec{a}_1|,$$

$$|\vec{c}_2| = \left(1 - \sqrt{\frac{|\vec{a}_2|^2\left|\vec{b}_1\right|^2 - (\vec{a}_2,\vec{b}_1)^2}{|\vec{a}_1|^2|\vec{a}_2|^2 - (\vec{a}_1,\vec{a}_2)^2}} - \sqrt{\frac{|\vec{a}_3|^2\left|\vec{b}_3\right|^2 - (\vec{a}_3,\vec{b}_3)^2}{|\vec{a}_1|^2|\vec{a}_3|^2 - (\vec{a}_1,\vec{a}_3)^2}}\right)|\vec{a}_2|,$$

$$|\vec{c}_3| = \left(1 - \sqrt{\frac{|\vec{a}_2|^2\left|\vec{b}_1\right|^2 - (\vec{a}_2,\vec{b}_1)^2}{|\vec{a}_1|^2|\vec{a}_2|^2 - (\vec{a}_1,\vec{a}_2)^2}} - \sqrt{\frac{|\vec{a}_3|^2\left|\vec{b}_3\right|^2 - (\vec{a}_3,\vec{b}_3)^2}{|\vec{a}_1|^2|\vec{a}_3|^2 - (\vec{a}_1,\vec{a}_3)^2}}\right)|\vec{a}_3|.$$

Thus, P_2 the illuminated place of the lighting set can be of arbitrary shape. We can find the installation location of the kit $P_1 \overset{*}{-} P_2$.

4 CONCLUSIONS

Thus, we have established that if the set of the P_1-triangle is the area that is being sanctified, the P_2-capabilities of the illuminating instrument are also a triangle and $Q \neq \emptyset$, then Qwill be a point or triangle similar to the P_1 set that must be installed by the illuminating instrument. In this work, we present our results for calculating the Minkowski difference of triangle-shaped sets. Despite the relatively simple nature of the problem, its complete solution and analysis of this solution allows us to solve more general problems on this topic. In fact, when working with triangles, it is clear that the shape of the triangle is not important, but its size. The results obtained can be applied in the implementation of the installation of lighting devices for residential buildings, offices and enterprises.

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