Option Pricing and Risk Hedging by Black-Scholes Model and Cox-Ross-Rubinstein Model for Unilever PLC

Sen Chen[1,†], Boyang Sun[2,*,†] and Jingzhe Lin[3,†]
[1]*The College of Liberal Arts and Sciences, Arizona State University, 336E Orange St Tempe, AZ 85281, U.S.A.*
[2]*Faculty of Science, University of Ottawa, 75 Laurier Ave E, Ottawa, ON K1N 6N5, Canada*
[3]*Faculty of Environment & Faculty of Mathematics, University of Waterloo, 200 University Avenue West, Waterloo, ON, Canada*

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Abstract: Currently, option pricing and risk hedging are interesting topics in the financial field within the violate world. This paper studies the performance of the different hedging strategies on options on stocks of Unilever PLC within the Fast-Moving Consumer Goods Industry, which is helpful for both individual and institutional investors to build their portfolios and choose a hedging strategy. In this study, implied volatility for each of Unilever's stocks is calibrated utilizing data on ten options on that stock. Then, with the Black Scholes Model and Cox-Ross-Rubinstein model calculated, a hedging portfolio is composed, containing one unit of a specific option and delta shares of the underlying stock for the company. Finally, the hedging performances of the options on the company's stocks are compared. The results in this paper benefit both individual and institutional investors in choosing the best-fit hedging strategy depending on the nature of the underlying asset to risk mitigation.

1 INTRODUCTION

To illustrate, a hedging strategy refers to a risk management strategy that offsets losses in an investment by taking the opposite position in a related asset. As a matter of fact, hedging strategies are widely used by individual investors as well as asset management companies to mitigate risks and reduce the extent of potential negative effects without significantly reducing the rate of return. In addition, as hedging strategies facilitate investors' investments in more diverse assets, they help increase the liquidity of their investments.

Within the numerous hedging strategies, using options to hedge risks towards equity portfolios is very critical and widely used, and its related topics have long been studied, Galai analyzed the components of the return from hedging options against stocks (D. Galai, 1983). Also, Platen and Schweizer provided a new explanation for the smile and skewness effects in implied volatility from hedging derivatives (E. Platen, 1998). Also, Bakshi, Cao, and Chen compared the pricing and hedging of short-term and long-term equity options (G. Bakshi, 2000), and Kumar studied the efficacy of option Greeks and their significance in risk hedging strategies (A. Kumar, 2018). Additionally, Howe and Rustem presented a robust hedging algorithm to hedge the risk of writing options (M.A. Howe, 1997). Moreover, Soner, Shreve, and Cvitanic proved that the least expensive method of dominating a European call in a Black-Scholes model with proportional transaction costs is the trivial strategy of buying one share of the underlying stock and holding it to maturity (H. M. Soner, 1995). Comparably, Gao, Li, and Bai et al. also proposed an optimal risk hedging strategy using put options with stock liquidity (R. Gao, 2019). Becker, Cheridito, and Jentzen introduced a deep learning method for pricing and hedging American-style options as well (S. Becker, 2019).

In this study, the same delta-hedging strategy utilizing the Black Scholes model and binomial tree model is applied to a specific option on Unilever

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[*] Corresponding author
[†] These authors contributed equally

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Corporation's stocks. And the hedging performance on the company's option is compared to give insights into the difference in performances utilizing the same hedging model across companies and sectors. The results of this study show that the delta hedging strategy utilizing the Black Scholes model and binomial tree model performs well on selected options.

2 DATA AND METHODS

2.1 Data

The data to be used are collected from Yahoo Finance (https://ca.finance.yahoo.com). Data are used of five call options and five put options on Unilever PLC stocks collected from June 16th to July 18th, 2022, which are used to calibrate implied volatility for the stock of Unilever PLC. Unilever PLC, a mass company with large volume and popularity in the Fast-Moving Consumer Goods Industry, is supposed to be representative of the FMCG industry. After gathering data, a hedging strategy is constructed for another option in this stock from June 27, 2022, to July 8, 2022. The ten options used for calibration are in-the-money, at-the-money, and out-of-the-money, with similar prices to ensure that the sum of the standard error is not so large that it is difficult to calibrate implied volatility. The options information collected to calibrate the implied volatility of Unilever's stock is shown below in the table.

Table 1: 10 options chosen for calibration from uilever plc.

	Call options				
Unilever PLC	UL220819C000 40000	UL220819C000 42500	UL220819C000 45000	UL220819C000 47500	UL220819C000 50000
	Put options				
Unilever PLC	UL220819P000 40000	UL220819P000 42500	UL220819P000 45000	UL220819P000 47500	UL220819P000 50000

The information on the option used for hedging is shown in the table below.

Table 2: Option that is chosen for hedging for unilever plc..

	Option
Unilever PLC	UL220819C00037500

In addition to this, Unilever's share price from June 17, 2022, to July 18, 2022, is shown below.

Table 3: Stock price of Unilever plc from June 16th,2022, to July 18th,2022.

Date (YYYY/MM/DD)	The stock price of Unilever PLC ($)
2022/6/16	43.96
2022/6/17	43.72
2022/6/21	44.37
2022/6/22	44.37
2022/6/23	44.57
2022/6/24	46.25
2022/6/27	46
2022/6/28	45.33
2022/6/29	45.56
2022/6/30	45.83
2022/7/1	46.29
2022/7/5	45.84
2022/7/6	46.18
2022/7/7	46.13
2022/7/8	46.13
2022/7/11	45.18
2022/7/12	46.02
2022/7/13	46.27
2022/7/14	45.93
2022/7/15	46.21
2022/7/18	45.94

These data are plotted as a linear graph as shown below.

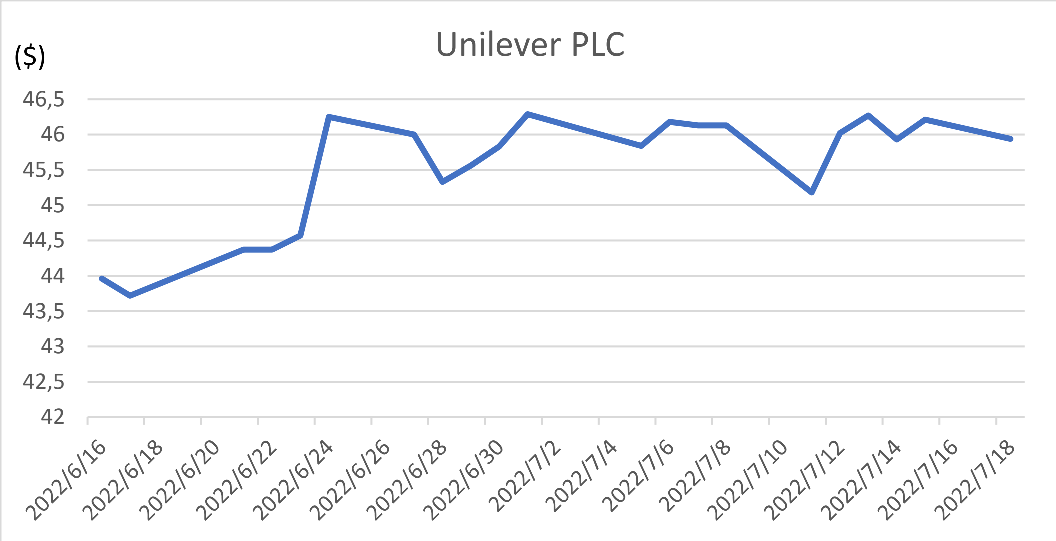

Figure 1: Stock price trend of unilever plc.

As shown in the figure above, Unilever's share price has a clear upward trend from June 14, 2022, to June 24, 2022, rising from $43.96 per share to $46.25 per share, after which the share price remains relatively stable, there was no obvious rise or fall, and some volatility appeared during the period. The stock price remained roughly at around $46 per share these days.

Table 4: Descriptive statistics of the rate of return of stocks of unilever plc.

	MEAN	STANDARD DEVIATION	MEDIAN	1ST QUARTILE	3RD QUARTILE
UNILEVER PLC	43.72	0.8288782	45.93	45.18	46.13

As shown in the figure above, the average value of this group of data is $43.72, while the median is $45.93. The average value is smaller than the median value, indicating that this group of data has an extreme minimum value, and it also implies that Unilever's stock price does rose.

2.2 Methods

There are two steps to do with this study. The first step is to bring the ten options used for calibration into the Black-Scholes model and the Cox-Ross-Rubinstein model to calibrate the implied volatility. The second step is to substitute the calibrated implied volatility into the formulas for calculating the delta value and calculating the profit to calculate the profit of the portfolio without hedging and the profit of the portfolio with hedging, to test the effectiveness of the hedging strategy that is constructed by the specific option mentioned in the data part.

Above all, the Black-Scholes model reduces the underlying asset and derivatives markets to a set of rules expressed through mathematical formulas. This model is widely cited around the world today and is the basis for most market analyses. This model makes pricing based on objective data. Objective data include the time value of the option, the current price of the asset on which the option is based; the strike price on the maturity date of the option; and the volatility of the asset price, which in turn can be regarded as the probability that the option can be executed. Despite many considerations, the model does not require a complex computational process to compute. However, the Black-Scholes model is not perfect. The model is limited to calculating European option prices. In some cases, the model cannot match actual market conditions, unrealistic factors include the following: the model assumes that interest rates are risk-free; volatility is known and constant; pricing does not take into account transaction costs or taxes; pricing does not take into account any dividends that may be received by holders of the underlying asset.

Implied volatility reflects the level of uncertainty or risk in the market and typically affects option prices. Implied volatility is calculated by substituting the traded option price into the price model and inversely deriving the volatility value. First, a Black-Scholes model is used to calibrate implied volatility. The stock prices, expiration date, and strike prices of all ten options used to calibrate implied volatility are collected, and all stock prices from June 16 to July 18 are collected, and their standard deviation is calculated to serve as the σ in the Black-Scholes model with the formula below. σ represents the implied volatility in the calibrated model but should be assumed as a number to be substituted in the calibration process, t represents the time to maturity, $S(t)$ represents the stock price, K represents the strike price of each option, r represents the interest rate of each option, here it is assumed to be zero as for the time to maturity is quite short. Equation (1) and equation (2) below calculate the prices of call options and put options respectively, equation (3) and equation (4) below represent the formula to calculate $d1$ and $d2$, which are the probability factors in the Black-Scholes model, and $N(d1)$ and $N(d2)$represent the normal distribution of d1 and d2 respectively (L.S. Lima, 2021; S. Ampun, 2021).

$$C = N(d1)S(t) - N(d2)Ke^{-rt} \tag{1}$$

$$P = N(-d2)Ke^{-rt} - S(t)N(-d2) \tag{2}$$

$$d1 = \frac{Ln\left(\frac{S(t)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \tag{3}$$

$$d2 = \frac{\ln\left(\frac{S(t)}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} = d1 - \sigma\sqrt{t} \tag{4}$$

The option prices calculated by the Black-Scholes model will be compared with the actual market prices of options, and the sum of standard error will be calculated by equation (5), where $P_i(B)$ represents option prices that are calculated by the Black-Scholes model, $P_i(M)$ represents market prices of options. The implied volatility will be calibrated by minimizing the sum of standard error.

$$\sum_{i=1}^{J} \frac{(P_i(B) - P_i(M))^2}{P_i(M)} \tag{5}$$

The second, the Cox-Ross-Rubinstein model, also named the binomial tree model, supposes that the stock price fluctuates only in two directions, up and down, and assumes that the range of up or down fluctuations in the stock price remains unchanged during the whole period. The model will divide the entire period into several stages, simulate all possible development paths of the underlying assets during the whole duration based on the historical volatility of the stock price, and calculate the option exercise profit and usage for each node on each path. The option price is calculated by the discount method. Compared with the Black-Scholes model, the option pricing by the Cox-Ross-Rubinstein model is more intuitive and simpler to calculate, it can be applied to the pricing of European options, American options, and some other options. Moreover, the Cox-Ross-Rubinstein model takes into account the interest rates and dividends available to the underlying holders. The disadvantage is that, when there are too many stages, that is, the step size is too large, which will cause calculation difficulties; when there are too few stages, that is, the step size is too small, will reduce the accuracy, and the gap between the market price and the actual price will inevitably be large.

To be started, a Cox-Ross-Rubinstein model is used to calibrate the implied volatility. Since the time to maturity is relatively short, the 2-step binomial option pricing model is used here, and the model has still assumed the risk-free rate, which is denoted by $r = 0$. In the Cox-Ross-Rubinstein model, Δt represents the expiration time corresponding to the options in each stage, u represents the multiplier when the stock rises, d represents the multiplier when the stock falls, and p represents the probability of the stock rising. Equation (9) is used to calculate the stock price in each layer, x represents the number of times the stock increases, and $S(0)$ is the original stock price corresponding to the option. After that, compare the calculated stock price with the strike price to calculate the profit of option execution or non-execution, calculate the probability of occurrence of each income through the binomial formula, and add the expected value of the profit to obtain the final option price.

$$u = e^{\sigma\sqrt{\Delta t}} \tag{6}$$

$$d = e^{-\sigma\sqrt{\Delta t}} = \frac{1}{u} \tag{7}$$

$$p = \frac{e^{r\Delta t} - d}{u - d} \tag{8}$$

$$S(x) = S(0)(u^x)(d^{(2-x)}) \tag{9}$$

The option prices calculated by the Cox-Ross-Rubinstein model are compared with the actual market prices of options, and the sum of standard error will be calculated by equation (5) above. Again, $P_i(B)$ represents option prices that are calculated by the Black-Scholes model, and $P_i(M)$ represents the market prices of options. The implied volatility will be calibrated by minimizing the sum of standard error.

Hedging strategies are formed against specific options in Unilever stock after calibrating for implied volatility. Then the collected Unilever PLC stock price from June 27, 2022, to July 8, 2022, and the implied volatility calibrated with each of the two models are substituted into the following equations. Equation (10) and equation (11) represent the portfolio value of day 1 and each day after the first day1, where $N(d1)$ in equation (11) uses the same $d1$ as the $d1$ in equation (3) and the algorithm is the same as the Black-Scholes model, $N(d1)$ represents the delta value of the option. The value of $N(d1)$ changes with the changes in data each day, thus it needs to be recalculated every day. Equation (12) and equation (13) represent the loss without hedging and the loss without hedging for each day.

Day1(June 27,2022)

$$Portfolio\ value\ X(1) = C(1, S(1)) \quad (10)$$

Days after day1(June 28,2022-July 8,2022)

$$portfolio\ value\ X(t) = X(t-1) + N(d1)(S(t) - S(t-1)) \quad (11)$$

$$Loss\ without\ hedging(t) = S(t) - K - C(1, S(1)) \quad (12)$$

$$Loss\ with\ hedging(t) = S(t) - K - X(t) \quad (13)$$

Compare the loss with hedging and the loss without hedging calculated by the equations above, to examine the difference between the two and the effect of hedging.

3 RESULTS AND DISCUSSION

3.1 Results

First, substituting the implied volatility calculated by the Black-Scholes model to calculate the profit before and after hedging within the specific option of Unilever PLC between June 27th, 2022, and July 11th, 2022. The comparison of trends for the two is shown below.

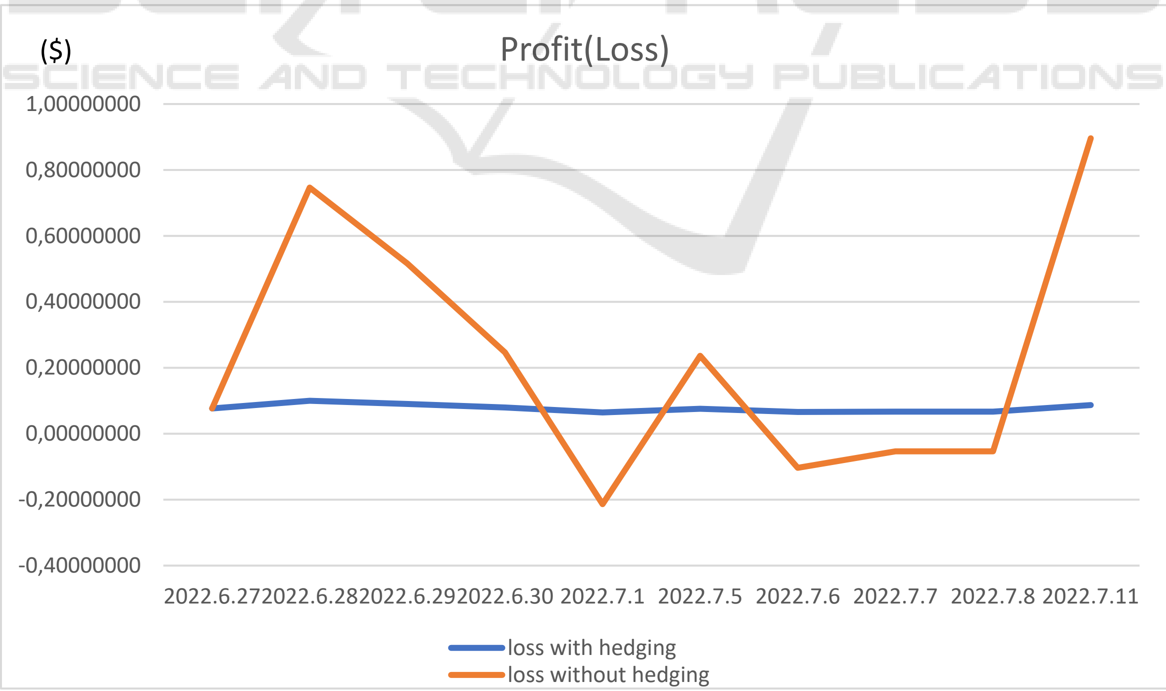

Figure 2: Profit(loss) of unilever plc portfolio with calibration by black-scholes model.

The yield curve with hedging is shown close to a straight line, parallel to the x-axis, which indicates that the effect of hedging is relatively good, and the volatility of returns is effectively reduced, which also implies that the risk of options is reduced, and the risk is close to zero, which avoids the loss of the option. However, the disadvantages also emerged. As the risk approaches zero, the option profit also decreases. The profit with hedging was basically below 0.2, while the high point of the income before hedging is around 0.8, and the frequency was twice. In the statistics of time, the profits with hedging are higher than the profits without hedging in only four days, and the profits without hedging of these four days are in a state of loss. However, although there are four days of negative option returns, it still does not affect the sum of benefits brought by high returns without hedging is higher than the sum of returns with hedging.

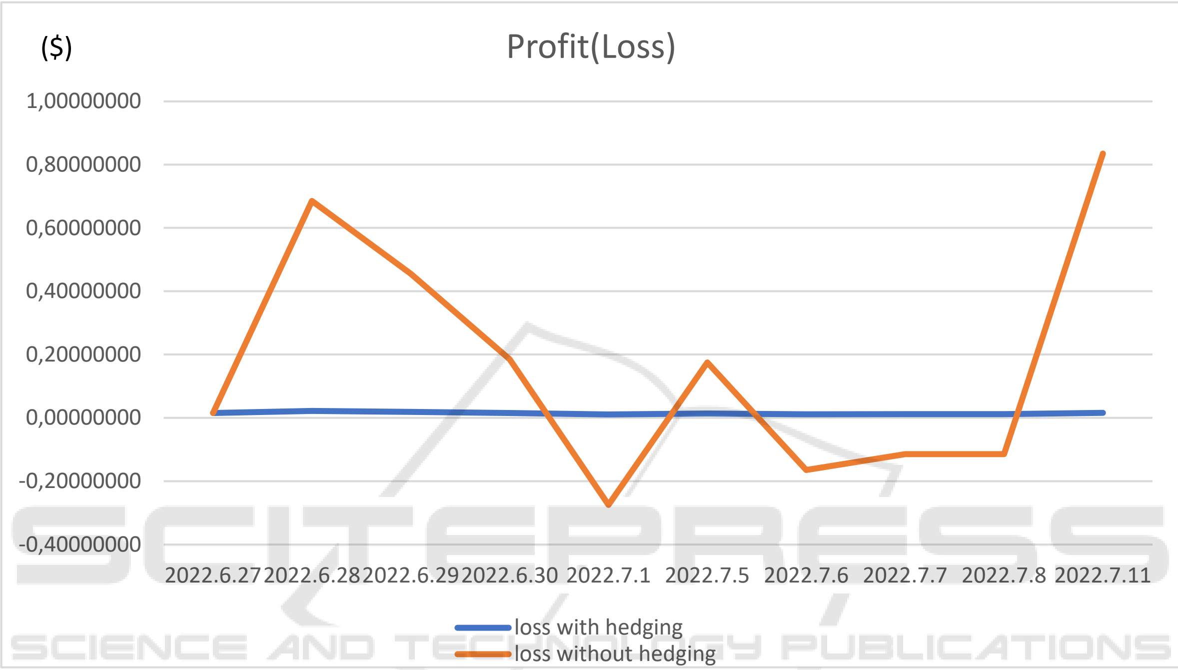

Figure 3: Profit(loss) of unilever plc portfolio with calibration by cos-ross-rubinstein model.

Option hedging strategies based on implied volatility calibrated with a Cox-Ross Rubinstein model also performed well. Its performance is roughly equivalent to the option hedging strategy based on the implied volatility calibrated by the Black-Scholes model, and profit with hedging is basically a straight line parallel to the x-axis, which proves that it successfully hedged the fluctuation of the option price and keeps the return value at a positive number, while it also means that the risk is reduced, and the possibility of high returns is also reduced at the same time. As shown in the figure, although the hedging returns continue to remain positive, the returns are close to zero. From the comparison of the curve amplitudes in the graph, the hedging strategies based on Cox-Ross-Rubinstein model-calibrated implied volatility seem to have lower returns than that of hedging strategies based on Black-Scholes model-calibrated implied volatility, which also means that their returns are lower than hedges former strategy.

Table 5: The comparison of profits with hedging and profits without hedging of the specific option.

($)	2022/6/27	2022/6/28	2022/6/29	2022/6/30	2022/7/1	2022/7/5	2022/7/6	2022/7/7	2022/7/8	2022/7/9	sum
Profit with hedging (Black-Scholes model)	0.076504	0.100322	0.090073	0.079625	0.064559	0.075890	0.066005	0.067168	0.067168	0.086779	0.774094
Profit without	0.076504	0.746504	0.516504	0.246504	(-)0.213496	0.236504	(-)0.103496	(-)0.053496	(-)0.053496	0.896504	2.295039

hedging (Black-Scholes model)											
Profit with hedging (Cox-Ross-Rubinstein model)	0.014980	0.022062	0.018612	0.015361	0.011089	0.013859	0.0112411	0.011513	0.011513	0.015781	0.146010
Profit without hedging (Cox-Ross-Rubinstein model)	0.014980	0.684980	0.454980	0.184980	(-)0.27502010	0.174980	(-)0.165020	(-)0.115020	(-)0.115020	0.834980	1.6797990

As can be seen from the figure, the total profit brought by the hedging strategy using the implied volatility calibrated by the Black-Scholes model from June 2, 2022, to July 9, 2022, is $0.774094, the non-hedging yield is $2.295039, which is higher than the yield after hedging. The total income brought by the hedging strategy using the implied volatility calibrated by the Cox-Ross-Rubinstein model from June 2, 2022, to July 9, 2022, is $0.146010, and the non-hedging yield is $1.6797990, which is also greater than the profits with hedging. In contrast, its profit with hedging is lower than that of the hedging strategy based on the implied volatility calibrated by the Black-Scholes model.

3.2 Discussion

As for the results, hedging strategies constructed from the implied volatility calibrated by the two models performed well on the specific options of Unilever PLC. This is reflected in the fact that both hedging strategies keep the option return at a positive value, and the return level is stable, a little higher than zero, which means that the risk is well hedged. In order to illustrate this point, first, volatility generally reflects risk, and a relatively stable yield curve can better reflect that risk has indeed been reduced. Second, delta can also be understood in options calculation as always, the probability of an option that can benefit or lose. For instance, for an at-the-money option, the delta value is generally around 50%. This is because the stock price is equal to the strike price of the option at a certain moment, and at the next moment, the stock price is equal to the strike price of the option. It may go up or down, and the probability of both is 50%. If the stock price increases, then this at-the-money call option becomes an in-the-money call option, and the delta value will be higher than 50%. This option has a greater than 50% probability of being exercised and profiting. If the stock price decreases at the next moment, this option will become an out-of-the-money call option, and the delta value is less than 50%, indicating that this option has a less than 50% chance of being exercised and benefiting. The option selected in this paper is an in-the-money call option with a delta value higher than 50%, which implies that it has a higher than 50% probability of being exercised and benefiting on the expiration date. At the same time, the object of this paper is Unilever PLC to represent the FMCG industry. From July 2022 to August 2022, the stock prices of most FMCG companies, including Unilever PLC, have a slight upward trend. It can be seen from the Unilever PLC stock price trend table in the data section of this paper that when Unilever PLC's share price rises, the delta value of the selected in-the-money call option will inevitably rise, and the possibility of profit is greater. When it is less than the stock price at a certain moment, the possibility of making a profit is also inevitable, which also shows that the options profit without hedging increases and is greater than the profits with hedging. The hedging strategy made in this paper is to neutralize the delta value and make it zero, which reduces the risk and also inhibits the possibility of profiting by options. Therefore, when the stock price is known to be rising and the object of the hedging strategy is in the case of in-the-money call options, it is inevitable that the return after hedging is less than the return before hedging, and the two hedging strategies with different implied volatility make the level of the return curves after hedging close to 0, indicating that they are well achieved for the purpose of hedging delta.

In addition, it can be seen from the Results section that the hedging strategies constructed by the implied volatility calibrated by the two different models have different returns. The implied volatility calibrated by the Black-Scholes model is considered to have higher accuracy because the Black -Scholes model itself has the function of calibrating the implied volatility, and the value of delta calculated in the hedging method used in this paper is different from Black-Scholes are closely related. The Cox-Ross-Rubinstein model may not be very accurate in calculating the data in the paper. This is because of its model characteristics. The more stages this model has, the more accurate the calibrated implied volatility will be. However, given that the option expiration time is too short, this paper adopts a 2- step volatility binary tree model, which may cause certain errors in the calculation of implied, hence the returns calculated by the two hedging strategies with different implied volatility are slightly different, however, it does not affect the good effect of the hedging strategy constructed by the two.

4 CONCLUSION

Currently, option pricing and risk hedging are interesting topics in the financial field. In this paper, we combine the two issues for the Unilever PLC stock. The empirical processes can be summarized as follows. First, relevant data on the targeted asset is carefully selected. Second, the Black Scholes Model and Binomial Tree model are applied. Finally, the hedging performance is compared for the two models, and the results show that the related investors may benefit from the hedging strategies when investing Unilever PLC. However, deficiencies exist. For example, option pricing models and hedging strategies have numerous alternatives, in this paper, limited methods are adopted, thus, applying other models deserves further investigations.

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