Option Pricing and Risk Hedging for High-Tech Company: The Case for Apple

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Abstract: In this study, we examine the profitability of various hedging techniques for Apple stock options, which is brand new to the field. The entire study is accomplished in two steps. In the first step, implied volatility is determined and calculated by analyzing data on 10 different options on Apple stock, and the next step is to construct a hedging portfolio based on the results of the first step, consisting of one unit of the specified option and the underlying stock's delta shares. In summary, it has been determined that the hedging approach investigated in this study has the potential to successfully cut risk. This study helps individual and institutional investors to effectively construct a portfolio of Apple's stocks and select the most appropriate hedging strategy. Use this as a reference. It should be noted that the Black-Scholes model used in this study has a zero-interest rate assumption. In addition, the Black-Scholes model ignores the transaction costs that exist in real markets; therefore, the results of this model may not be fully accurate, but additional research in the near future would be beneficial.

1 INTRODUCTION

In today's economy, there simply aren't any ways to profit from price differences. In a world without risks, there is a certain likelihood that the stock price would increase, and this is known as the risk-neutral probability. We do not, however, presume that all market participants are risk averse or that high-risk investments would provide returns equivalent to those on safer investments. This theoretical value is a measurement of the probabilities associated with purchasing and selling the assets assuming all probabilities were equal (Victoria, 2021). This is the premise of the study for hedging.

In finance, hedging refers to the action of protectingb your position, and or profit from the potential risks (Cvitanic, 2004). No matter which positions you take in the financial market, there will always exist different kinds of risk referring to your decision. It could be liquidity risk, interest rate risk, and or default risk, depending on which financial asset you chose to protect your current position. Thus, financial derivatives were invented, and were utilized to mitigate some of the risk that you might encounter.

Following on, within all these different kinds of financial derivatives, options were most widely and commonly used in hedging strategies. Option-hedging strategies were one of the most well-known and thoroughly studied strategy in the financial industry as well.

To demonstrate, option hedging strategy first starts when option contracts appear, in which option pricing theory such as the Black-Scholes theory were also investigated. Following on, different scholars started to investigate in different option hedging strategies that could outperform the market. For instance, Sami Vähämaa studied the strategy which was called “Delta Hedging with Smile”, which utilized the volatility smile as one of the hedging components, and, moreover, demonstrated a better performance on hedging instead of delta hedging (Vähämaa, 2004). Subsequently, there were also statisticians and scholars such as Roland Langrock, Thomas Kneib,

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Richard Glennie & Théo Michelot who investigated in "Markov-switching models", which this statistical model were then widely utilized in producing Markov-switching deltas in different hedging strategies as well (Langrock, 2017). Besides, many scholars decided to study delta hedging with respect to different kinds of options as well. For instance, Alexander and Iryna decided to look for potential delta hedging strategies in exotic options, based on "Price trajectories are d-dimensional continuous functions whose path wise quadratic variations and covariation are determined by a given local volatility matrix" (Schied, 2016).

On the other hand, researchers have dedicated inspections on some fields. For instance, Jaska Cvitanic and Ioannis Karatzas applied the continuous-time model in order to do hedging for an arbitrary contingent claim, with proportional transaction costs (JAKSA, 1996). Also, Avellaneda Marco and ParaS Antonio developed strategies, for the hedge of derivative securities, under the presence of transaction costs (AVELLANEDA, 1994).

However, limited research scrutinized the performance of hedging strategies in companies' sector. This paper will orient on such matter, selecting Apple Inc. as our analysis target. Its leading position in technology sector enables it to be a representative company as chosen. In this study, results are given by the act of delta hedging strategy applying the Black-Scholes Model on Apple Inc.

This paper is organized as follows: Section 2 Data and Method; Section 3 Result and Discussion; and Section 4 Conclusion.

2 DATA AND METHOD

2.1 Data

The data used is all from Yahoo Finance (https://finance.yahoo.com/). We choose the assets of apple. The reason why we choose these assets as follows Apple's products are consistently ranked among the most sought after throughout the globe, and the firm is now venturing into new areas of the technology sector to provide even more ground-breaking instruments and services, which means Apple has made preparations to maintain its competitive advantage. We choose five different call options and put options that all mature on 2022 July 1st with different strike prices, to calibrate an implied volatility. After that, a hedging strategy was constructed for another option on Apple stock from June 1st, 2022, to June 14, 2022. In detail, the option contracts collected to calibrate the implied volatility of Apple Inc. stock are shown in the table below.

Table 1: The 10 options chosen.

	Call options chosen for calibration				
Apple Inc.	AAPL220701C00125000	AAPL220701C00126000	AAPL220701C00127000	AAPL220701C00128000	AAPL220701C00129000
	Put options chosen for calibration				
Apple Inc.	AAPL220701P00125000	AAPL220701P00126000	AAPL220701P00127000	AAPL220701P00128000	AAPL220701P00129000

And the option contract selected for Apple inc. for hedging is shown in the table below.

Table 2: The option chosen for hedging.

	Option selected for hedging
Apple Inc.	AAPL220708C00125000

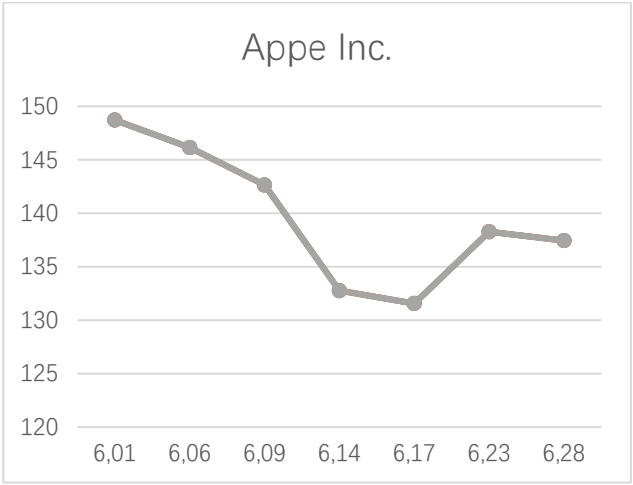

Figure 1. The stock price trend of Apple Inc.

As shown above, Apple Inc's stock price had small reduction from Jun. 1st, 2022, to Jun.28th,2022, from $148.71 to $138.44. there was some minor volatility as Apple's share price declined during the study period.

2.2 Method

The Black-Scholes formula is used as a model for valuing European or American call and put options on a non-dividend paying stock. In option pricing theory, the Black-Scholes equation is one of the most effective models for pricing options (Mehrdoust, 2014). Black Scholes model take into account the risk factor of the underlying stock, the strike price of the option contract, the initial stock price and the time value of money, which are all significant factors, while pricing each option contract. Its comprehensiveness and completeness also enhances its accuracy while computing different option prices. All options have limited downside and rely on price volatility for upside, thus a rise in volatility raises the value of both calls and puts (Yacine, 2003). However, the Black Scholes model, similar to other option pricing models, is not a perfect one as well. In fact, there are some unrealistic assumptions that the Black Scholes model makes in order for its convenience of computation and conciseness. First, the Black Scholes model assumes that volatility of the stock and the market's risk-free rate are all constant throughout the whole maturity time, which is not true in the real case. Second, the Black Scholes model also assumes that no option could be exercised before the maturity time, which makes it only effective while pricing European option contracts. Third, the Black Scholes assumed that the market is a frictionless one, which means there would not exist any transactional costs while trading these financial derivatives, which is also unrealistic, too. Last but not least, although all these up-mentioned assumptions might affect the accuracy of the Black Scholes model, we could not deny that it is still one of the most common and conservative way for pricing European option contracts.

Options Spread is a trading strategy for options in the financial market that entails taking opposite directional positions in the price of a pair of options belonging to the same asset class but with different strike prices and expiry dates. A spread position involves two options with distinct strike prices and expiry dates (Abhilash, 2021). The hedging strategy is completed with two steps in the research as below. First, the implied volatility of the stock is calibrated with the price of the stock and that of ten options as selected, on the stock using Black Scholes model. Second, applying the implied volatility calculated, a hedging strategy is constituted for a chosen option on the stock.

Following on, the implied volatility utilized in the Black-Scholes model is an estimated value estimated by calculating the market stock prices' standard deviation or other estimating and forecasting techniques such as maximum likelihood estimation, moment estimation, generalized linear model, Bayesian point estimation. In our study, we decided to estimate the volatility in two different ways: the traditional market stock prices' standard deviation and minimizing the difference between real market option price and Black-Scholes price calculated utilizing the Black-Scholes formula. The whole pricing process stars as the following. First, we choose the 2022 June 1st AAPL open price as our initial stock price S (0). Second, we choose five different call options and put

options that all mature on 2022 July 1st with different strike prices. After we finish computing our Black-Scholes price for every option, we start calibrating our standard deviation. As mentioned, we first calibrated our sigma by calculating the standard deviation between the market stock prices from May 1st to May 31st. The second way is to calculate the sum of squared errors between the BSM options prices and the real-market option prices calculated for every option from June 1st, 2022, to July 1st, 2022. Subsequently, the sum of squared errors for the 10 options for the AAPL stock are minimized to calibrate the most accurate implied volatility. Third, we plug in these numbers into the Black-Scholes formula listed below with t representing the time to maturity, the time difference between July 1st and June 1st, S(t) representing the initial stock price, K representing the strike prices, 125 to 129, and representing the risk-free interest rate, which is assumed to be 0 for computational conveniency. In the Black-Scholes formula, we have to first compute d_1 and d_2 as the following equation.

$$d_1 = \frac{1}{\sigma\sqrt{(t)}} \ln\left(\frac{S(t)}{K} + \left(r + \frac{\sigma^2}{2}\right)t\right) \tag{1}$$

$$d_2 = \frac{1}{\sigma\sqrt{(t)}} \ln\left(\frac{S(t)}{K} + \left(r - \frac{\sigma^2}{2}\right)t\right) = d_1 - \sigma\sqrt{t} \tag{2}$$

Following on, we have to plug d_1 and d_2 into the normal distribution operator in order to calculate its associating probability. Lastly, we can implement the European Call option pricing formula under Black-Scholes in the following.

$$C(t, S(t)) = S(t)N(d_1) - Ke^{-rt}N(d_2) \tag{3}$$

Similarly, for the price of the put option, we can calculate it by either the Black-Scholes formula listed below, or simply by implementing the Put-Call Parity formula.

$$P(t, S(t)) = Ke^{-rt}N(-d_2) - S(t)N(-d_1) \tag{4}$$

After we finished computing our Black-Scholes price for every option, we start calibrating our standard deviation. As mentioned, we first calibrated our sigma by calculating the standard deviation between the market stock prices from May 1st to May 31st. The second way is to calculate the sum of squared errors between the BSM options prices and the real-market option prices calculated for every option from June 1st, 2022, to July 1st, 2022. Subsequently, the sum of squared errors for the 10 options for the AAPL stock are minimized to calibrate the most accurate implied volatility.

Table 3: Comparison for Statistical Indicators.

	Sigma	Call option	Put option
SSE	61.3	59.59	1.71
Historical Value	7.9%	55.53	1.12

Following the implied volatility calibration, a hedging strategy is created for a particular option on the stock's equity. First, the strike price and maturity date for the selected option contract, as well as the stock price information from 1st to 14th June 2022, are collected. The prices of the selected option contract are then determined by applying the Black Scholes model and the calibrated implied volatility determined by the aforementioned formula.

Day 1:

$$Portfolio\ value\ (t) = C(1, S(1)) \tag{5}$$

Day 2-14:

$$Portfolio\ value\ (t) = portfolio\ value\ (t-1) + N(d_1)(S(t) - S(t-1)) \tag{6}$$

Second, the hedging portfolio consists of N(d1) units of shares and one unit of a call option contract, where N(d1) represents the delta for the BSM model. The value of N(d1) on each day can be computed. The methodology can be used to calculate the portfolio value, loss or profit without hedging, and loss or profit with hedging for each day between June 1 and June 14, 2022.

$$Loss\ without\ hedging(t) = S(t) - K - C(1, S(1)) \tag{7}$$

$$Loss\ with\ hedging(t) = S(t) - K - portfolio\ value(t) \tag{8}$$

The effectiveness of the hedging strategy could be evaluated in comparison across businesses and industries as the loss or profit of the portfolio with or without hedging is assessed.

3 RESULTS AND DISCUSSION

3.1 Results

In our study, after processing the comparisons of profits or losses of the portfolio values with and without hedging, from June 1st to June 14th, 2022, the result shows obviously that the decision for holding Apple Inc.'s option with hedging is more preferable, despite both strategies obtained negative outcomes instead.

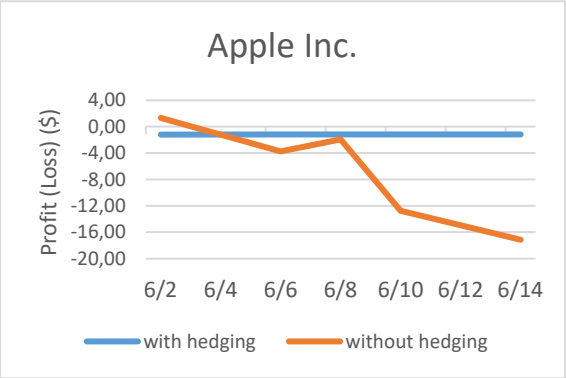

Figure 2: The comparison of the profit/loss holding one unit of the option on Apple Inc.'s stock with or without hedging.

The graph illustrates that the hedging strategy completed better on the option on Apple Inc.'s stock. With hedging, the loss is stable in the period; without hedging, the loss declines rapidly within 10 working days although there is a fluctuation in the middle. If adding the profits and losses up for these 10 days, the loss with hedging is $11.90, while that of without hedging is $66.48. The differed amounts indicate the hedging strategy has notably diminished the risk.

3.2 Discussion

The delta hedging strategy above-mentioned performs pretty well on the Call option contract for the Apple company and stock. We can tell that it has an outstanding effect on helping us reduce and mitigate our loss while investing in the Apple company's stock and option. In other words, the strategy implemented in the study helped hedge the delta risk which is the risk associated with the price movement of the underlying stock of the option. Theoretically, delta could also be defined as the derivative of the option price with respect to the market price of the underlying stock. In short, as we hold one unit of the option contract, and delta shares of the underlying stock, the portfolio reaches delta neutral, and thus, helps us hedge the potential risk induced by the price movement.

The Apple Inc Corporation is one of the most essential companies not only in the technology sector, but also in the US NASDAQ index. Moreover, the Apple company is also one of the most actively traded options, which also indicates that it would have more complete and continuous data while implementing our hedging strategy. Statistically speaking, more continuous and complete data helps empower our estimation and calculation of the volatility of the stock and Black-Scholes option price. Moreover, in a financial perspective, a more actively traded option also enables the liquidity of our transaction and also indicates that there would not be many options that are far out of the money.

The Apple Inc's has an average option trading volume of four hundred thousand per day, which is very liquid and commonly traded. Many other technology companies such as Microsoft, Meta, TESLA, and or Netflix also possess high trading volume. In contrast, companies from the industrial, financial and retail sectors do not trade in such large volume, which also make these option contracts difficult to liquidate. Therefore, for our Black-Scholes model to be stable and complete initially, an option which is commonly traded is an important factor. Subsequently, a commonly traded option could also enhance the accuracy of our volatility calibration and Black Scholes option pricing. In sum, the Apple company is a distinguished example which fit our study needs and also perform well in our hedging strategy.

4 CONCLUSION

This paper examines the performance of hedging strategies for options on Apple stock. Although researchers have studied hedging strategies using options and sector-specific hedging strategies, the topic discussed in this paper has not been studied before. Therefore, in this paper, the delta hedging strategy is used for options on Apple Inc. and its hedging performance is analyzed. First, the implied volatility is calibrated by using data from ten options on this Apple stock. Then, based on the calculated implied volatilities, a hedging portfolio was composed containing one unit of a specific option and the underlying stock's Delta shares. Finally, it is calculated that the hedging strategy in this study effectively reduces the risk. This study provides an in-

depth analysis of the performance of Apple's stock options using a hedging strategy, which helps individual and institutional investors to effectively build an Apple portfolio and choose the most suitable hedging strategy, as well as being a reference for the study of hedging strategies on other stocks.

It should be noted that the interest rate used in the Black-Scholes model in this study is assumed to be zero, and the Black-Scholes model also ignores the transaction costs that exist in the real market, so the results of the model may not be fully accurate and deserve more research in the future.

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