Performance of Delta-Neutral Hedging Strategy on Moderna Inc Stock

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Abstract: This paper investigates the effectiveness of delta-neutral hedging strategy. The goal of this paper is to hedge an option contract on Moderna Inc stock. The result of this paper is useful for investors, especially beginners, to use as a reference when building a portfolio. This study is divided into two parts. The first one is to calibrate volatility of stock using three different models: the Black-Scholes model, binomial tree model, and historical return model. With implied volatility in hand, a delta-neutral portfolio is built to hedge a put option on Moderna Inc stock. The performance of the hedging strategy can be observed by comparing portfolio return with the return of the option contract alone. The result of this study indicates that delta-neutral hedging strategy does reduce loss in investment. Such result is beneficial for individual investors in formulating a simple portfolio.

1 INTRODUCTION

Option pricing calculates implied value of option contract with the aid of mathematical models. Two commonly used derivative pricing models are the binomial tree model and the Black-Scholes model. The Black-Scholes model is a well-known derivative pricing strategy. The significance of the Black-Scholes model is it lays a foundation for a new field of finance called the continent-claims analysis (Gilster, 1997), which is useful in pricing complex financial securities. The binomial tree model values options at a discrete set of nodes. Binomial tree model has more applications than the Black-Scholes model because it works for both American options, European options, and options with dividend-paying underlying stock.

Hedging strategy is a risk management strategy, and it generates value for investors by reducing loss of portfolio. Investments like options, futures, and other derivatives are most used by investors when formulating a hedging strategy. Delta hedging is a commonly used strategy, where delta measures the fluctuation in portfolio value with respect to the change in the underlying asset price (Ajay, 1997). The goal of delta-neutral hedging strategy is that value of portfolio does not vary much as stock price changes. Such a goal can be achieved by building a portfolio that has zero value for delta (Capinski, 2003). One problem with delta-neutral hedging strategy is that it requires constant rebalance to ensure delta is equal to zero (Robins, 1994). However, in the real world, market is not frictionless. Rebalance results in transaction cost, which is not taken into consideration by delta-neutral strategy. Even though delta hedging might not be an optimal strategy, it's still commonly used due to its simplicity.

Within the field of financial engineering, much research has been done on different hedging strategy and option pricing strategy. For example, Hauser and Eales analyzed option hedging strategies (Hauser, 1987); Schweizer researched on mean-variance hedging (Schweizer, 1992); Wang, Wu, and Yang studied hedging with futures (Wang, 2015); Schied and Staje wrote about the robustness of delta hedging (Schweizer, 1992). Moreover, for option pricing, Merton analyzed the theory of rational option pricing (Merton, 1973); Kremer and Roenfeldt compared jump-diffusion pricing model with the Black-Scholes model (Kremer, 1993); Schaefer investigated the development of derivative pricing method (Schaefer, 1998) etc. As the topics are of interests in the financial field, this paper also focuses on the issue.

This paper combines option pricing and risk hedging and specifically looks into the implied volatility by three different methods on the same stock

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during the same time period, and compares return with and without hedging.

The paper is divided into four sections: section 2 organizes data and explains method used; section 3 displays results and discuss; section 4 concludes the discussion.

2 DATA

In this paper, Moderna stock is chosen because Moderna is a dominant player in the field of mRNA vaccine (Dolgin, 2021). During the Covid-19 pandemic, it was the second pharmaceutical company to develop a mRNA vaccine for Covid-19. Moderna's vaccine reduces the chance of getting infected by Covid-19, and many countries around the globe have adopted Moderna's mRNA vaccine during the pandemic. For example, British had ordered around 17 million doses of Moderna before January 2021 (BBC, 2021).

Stock open price and option contract price is collected from Yahoo Finance (www.yahoo.com/finance). Open price of Moderna Inc stock from July 25th, 2022, to Aug 5th, 2022, is recorded and shown in the graph below.

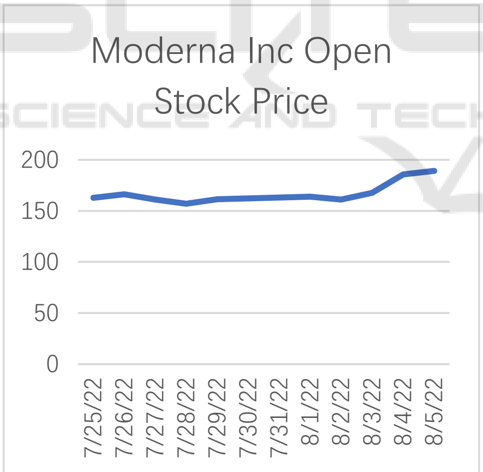

Figure 1. The stock price trend

In order to calibrate volatility, price of 5 call options and 5 put options on July 25th, 2022, is recorded. Each option has Moderna Inc stock as the underlying asset. Option contracts chosen are shown in the table below.

Table 1: 10 Options chosen.

Contrast Name	Option Price
Call Option	
MRNA220729C00160000	5.15
MRNA220729C00157500	7.87
MRNA220729C00155000	8.76
MRNA220729C00152500	11.75
MRNA220729C00150000	11.50
Put Option	
MRNA220729P00160000	3.70
MRNA220729P00162500	5.00
MRNA220729P00165000	8.00
MRNA220729P00167500	10.04
MRNA220729P00170000	11.69

After acquiring calibrated volatility, a delta hedging strategy is formulated to hedge a new put option, MRNA220805P00170000, from Aug 1st, 2022, to Aug 5th, 2022. Market put option price, from Aug 1st to Aug 5th, is shown in the table below.

Table 2: Put option price.

08/01	**08/02**	**08/03**	**08/04**	**08/05**
10.07	10.20	3.21	0.36	0.07

3 METHOD

In this paper, the hedging strategy is made up of two parts. The first one is to calibrate volatility using three different models: Black-Scholes model, binomial model, and historical return model. The reasons are shown below. First, the Black-Scholes model is shown to be a highly accurate prediction of future volatility; Second, the binomial tree model suits the discrete-time case. Next, this paper builds a delta-neutral portfolio to hedge a new option using delta hedging strategy.

3.1 The Black-Scholes Model

The Black-Scholes model, a derivative pricing model, measures the price of European put and call option. The Black-Sholes model assumes that price of European option is a function of strike price, time to maturity, underlying stock price, volatility, and interest rate of the return of the underlying stock. First developed by Fischer Black, Robert Merton, and Myron Scholes in 1973 (Manaster, 1982), the Black-Scholes model is still widely used today to price option contract. Although Black-Scholes model is an easy method to calculate option price, this method has some limitations. First, the Black-Scholes model only works for European options since one assumption is options can only be exercised at their maturity date. Second, this model assumes that stocks do not pay dividends and no interest is paid. Third, it assumes a frictionless market, which means this model does not take various transaction cost, like commissions and taxes, into consideration. Fourth, it assumes that the risk-free interest rate remains constant. However, above assumptions are hardly ever the case in reality. Details of the Black-Scholes model is shown below.

Time to maturity (t), strike price (K), interest rate (r), and stock spot price (S(t)), are all known for each of the ten options. The only unknown variable is volatility (σ). To calculate option price using the Black-Scholes model, first assume that volatility equals to 0.3. With above information, implied prices of ten options are calculated using the below equations. For clarification, $C(t)$ and $P(t)$ denote price of European call and put option, and $N(d_1)$ represents the cumulative standard normal probability of the value d_1.

$$cd_1 = \frac{1}{\sigma\sqrt{(t)}}\ln\left(\frac{S(t)}{K} + \left(r + \frac{\sigma^2}{2}\right)t\right) \quad (1)$$

$$d_2 = \frac{1}{\sigma\sqrt{(t)}}\ln\left(\frac{S(t)}{K} + \left(r - \frac{\sigma^2}{2}\right)t\right) \quad (2)$$

$$C(t) = S(t)N(d_1) - Ke^{-rt}N(d_2) \quad (3)$$

$$P(t) = Ke^{-rt}N(-d_2) - S(t)N(-d_1) \quad (4)$$

Then, the sum of squared errors (SSE) is used to measure the discrepancy between the implied prices and the market option prices. Equation (5) is the formula for calculating SSE. In equation (5), P_n^m stands for actual option price in the market. P_n stands for the theoretical option price. Lastly, minimize SSE by plugging in different values for volatility, and mark the value that yields minimum SSE.

$$SSR = \frac{(P_1 - P_1^m)^2}{P_1^m} + \frac{(P_2 - P_2^m)^2}{P_2^m} + \cdots + \frac{(P_3 - P_{10}^m)^2}{P_{10}^m} \quad (5)$$

3.2 Binomial Tree Model

Binomial tree model is a simple discrete-time model used to determine value of option. Under the binomial tree model, the lifetime of one option contract is divided into discrete many intervals (Breen, 1991). During each interval, the value of underlying asset either goes up or goes down. The multiplicative parameters of the movements are denoted by u and d, and p denotes the probability of price of underlying asset going up. Mechanism of binomial tree model is that the value of option at certain node relies on the possibility of stock price moving up or down. One advantage of binomial tree model is that it works for both American and European options. Also, it is appliable for dividend paying options. However, one fundamental assumption of binomial tree model is that the underlying asset can only take one of the two suggested values, which is more than idealized.

Binomial tree model is similar to the Black-Scholes model when time interval is small enough (Cvitanić, 2004). Use below equations to find theoretical option price. Similarly, calculate SSE and mark the volatility that yields minimum SSE. Detailed model specifications are shown by the follow equations.

$$u = e^{\sigma\sqrt{\Delta t}} \quad (6)$$

$$d = \frac{1}{u} \quad (7)$$

$$p = \frac{e^{rt} - d}{u - d} \quad (8)$$

3.3 Historical Return Model

The historical return model utilizes historical stock returns to predict sigma. Among the three models, volatility calibrated by the historical return model, in theory, deviates most from the actual volatility. Because stock price is highly volatile and follows no discernible trend. Past return is not a good indicator of return in the future.

For the historical return model, plug stock prices from July 25th to July 29th into equation (9).

$$\sigma = \sqrt{251Var[\ln(S_{t+1}) - \ln(S_t)]} \quad (9)$$

3.4 Delta-neutral Hedging Strategy

Delta-neutral hedging strategy is a commonly used risk managing option trading strategy. Delta measures the fluctuation in the value of portfolio as the underlying asset price moves. Mathematically, delta can be expressed as the partial derivative of the portfolio value with respect to the underlying asset price. The purpose of the hedging strategy is to build a portfolio that reaches a delta neutral position, which means the delta of the portfolio is zero. Following equations shows how to calculate the overall profit or loss of a delta neutral portfolio. Lastly, calculate loss without using hedging and compare the loss with hedging and loss without hedging. This step verifies that delta hedging strategy reduce the loss in option trading.

The goal of the paper is to hedge one share of put option, MRNA220805P00170000, with some shares of stock. Equation (10) gives the delta of option, ΔP. The delta-neutral portfolio consists of one share of put option and -ΔP shares of stock. In addition, equation (11) and equation (12) calculate the loss with and without hedging strategy at maturity (T).

$$\Delta P = -N(-d_1) \quad (10)$$

$$Loss\ with\ hedging = Max\{0, K - S(T)\} - \Delta P[S(T) - S(0)] - P(t) \quad (11)$$

$$Loss\ without\ hedging = Max\{0, K - S(T)\} - P(t) \quad (12)$$

4 RESULT

The below chart shows three parameters of the Black-Scholes model. Interest rate, 0.028, is collected from Federal Reserve website. Because there is 251 trading days in 2022, time to maturity (t) is $\frac{5}{251}$.

Table 3: Parameters of black-scholes model.

Parameter	**r**	**t**	**S(t)**
Value	0.028	0.020	162.75

The volatility calibrated, and the return from three different methods are shown in the table III. The return of delta-neutral hedging is calculated using three different volatilities.

Table 4: Result using three different models.

Method	**SSE**	**volatility**
Black-Scholes	8.162	0.389
Binomial	1.706	0.481
Historical Return	-	0.493

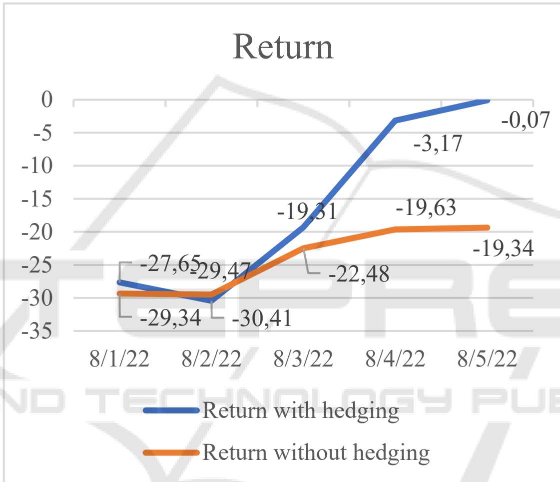

Figure 2: Trends of return with and without heding.

5 DISCUSSION

As shown in the previous section, the volatilities calibrated ranges from 0.389 to 0.493, which means Moderna stock is volatile. Price of security is closely related to the value of the firm. Modern belongs to the healthcare sector, and its firm value is closely related to the change in sales, and the launch of new product. Moderna specializes in developing mRNA vaccine. Because, before a vaccine got approved by the FDA, its research process is highly costly. Currently, mRNA vaccine for COVID-19 is the only type of mRNA vaccine in the market, and Pfizer is a strong competitor in the Covid-19 mRNA vaccine market. Due to these factors, it's reasonable that Moderna stock is highly volatile.

In figure (2), there is only one curve showing the return with hedging. Because the difference between the returns calculated using three different volatility is too small that it's reasonable to ignore the difference. The Result section also shows that the loss with hedging is lower than the loss without hedging on every day except for Aug 2nd. The most significance difference between the two approach is 19.27. In this case, the hedging strategy indeed reduces the overall risk of the portfolio. On Aug 2nd, the loss without hedging is lower than the loss with hedging, because the goal of delta-neutral hedging strategy is to reduce the fluctuation in overall portfolio value with respect to variation in stock price. But in some cases, change in stock price leads to a larger increase in option return than increase in overall portfolio return. To avoid having lower return with hedging strategy, when trading with delta-neutral strategy, it's necessary for traders to closely monitor and constantly rebalance the portfolio.

6 CONCLUSION

This paper examines the performance of delta-neutral hedging strategy using three different implied volatility. Because the limitations of each model the implied volatility might deviate from the actual volatility of the stock. No transaction cost is taken into consideration, and very option studied in paper is treated as European option. Moreover, despite the actual interest rate futurates daily, for simplicity of this paper, 0.028 is chosen to be the interest rate. Therefore, the implied volatility might deviate from the actual volatility of the stock, and the results shown in this paper might not be perfectly accurate. Another potential problem is that calibration using the Black-Scholes model and binomial tree model only uses 10 options; calibration using the historical return method only collects historical open price for five consecutive trading days. For improvement, more data should be used in the process of calibration. Further study is needed for a more accurate result.

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