A Comparative Study of Traditional Linear Models and Nonlinear Neural Network Model on Asset Pricing

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Abstract: Models in traditional asset pricing theories, such as CAPM and the Fama-French three-factor model, explain the linear relationship between market returns, company size, company type, and return on assets. But in a more complex financial market, the linear relationship contained in the above model may not hold. Therefore, the main focus of this paper is to analyze the nonlinearity between stock excess return and its influencing factors. The existence of nonlinearity is confirmed via the RESET test proposed by Ramsey. Then, the nonlinear neural network model is used to further study the nonlinear relationship. Based on the data of the A-share market, it is verified that there is a nonlinear relationship between stock excess returns and their influencing factors, and the nonlinear neural network model shows better prediction performance than traditional linear models.

1 INTRODUCTION

Modern asset pricing theory mainly focuses on the difference between expected returns of different assets and the dynamics of the market risk premium. Among the large number of theoretical models in this field, the capital asset pricing model (CAPM) undoubtedly occupies an important position. It is the cornerstone of modern financial economics and the pillar of financial market price theory. The model was developed from the theory of modern portfolio selection (Sharpe, 1964; Lintner, 1969; Fischer, 1972).

With the continuous development in the research fields of asset pricing theory, academic circles gradually discovered that, in addition to a single risk factor, the return on assets is also affected by the company's market value and book-to-market ratio. Combining these new findings, Fama and French (Eugene, 1996) proposed a three-factor model that combines the risk factor, size factor, and value factor as an improvement of CAPM.

However, most of the traditional asset pricing models, such as the CAPM model and the Fama-French three-factor model, adopt a linear form and usually have a problem with poor prediction of stock returns. Therefore, the academic community has gradually begun to explore the nonlinear relationship in asset pricing models. According to empirical research, there are complex internal structures in asset price time series such as non-normal distribution with fat tails, volatility clustering phenomenon, and seasonal effects (Edgar, 1996; Xu, 2001; Michael, 1976). Faced with these nonlinear characteristics, it is natural that reducing strict assumptions in traditional models and building nonlinear models becomes a new research direction in the field of asset pricing (Xing, 2019; James, 2002).

This paper conducts an empirical analysis of the traditional CAPM model, the Fama-French three-factor model, and the neural network model based on the A-share market data. Since it is confirmed that the Fama-French three-factor model is more suitable for the Chinese market than the Fama-French five-factor model (Zhao, 2016), the Fama-French five-factor model is not selected in this paper.

The Ramsey RESET method (James, 1969; Ruey, 2005) is used in the process to test whether there is a nonlinear relationship between stock excess returns and relevant factors under the single-factor assumption and three-factor assumption, respectively. Subsequently, the predicted stock return of each model is compared via out-of-sample R2 and mean absolute error (MAE).

In this paper, the nonlinear neural network model is introduced to price assets in order to analyze the

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nonlinear relationship between traditional pricing factors and stock returns in the Chinese market. As a result, the nonlinear problem existing in the traditional linear asset pricing model is verified, and the effectiveness of the neural network model applied to the field of asset pricing is proven. The conclusions obtained in this paper help to provide some guidance for the improvement of the asset pricing model for the Chinese market in the future.

2 MODELS

This section will introduce the pricing model used in this paper. Pricing models are divided into two categories: linear pricing models and nonlinear pricing models.

2.1 Traditional Linear Models

Traditional asset pricing models generally take a linear form. In this paper, the most classic CAPM and the Fama-French three-factor models are selected for empirical research.

CAPM. The single-factor CAPM formula (including the market risk premium factor) is shown below:

$$R_{it} - R_{ft} = \alpha_i + \beta_i(R_{mt} - R_{ft}) + \varepsilon_{it}$$

Among the equation above, R_{it} represents the rate of return of stock i at time t; R_{ft} is the risk-free interest rate at time t; α_i and β_i are parameters to be estimated; R_{mt} is the rate of return of the market index at time t; ε_{it} is the regression residual.

Fama-French three-factor model. Compared with CAPM, the Fama-French three-factor model includes two additional factors: stock market value and book-to-market value. The formula of the model (including market risk premium factor, size premium factor and value premium factor) is shown below:

$$R_{it} - R_{ft} = \alpha_i + {\beta_i}^M(R_{mt} - R_{ft}) + {\beta_i}^{SMB}SMB_t + {\beta_i}^{HML}HML_t + \varepsilon_{it}$$

In the equation above, R_{it} is the rate of return of stock i at time t; R_{ft} is the risk-free interest rate at time t; α_i, ${\beta_i}^M$, ${\beta_i}^{SMB}$ and ${\beta_i}^{HML}$ are parameters to be estimated; R_{mt} is the rate of return of the market index at time t; SMB_t represents the size premium at time t, which is the difference between the return of a portfolio of stocks with small market value and a portfolio of stocks with large market value; HML_t represents the value premium at time t, which is the difference between the return of a portfolio of value stocks and a portfolio of growth stocks at time t; ε_{it} represents regression residuals at time t.

2.2 Nonlinear Neural Network Model

In order to further explore the nonlinearity between stock returns and their influencing factors, the nonlinear neural network model is introduced in this paper.

Principle of the model. A neural network is one of the most powerful nonlinear models in all kinds of machine learning methods. The principle of the model is to imitate the structure and function of a biological neural network. As for the traditional feedforward neural network, it mainly consists of an input layer for raw data input, several hidden layers for processing the input data, and an output layer that outputs the final prediction results. Similar to the axons of a biological brain, different layers in a neural network model represent a group of neurons, and each layer is connected by "synapses" that transmit signals between neurons in different layers.

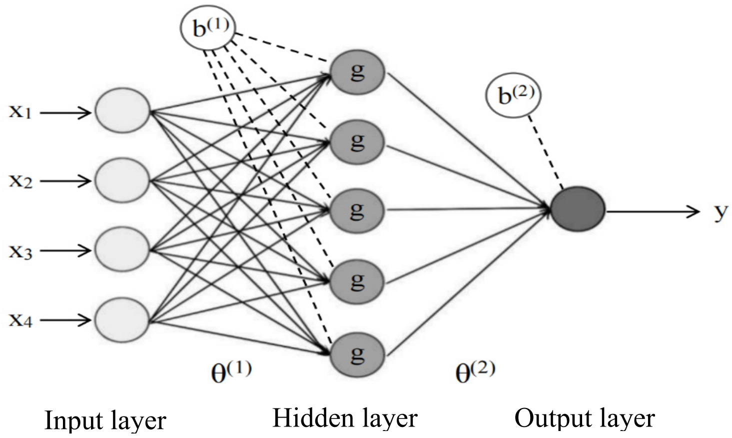

Figure 1: A single hidden layer neural network structure.

Figure 1 shows a single-hidden-layer neural network structure. In this neural network structure, x_1, x_2, x_3 and x_4 are the input data; $\theta^{(1)}$ and $\theta^{(2)}$ are the weight matrices mapped from the first layer to the second layer and from the second layer to the third layer, respectively; $b^{(1)}$ and $b^{(2)}$ are bias between the first layer and the second layer, and between the second layer and the third layer, respectively; g is the nonlinear activation function; y is the final output value. The overall structure of the model can be expressed as equations below:

$$x_i^{(1)} = g(\theta_{i,0}^{(1)} b^{(1)} + \sum_{j=1}^{4} \theta_{i,j}^{(1)} x_j)$$

$$y = \theta_0^{(2)} b^{(2)} + \sum_{i=1}^{5} \theta_i^{(2)} x_i^{(1)}$$

The form of the model. First, relevant data used in asset pricing is expressed in vector form:

$$X_t = [x_1 \quad \cdots \quad x_m]$$

$$Y_t = [y_1 \dots y_n]$$

In the equations above, X_t is a dataset containing m factors that impacts stock prices at time t; x_i represents the ith factor at time t; Y_t is the stock excess return of n stocks at time t; y_i represents the excess return of stock i at time t.

Introducing the neural network into asset pricing, a nonlinear model can be obtained as follows:

$$Y_t = f(X_t; \theta)$$

In this model, f is the nonlinear function that maps the stock factor dataset X_t to the excess return Y_t, and θ is the parameter set to be estimated.

3 EMPIRICAL ANALYSIS

The monthly data of 163 stocks selected from the A-share market from January 2000 to May 2022 is used in the paper to carry out the empirical experiment. All stock data is obtained from the CSMAR database. Stocks carrying “ST” (special treatment) or “*ST” tags (which have suffered losses for two consecutive years or more) are excluded. The market index used here is the CSI 300 index, which includes the 300 A-share stocks traded on the Shanghai and Shenzhen stock exchanges, and the risk-free interest rate is the one-year short-term treasury bond rate. The stock data from January 2000 to December 2020 is used as the training set of the pricing model, and the stock data from January 2021 to May 2022 is used as the test set.

3.1 Performance Evaluation

In order to compare the predictive ability of different models, this paper selects two quantitative indicators: R^2 and mean absolute error (MAE).

Out-of-sample R^2. The predictive ability of each model is evaluated by the out-of-sample R^2:

$$R_{OOS}^2 = 1 - \frac{\sum_{(i,t)} \left(r_{i,t+1} - \hat{r}_{i,t+1}\right)^2}{\sum_{(i,t)} r_{i,t+1}^2}$$

In the formula above, $r_{i,t+1}$ represents the actual excess rate of return of stock i at time t+1, and $\hat{r}_{i,t+1}$ represents the excess rate of return of stock i at time t+1 predicted by pricing models. Considering that there is too much noise in the historical average excess return of a single stock, it is better to directly use 0 as the benchmark (Gu, 2020). Therefore, the denominator of the out-of-sample R^2 defined here is without demeaning, which means the historical mean is replaced by 0.

Mean absolute error. In addition to R^2, the predictive ability of each model is also evaluated by mean absolute error (MAE):

$$MAE = \frac{1}{n} \sum_{i=1}^{n} \left| r_{i,t+1} - \hat{r}_{i,t+1} \right|$$

In the formula above, $r_{i,t+1}$ represents the actual excess rate of return of stock i at time t+1, and $\hat{r}_{i,t+1}$ represents the excess rate of return of stock i at time t+1 predicted by pricing models.

3.2 Nonlinearity Test

In this paper, regression specification error test (RESET) is selected to detect the possible nonlinear relationship in the model.

Testing method. The RESET test is a commonly used test method in econometrics. It is a specification test for linear least-squares regression analysis proposed by Ramsey (1969). The basic idea of the RESET test is that if there is no nonlinearity, the coefficient of the multinomial term of the regression model should be 0. In other words, the null hypothesis of the RESET test is that the coefficient of the higher-order term is equal to 0. This can be tested by the F test.

Test result. The monthly excess returns of the 163 stocks selected in this paper are tested by RESET under the assumption of a single factor (i.e. market risk premium factor) and three factors (i.e. market risk premium factor, size premium factor, and value premium factor) respectively. The P values obtained by the F test are shown in Table 1 and Figure 2.

Table 1: Partial results of P value obtained by F test.

Stock code	P value for single-factor	P value for three-factor	Stock code	P value for single-factor	P value for three-factor
000012	0.0406	0.0170	000551	0.0118	0.0441
000021	0.0418	0.0368	000559	0.0664	0.0201
000026	0.0350	0.0681	000570	0.0062	0.0249
000039	0.0370	0.0292	000573	0.0434	0.0366
000055	0.0359	0.0493	000581	0.0140	0.0173
000060	0.0259	0.0110	000589	0.0350	0.0014
000078	0.0289	0.0466	000597	0.0072	0.0681
000089	0.0372	0.0135	000598	0.0197	0.0508
000402	0.0178	0.0333	000599	0.0377	0.0475
000404	0.0312	0.0242	000632	0.0639	0.1399
000417	0.0481	0.0342	000637	0.0129	0.0246
000419	0.0491	0.0161	000661	0.0159	0.0231
000422	0.0185	0.0023	000667	0.0090	0.0220
000425	0.0241	0.0148	000680	0.0378	0.0485
000507	0.0244	0.0242	000685	0.0274	0.0278
000521	0.0484	0.1164	000701	0.0083	0.0125
000528	0.0112	0.0483	000702	0.0080	0.0118
000543	0.0043	0.0171	000729	0.0178	0.0352
000548	0.0142	0.0034	000733	0.0242	0.0296

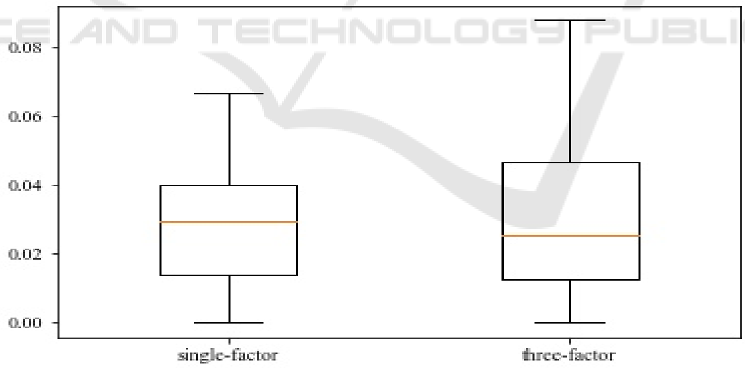

Figure 2: p value of the RESET test.

As shown in Table 1 and Figure 2, among the 163 selected stocks, P values of most stocks' excess returns are less than 0.05. 92.64% of the stocks have a P value less than 0.05 under the single factor assumption, and 84.66% of the stocks have a P value less than 0.05 under the three-factor hypothesis. Hence, there are sufficient reasons to reject the null hypothesis, that is, there are nonlinear terms in the model under the single-factor and three-factor assumptions. Consequently, it is reasonable to use nonlinear neural network models to predict stock excess returns.

3.3 Model Training

The stock data from January 2000 to December 2020 is used as the training set to estimate the parameters of three different models, and the stock data from January 2021 to December 2022 is used as the test set to compare the performance of models via out-of-sample R^2_{OOS} and MAE.

Linear Regression for Traditional Models. The linear regression is performed on the training set data to calculate the parameters of the CAPM and Fama-French three-factor models. Part of results of each model are shown in table 2 and table 3 respectively.

As can be seen from table 2 and table 3, the regression coefficients of the market risk premium factor are generally around 1, while the coefficients of the size premium factor and the value premium factor are basically distributed between 1 and -1.

Table 2: Partial CAPM Model Regression Coefficients.

Stock code	β	Stock code	β
000012	1.3623	000551	1.0514
000021	1.0567	000559	1.1099
000026	1.0829	000570	1.1503
000039	1.0150	000573	1.1043
000055	1.1454	000581	0.9272
000060	1.4895	000589	1.0393
000078	1.2271	000597	1.0332
000089	0.9121	000598	1.0690
000402	0.9913	000599	1.1078
000404	1.0987	000632	0.9774
000417	0.9409	000637	0.9588
000419	1.0821	000661	1.0209
000422	1.1367	000667	1.1438
000425	1.0501	000680	1.3344
000507	1.1369	000685	1.3469
000521	1.1357	000701	1.1495
000528	1.2415	000702	0.9878
000543	1.2621	000729	0.6887
000548	1.2354	000733	1.1190

Table 3: Partial FF three-factor model regression coefficients.

Stock code	βM	βSMB	βHML	Stock code	βM	βSMB	βHML
000012	1.2984	0.8745	0.2684	000551	0.9667	1.0525	0.1440
000021	0.9835	0.6161	-0.4609	000559	1.0265	1.0905	0.2469
000026	0.9729	1.0106	-0.5234	000570	1.0791	0.8716	0.0944
000039	1.0365	-0.1216	0.2529	000573	1.0465	0.9281	0.5163
000055	1.0565	1.0752	0.0905	000581	0.8934	0.3855	-0.0135
000060	1.4956	0.0598	0.2589	000589	0.9865	0.8760	0.5254
000078	1.1373	0.9139	-0.2511	000597	0.9531	1.1435	0.4302
000089	0.8898	0.2253	-0.0658	000598	1.0308	0.6422	0.3988
000402	1.0414	-0.4816	0.1966	000599	1.0207	1.1217	0.2238
000404	1.0260	1.0489	0.4129	000632	0.8818	1.0587	-0.0961
000417	0.8932	0.7500	0.3928	000637	0.8985	0.8616	0.3242
000419	1.0099	0.7348	-0.1998	000661	0.9102	1.1682	-0.2256
000422	1.1040	0.6689	0.5791	000667	1.1074	0.7903	0.7325

000425	1.0329	0.4887	0.5753	000680	1.3322	0.1623	0.2709
000507	1.0981	0.5070	0.1139	000685	1.3244	0.3403	0.1592
000521	1.0679	0.9031	0.2354	000701	1.0545	1.0158	-0.1671
000528	1.2709	-0.2598	0.1607	000702	0.9178	1.0581	0.4926
000543	1.2454	0.4936	0.5963	000729	0.6727	0.1638	-0.0433
000548	1.1806	0.7695	0.2683	000733	1.0211	0.6783	-0.9043

Linear Regression for Traditional Models. Considering the limited amount of data, the neural network models used here have only up to 4 hidden layers. Four neural network structures (NN1, NN2, NN3 and NN4) are selected respectively according to the geometric pyramid rule [13]. NN1 has a single hidden layer with 32 neurons; NN2 has two hidden layers with 32 and 16 neurons, respectively; NN3 has three hidden layers with 32, 16, and 8 neurons, respectively; NN4 has four hidden layers with 32, 16, 8, and 4 neurons respectively. Under the single-factor assumption, the input layer of each neural network model receives market risk premium data; under the three-factor assumption, the input layer receives market risk premium, size premium and value premium data.

3.4 Model Comparison

The R^2_{OOS} and MAE of CAPM and the neural network model under the single-factor assumption is shown in Table 3. It can be seen that the performance of neural network models with all four different structures is better than CAPM. The R^2_{OOS} and MAE of the Fama-French three-factor model and the neural network under the three-factor assumption are shown in Table 4. The neural network model also shows better performance than the linear three-factor model does.

Table 4: R^2_{OOS} and MAE under the single-factor assumption.

Model	CAPM	NN1	NN2	NN3	NN4
R^2_{OOS}	0.1271	0.1307	0.1308	0.1300	0.1300
MAE	0.0799	0.0795	0.0795	0.0796	0.0795

Table 5: R^2_{OOS} and MAE under the three-factor assumption.

Model	FF	NN1	NN2	NN3	NN4
R^2_{OOS}	0.2261	0.2287	0.2274	0.2278	0.2302
MAE	0.0741	0.0740	0.0737	0.0737	0.0736

4 CONCLUSION

Through empirical method, according to A-share market data, the Ramsey RESET method is used in this paper first to confirm that there is a nonlinear relationship between the stock excess return and its influencing factors. Then, from the results of both R^2 and MAE, we can see that all four nonlinear neural network models with different structures show better performance than traditional linear asset pricing models. Hence, it is further verified that the non-linear form may be a better option when it comes to predicting asset returns.

However, the improvement of the neural network model over traditional linear models is not very significant. In addition, it is difficult to give an intuitive explanation of the specific nonlinear relationship between expected returns and their influencing factors via a neural network model because it is like a "black box". Consequently, other nonlinear models are needed to further explore the nonlinear relationship between stock excess return and relevant factors.

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