Option Pricing and Delta Hedging for Moderna Inc. on Different Models

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Abstract: The aim of this paper is to compare the performance of the same delta hedging strategies using three different models in the process of pricing options for Moderna Inc., which has reference value for investors to compare such models and build their hedging portfolios. Historical volatility is estimated from past open prices and two implied volatilities are calibrated utilizing the selected 10 options on the Moderna Inc.'s stock. Then the delta hedging strategy using three different models with volatilities above, containing one unit of a specific option with different maturity and delta shares of the stock, is applied to obtain daily profit/loss. Finally, the trends of daily gain/loss for such three models are visualized compared with the trends without hedging. To conclude, the hedging strategy performs all well for the three models. This results in this study benefits investors and researchers in choosing the relatively suitable option pricing model and the best-fit hedging strategy for specific companies or sectors.

1 INTRODUCTION

Option pricing has been a focus of mathematical research in finance since the publication of the Black-Scholes formula in 1973 (Davis, 1993). Basically, it provides an evaluation of an option's value, which would be involved into investors' strategies. Options are used for hedging and speculation (Amir, 2018). Hedging is a term used nowadays primarily in conjunction with financial markets (Rata, 2009), which refers to the entrepreneur's financial strategy of mitigating market price risks through selling or buying futures contracts for the commodity that is the object of his activity (Rata, 2009). Hedging strategies limit the losses to a great extent and meanwhile, provide a flexible price mechanism. In recent years, hedging with options, no matter what kind of options, has been a crucial part in mathematical finance and widely used. The related topics have been continuously studied.

To demonstrate, on the one hand, researchers have improved classical model and hedging strategy to fit them better in realistic situations. First, based on the exist theory under the model without considering liquidity, Gueant, Olivier, and J. Pu modeled a new framework which considers stochastic optimal control to price and hedge a call option with execution costs and market impact (Gueant, 2017). Also, to improve delta hedging for options, Hull, J., and A. White determined empirically a model for minimizing the variance of changes in the value of a trader's position (Hull, 2017). Moreover, Ye, M., et al. improved Black-Scholes model for crop price insurance premium (Ye, 2017). Also, Imaki et al. proposed a new neural network to facilitate fast training and accurate optimal hedging strategies. In addition, Kim et al. extended binomial model in two ways, developed the one with time-dependent parameters and derived a hedging strategy for a trinomial model (Kim, 2017).

On the other hand, there are also some scholars who try to test the performance of different models for specific companies or fields, or for different kind of options. Lassance, Nathan and Vrins, Frederic compared the hedging ability of several popular models on the Apple stock (Lassance, 2017). Also, Doffou tested three parametric models in pricing and hedging higher-order moment swaps (Doffou, 2019). Besides, Bollin and Lepaczuk explored the performance of several option pricing models in hedging the exotic options (Bollin, 2020)

To sum up, this topic is of interest to the researchers in the financial field. This paper also focuses on this topic and two main parts are included in the entire study. First, the three volatilities are

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calibrated. In detail, one historical volatility is estimated by historical stock open prices, and the Black-Scholes model as well as Binomial model are applied to calibrate the other two implied volatilities by minimizing the SSE. Then, the same delta hedging strategy using Black-Scholes model and Binomial model is applied to a specific option. Moreover, the replicate processes display the daily profits/losses with hedging and without hedging in three different models, which are visualized to compare. As a result, the hedging strategies are successful utilized on the three calibrated models and all perform well, while the gain/loss gap between BSM and Binomial model is quite large.

This paper is structured as follows. Section 2 shows the data and methods. In Section 3, the results are displayed, and Section 4 refers to the conclusion of this paper.

2 DATA AND METHODS

2.1 Data

Moderna Inc. are chosen to be the target company because it delivered a strong first quarter performance and had a tremendous financial performance throughout the past years. Besides, the Moderna vaccines has been broadly used by many countries to prevent Covid-19, and recently, the Moderna Covid-19 vaccine was authorized by FDA for children down to 6 months of age, which makes the Moderna stock trending higher. Thus, the company's stock and underlying options are worth studying.

The data used is all extracted from the Yahoo Finance (https://ca.finance.yahoo.com). Five call options and five put options on Moderna Inc. are chosen with a same maturity date, July 15th, but different strike prices to calibrate two implied volatilities for the stock using two different models. The data is around the current price at the day when the data was selected to prevent the edge cases of volatility smile from the calibrated results. Besides, the historical stock open prices are collected from March 21st, 2022, to June 22nd, 2022, to estimate a historical volatility. Then, a Delta hedging strategy is constructed for another new option, whose maturity is July 29th, 2022, from June 23rd, 2022, to July 7th, 2022, using the three different volatilities and corresponding models. In details, options contracts that are selected to calibrate the implied volatility for Moderna Inc., followed by their strike prices, are shown below in the table 1 below.

Table 1: The 10 options selected for the calibration.

Call options chosen for calibration	Strick prices
MRNA220715C00165000	165
MRNA220715C00170000	170
MRNA220715C00175000	175
MRNA220715C00180000	180
MRNA220715C00185000	185
Put options chosen for calibration	Strick prices
MRNA220715P00160000	160
MRNA220715P00165000	165
MRNA220715P00170000	170
MRNA220715P00175000	175
MRNA220715P00180000	180

And the option contract which is used in the hedging section is shown in the table 2 below.

Table 2. The option chosen for hedging

	Option chosen for hedging
Moderna Inc.	MRNA220729P00135000

Furthermore, the historical stock open price trends for Moderna Inc. 's are shown in the figure below.

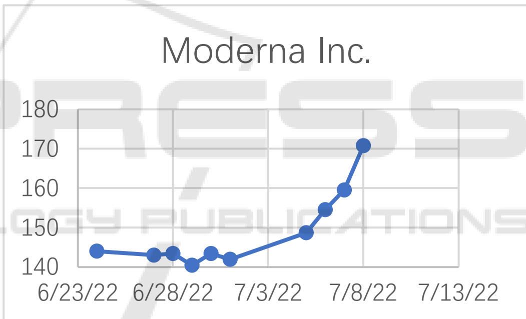

Figure 1: The stock price trend for Moderna Inc.

As shown above, Moderna Inc. 's stock open price has increased dramatically from June 30th to July 7th from \$143.4 to \$159.5, while prior to that, the prices fluctuated around \$143. It is interesting to find that the slopes during the increasing prior are almost same.

2.2 Method

To begin with, the Black-Scholes model and Binomial model are two of the most important concepts which are widely applied in modern finance to solve the option pricing problems, and each has its own pros and cons. Thus, they have been compared to judge which one is better. On the one hand, the 5 inputs: the strike price, the current stock price, the time to maturity, the risk-free rate and the volatility are taken into account by the Black-Scholes model, and it will return the numerical result based on these arguments, while the multiple-stage Binomial model can show a binomial tree structure with options and their possibilities. On the other hand, the Black-Scholes model can be only applied to European options but not American options, but the Binomial model is able to utilize multiple periods to value American options. Besides, the assumption of the Black-Scholes model is that the returns of the underlying asset are normal distributed.

In this study, the process can be divided into two parts. First one historical and two implied volatilities of the stock are calibrated using three different methods. In details, the historical volatility is approximated by the standard deviation of the natural log of the ratio of consecutive past stock open prices from the assumption that the stock follows the Black-Scholes model.

$$Var\left[\log\left(S(t_{k+1})\right) - \log\left(S(t_k)\right)\right] = (t_{k+1} - t_k)\sigma^2 \quad (1)$$

Then the Black-Scholes model and Binomial model are used to calculate theoretical prices of the selected 5 call and 5 put options. The calculated prices are compared with the real market prices to calibrate the implied volatilities by minimizing the SSE. For the Black-Scholes model, initially, the stock price, the price, the strike price, the maturity date for 5 call and 5 put options on the stock are collected from June 27th, 2022, to July 6th, 2022. The Black-Scholes option prices are calculated using Black-Scholes model with the formula below, where σ represents to the implied volatility, t represents time to maturity, S(t) represents to the stock price at time t, K represents to the strike prices of the option, r is the risk-free rate, which is 2.8% in annual term obtained from the Federal Reserve. The call option price and put option price are calculated by equation (2) and (3) respectively, where N represents the standard normal distribution.

$$C\left(t, S(t)\right) = S(t)N(d_1) - Ke^{-rt}N(d_2) \quad (2)$$

$$P\left(t, S(t)\right) = Ke^{-rt}N(-d_2) - S(t)N(-d_1) \quad (3)$$

$$d_1 = \frac{1}{\sigma\sqrt{(t)}}\ln\left(\frac{S(t)}{K} + \left(r + \frac{\sigma^2}{2}\right)t\right) \quad (4)$$

$$d_2 = \frac{1}{\sigma\sqrt{(\mathrm{t})}}\ln\left(\frac{S(t)}{K} + \left(r - \frac{\sigma^2}{2}\right)t\right) = d_1 - \sigma\sqrt{\mathrm{t}} \quad (5)$$

Moreover, one-step Binomial model is constructed to calculate the option price as well, and the same calibration approach is applied in the Binomial model by minimizing the SSE of calculated option prices. For the Binomial model, up and down factors, which are represented by u and d respectively, are needed in the formula. The general Binomial model formula is shown below, where T represents to the time to maturity, p represents the possibility to move up, n represents to the number of time steps of binomial trees. The equation (7) calculate the option price and it is interesting to notice that call and put options use the same formula.

$$S_0 = e^{-rT}[S_0 u \times p + S_0 d \times (1 - p)] \quad (6)$$

$$c_0 = e^{-rT}[f_u \times p + f_d \times (1 - p)] \quad (7)$$

$$u = e^{\sigma\sqrt{\Delta t}}, d = \frac{1}{u}, p = \frac{e^{rT} - d}{u - d}, \Delta t = T/n \quad (8)$$

After the theoretical option prices are calculated from the two models. The sum of percentage squared errors between the calculated option prices and real-market prices is calculated for every option on every day from June 27th, 2022, to July 6th, 2022, except for some losing data, shown in the equation (9), where p_i^c represents the calculated option and p_i^m represents the real-market prices. Then the SSE is minimized to calibrate the final implied volatility. And this process is repeated for both models.

$$\mathrm{SSE} = \sum \frac{(p_i^c - p_i^m)^2}{p_i^m} \quad (9)$$

At the end of the calibration section, the three different volatilities and the corresponding methods are compared. Since the similar amount of data for calculated option price, SSE from two methods for implied volatility can be compared as well.

After the calibration of the historical and implied volatilities, a Delta hedging strategy is applied for a new selected option on the same stock, which has a further maturity on July 29th, 2022. Its strike price is \$135. The Black-Scholes models for both historical volatility and implied volatility follow the same strategy but with different sigma value. The price of the option is calculated using the Black-Scholes model with the parameters and calibrated volatility provided above. The portfolio contains one unit of call option contract, and Δ_t shares of stock, where Δ_t is equal to $N(d_1)$ for call option. Thus, the following formula can be applied to calculate the portfolio value at time t, the profit or loss with hedging as well as profit/loss without hedging can be calculated every day from June 23rd, 2022, to July 7th, 2022.

Day 1:

$$X(t) = C(0) \quad (10)$$

Day 2-10:

$$X(t) = X(t-1) + \Delta_{t-1}(S(t) - S(t-1)) \quad (11)$$

$$Loss\ without\ hedging(t) = S(t) - K - C(0) \quad (12)$$

$$Loss\ with\ hedging(t) = S(t) - K - X(t) \quad (13)$$

Besides, for the one-step Binomial model, the strike price, stock price from June 23rd, 2022, to July 7th, 2022, upper factor, down factor, and the possibility to move up are collected and computed. The option prices are composed from the formula below.

$$Hedge\ Ratio = \frac{C^+ - C^-}{uS - dS} \quad (14)$$

$$C^+ = \max(0, uS - K), C^- = \max(0, dS - K) \quad (15)$$

$$Payoff = Hedge\ Ratio * uS - C^+ = Hedge\ Ratio * dS - C^- \quad (16)$$

$$C = Hedge\ Ratio * S - Payoff * e^{-rT} \quad (17)$$

Then Loss/profit with hedging can be calculated every day from June 23rd, 2022, to July 7th, 2022, using the following formula.

$$Loss\ with\ Hedging = S(t) - K - [Hedge\ Ratio * (S(t) - S(0)) + C] \quad (18)$$

$$Loss\ without\ Hedging = S(t) - K - C \quad (19)$$

Thus, starting at different date, the gain/loss with or without hedging can be composed every day from June 23rd, 2022, to July 7th, 2022.

As the profit or loss of the portfolio with or without hedging using two models and three volatilities is calculated, the performance of the hedging strategy can be compared among them.

3 RESULT

From the process of the calibration, one historical volatility and two implied volatilities using Black-Scholes model and Binomial model are shown in the table below followed by their SSE.

Table 3. The calibrated volatilities

Method	Calibrated volatility	SSE
Historical volatility	0.696	
IV from BSM	0.652	31.903
IV from Binomial model	0.549	61.713

Furthermore, to test the influence of number of historical stock prices to the calibrated historical volatility, the trends of historical volatility as the amount of historical stock prices used increases are shown in the figure below.

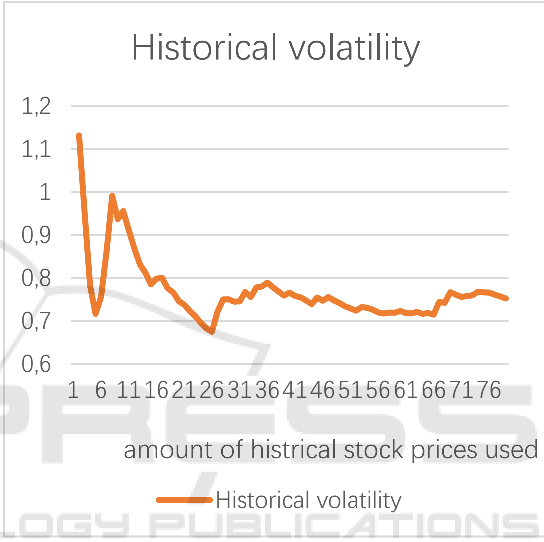

Figure 2: The historical volatility trend.

As shown in the figure, when the used historical data reaches a certain amount, around 28 in this study, the change of historical volatility becomes smaller, and the values of historical volatility fluctuate within a relative narrow range. But eventually, the chosen historical volatility is obtained from around 64 historical prices, which is close to the number of calculated terms in calibrating implied volatility to make the results comparable.

As shown in the Table 3 above, historical volatility is largest, followed by the implied volatility calibrated from Black-Scholes model. They are similar, while implied volatility using binomial model is relatively small with much larger SSE, compared with the one in Black-Scholes model.

In terms of hedging, the profits or losses of the portfolio with and without hedging for the chosen period from June 23rd, 2022, to July 7th, 2022, using the historical volatility and the two implied volatility are compared and shown in the following graphs and table. It’s clear that the Delta hedging strategy on Moderna Inc. has performed better than merely holding the call option without hedging.

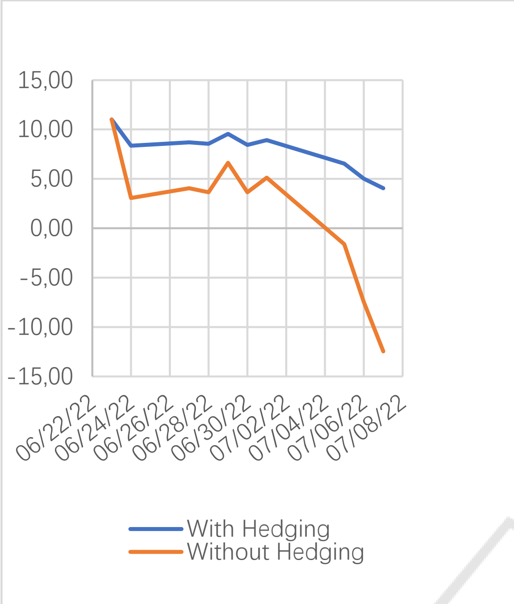

Figure 3: The Comparison of the Profit/Loss holding one unit of option on Moderna Inc. with/without hedging using Implied Volatility in BSM.

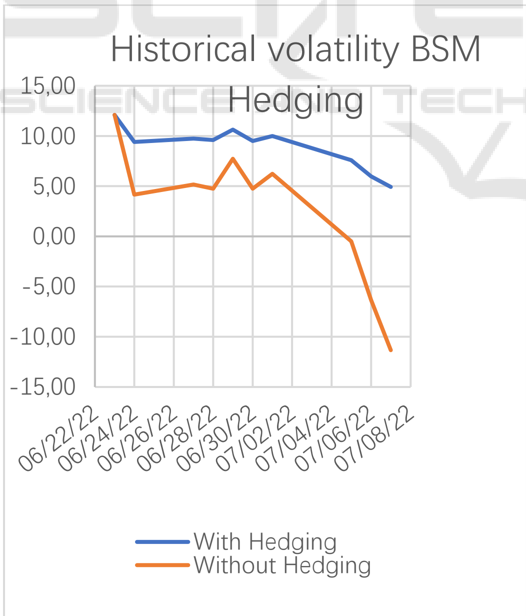

Figure 4: The Comparison of the Profit/Loss holding one unit of option on Moderna Inc. with/without hedging using Historical Volatility in BSM.

As shown in the two figures above, the hedging strategy works very well on the option on Moderna Inc. using both historical and implied volatility in Black-Scholes model. Besides it shows that the two figures follow the similar trends no matter with or without hedging. Overall, the profit with hedging for each trading day is comparatively stable, while without hedging, it remains profitable during the early days in the period, but the loss appears at the end of the period. Besides, two methods indicates that the difference between profit/loss with and without hedging becomes larger gradually.

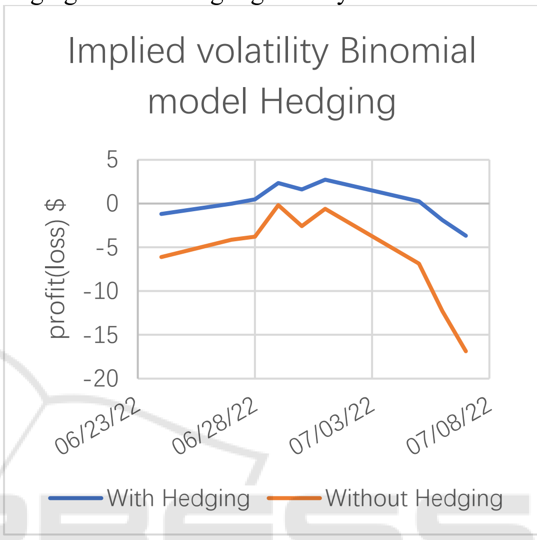

Figure 5: The Comparison of the Profit/Loss holding one unit of option on Moderna Inc. with/without hedging using Implied Volatility in Binomial Model.

As shown in the figure, the line corresponding to the strategy with hedging is always above the other line which displays the gain/loss without hedging, which is same as the previous results. Besides, the same conclusion can be presented that the profit/loss with hedging is relatively more resisting than the loss by only holding the option. However, the overall profit or loss is less than the profit/loss using Black-Scholes model. During the period, the hedging strange only guarantee that the profit or loss is around 0.

4 CONCLUSION

This paper studies the difference between two kinds of models: Black-Scholes model and Binomial model, as well as the disparities between historical and implied volatilities. In this study, first, the one historical volatility and two implied volatilities in the two model are calibrated by utilizing the data on past stock prices and ten options respectively. Then, a delta hedging strategy is constructed, including one unit of the new selected option and delta shares of the stock to make delta neutral. Finally, the hedging performances for three different volatilities in two models are compared. The hedging strategy on all the three situations performs well, while the results show the different amount of profit or loss, especially the two in BSM with the one in Binomial model.

Nevertheless, there is a limit in the paper, for example the simple one-step binomial model is considered merely, but not a higher steps model. The one-step model makes the calculation much easier when dealing with long-time period in a replicated process, while the more steps may make the results more accurate. Besides, since the interest rate used in this study is fixed, the transaction costs are ignored and the options chosen are assumed to be European, these deficiencies need to be researched comprehensively in the future.

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