Option Pricing and Risk Hedging in Current Financial Market: A Case for Pfizer

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Abstract: The research background of this paper is that in stock option trading, the risk is weakened, and the profit level is improved by means of hedging. This paper discusses and analyzes the differences in the hedging results of stock options of the same company based on different option pricing models. The data adopts the stock option trading data of Pfizer, and uses the historical data model, BS (black-Scholes) model, and BT (Binomial Tree) model to construct an option pricing model and calibrate it, and finally perform delta hedging. The results show that the historical data model shows the best results in the final hedging, the hedging results of the BS model are in the middle level, while the hedging results of the BT model are not satisfactory in this study. This research appears in stock option trading, and the investment behavior of single company stock option as the target weakens the comprehensive risk and improves the comprehensive profit level.

1 INTRODUCTION

Beginning in the second half of 2019, Covid-19 spread recklessly around the world, which has brought huge negative impact on people's daily life and production and construction. With the development and production of vaccines and antiviral drugs, people are working hard to fight and defeat the virus. In this process, pharmaceutical companies have become bridgeheads to overcome difficulties, and have introduced a large amount of funds for scientific research and development and drug production. Therefore, in the post-epidemic era, companies in the field of big health are generally beneficial. As one of the top pharmaceutical companies, Pfizer has become the most favored company by investors in the pharmaceutical industry. However, with the influx of more and more investors, the phenomenon of blind investment and follow-up buying continues to appear. People are superstitious about the long-term benefits of the pharmaceutical industry, while ignoring the objective risks in stock option trading. Therefore, it is very important to choose a suitable option pricing model and a reasonable hedging strategy, which will help to reduce risks and improve returns.

The research on stock option pricing model and hedging strategy is not a new topic in the industry. Around this center, in the current field, many scholars have expressed their views on option pricing models and their understanding of hedging strategies. Li Xu, Shijie Deng, Valerie M. Thomas co-published article and hold the idea that options that the impact of market volatility on option prices is divided into effectiveness and destructiveness (Xu, 2016). Ghulam Sarwar wrote that there is contemporaneous positive feedback between the volatility of the exchange rate and the trading volume of call and put options (Sarwar, 2003). The paper completed by Jie cao and Bing Han pointed out that when using delta to hedge stock options, as the heterogeneous volatility of the underlying stock increases, returns show a monotonically decreasing trend (Cao, 2013). Research by Gurdip Bakshi and Nikunj Kapadia found that delta-hedged portfolio returns are correlated with changes in the volatility risk premium (Bakshi, 2003). When Erik Ekström and Johan Tysk studied stochastic volatility and the Black-Scholes equation, they found that option prices are the only classical solution to parabolic differential equations, and this solution is bounded (Ekström, 2010). Yisong Tian's research pointed out that the calibrated binomial model can recalibrate the binary tree through the skew parameter, this method allows the

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location of barrier location nodes based on different option form prices (Tian, 1999).

The propositions and findings of the above scholars are worth studying and learning. On this basis, this paper sets the research focus on the differences in the performance of hedging strategies based on different option pricing models when investing in stock options of the same company. Taking Pfizer's stock option positioning data as the object, using historical data model, Black-Scholes model and Binomial Tree model to construct and calibrate three different option pricing models, and then perform hedging on this basis, and finally compare the results.

2 DATA COLLECTION

The data of this project is all from Yahoo Finance (https://ca.finance.yahoo.com), and the stock price data of Pfizer Inc. during the ten working days from June 6 to June 17, 2022, and the transaction on June 17, the maturity time is unified There are five calls and puts on July 1. In addition, option A selects a put option on July 29 with a relatively late maturity date. The reason why choose Pfizer Inc. is, Pfizer Inc. is a stock that is publicly traded in the United States, and its stocks have corresponding option trading behavior. The selected options data and historical stock prices are as follows in table 1.

Table 1: The 5 call and 5 put options chosen.

	Call options chosen	Put options chosen
Pfizer Inc.	PFE220701 C00045000	PFE220701 P00045000
	PFE220701 C00046000	PFE220701 P00046000
	PFE220701 C00046500	PFE220701 P00046500
	PFE220701 C00047000	PFE220701 P00047000
	PFE220701 C00047500	PFE220701 P00047500

Table 2: The Option A

Option A	PFE220729P0050000

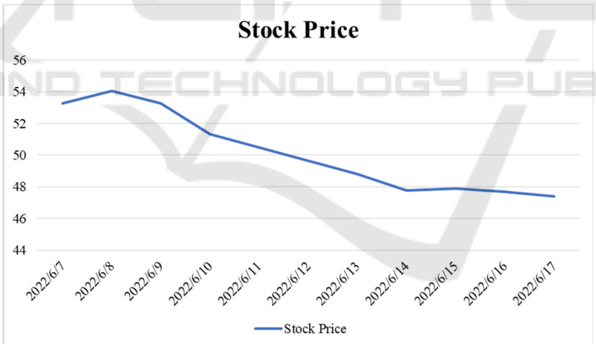

Figure 1: The historical stock price if Pfizer Inc.

This icon shows and indicates that Pfizer's stock traded continuously during the trading day from June 7th to June 17th, and the stock traded at relatively random prices.

3 METHODS

After selecting the above data, use the data to construct three different option pricing models. According to the different theoretical basis of the models, the three models are divided into historical data model, BS model, and BT model. The construction ideas of the three models are roughly similar, the difference is that the volatility is slightly different. The historical data model uses historical volatility, while the BS and BT models use implied volatility.

The Black-Scholes model, also known as the Black-Scholes-Merton (BSM) model, is one of the most important concepts in modern financial theory. This formula is based on consideration of the theoretical value of derivatives of investment instruments, other risk factors that affect value, such as time, are also considered (Hayes, 2022). Since its birth in 1973, it has been an important application tool in the history of options contract pricing. The binomial option pricing model, another important method for option valuation, was developed in 1979. The core of this model is the usage iteration process, which allows arbitrary nodes or points in time within the time span between the valuation date and the option expiration date (Chen, 2019). In this study, three different option pricing models are used to complete the hedging of option A to minimize the impact of errors on the final data.

After the option pricing model is established, the model needs to be calibrated with data, and all three models use $\boldsymbol{\sigma}$ calibration. In the calibration model stage, the ideas of the three model calibrations are relatively unified, but they tend to be diversified in performance. The idea is to bring the variable $\boldsymbol{\sigma}$ into one or more formulas that can reflect the size of the error value and bring it in according to the collected data to form a formula based on the data and the value of $\boldsymbol{\sigma}$. The result of the reaction is that under different data, the error The size of the value. After that, set $\boldsymbol{\sigma}$ as the independent variable, set the formula representing the error value and its result as the dependent variable, perform the calibration operation, and obtain the $\boldsymbol{\sigma}$ when the minimum error value is obtained, which is the final purpose of this step. Detailed model specifications are shown below.

In Historical Data Model, S represents the stock price, and k represents the date, which starts from June 6 to June 17. Then k+1 represents the second day based of the k. First, use equation (1) to get the logarithm values of stock prices on each date. Then use equation (2) to calculate the difference of logarithm values between every two days. All the results of equation (2) are used to take the variance value of equation (3). Because the trading of stocks and options only occurs on trading days, assuming that there are 251 trading days in a year, bring it into equation (4) to get the final $\boldsymbol{\sigma}$ value.

$$\mathrm{A} = \log S_k \tag{1}$$

$$\mathrm{R} = \log(S_{k+1}) - \log S_k \tag{2}$$

$$Var = var(\log(S_{k+1}) - \log S_k) \tag{3}$$

$$\sigma = \sqrt{Var \times 251} \tag{4}$$

In BS Model, σ represents implied volatility, t represents time to maturity, S(t) represents the stock price, K represents the strike price of the option, and r represents the interest rate, which is assumed to be 2.8% in this model (The Fed - Selected Interest Rates (Daily), 2020). First use equation (5) and equation (6) to calculate value of d_1 and d_2 which are the required elements for further calculation. Equation (7) and (8) calculate the call and put price of the option, and the $N(d_1)$ represents the normal distribution of d_1 , $N(d_2)$ represents the normal distribution of d_2. After getting the values of the call and put options, put them into equation (9) to calculate the SSE (Residual sum of squares) error. C represents the call option prices, which calculate from equation (7), P represents the put option prices, which calculate from equation (8), and P_m represents the corresponding market values. At last, when the SSE value is the smallest, the desired σ value can be obtained.

$$d_1 = \frac{1}{\sigma\sqrt{(t)}} \ln\left(\frac{S(t)}{K} + \left(r + \frac{\sigma^2}{2}\right)t\right) \tag{5}$$

$$d_2 = \frac{1}{\sigma\sqrt{(t)}} \ln\left(\frac{S(t)}{K} + \left(r - \frac{\sigma^2}{2}\right)t\right) = d_1 - \sigma\sqrt{t} \tag{6}$$

$$C\big(t, S(t)\big) = S(t)N(d_1) - Ke^{-rt}N(d_2) \tag{7}$$

$$P\big(t, S(t)\big) = Ke^{-rt}N(-d_2) - S(t)N(-d_1) \tag{8}$$

$$SSE = \frac{(P_1 - P_1^m)^2}{P_1^m} + \frac{(P_2 - P_2^m)^2}{P_2^m} \tag{9}$$

In the BT Model, u represents the up coefficient in binomial tree, d represents the down coefficient, t represents time to maturity, p represents option prices, and r represents the interest rate, which is assumed to be 2.8% in this model. Calculate the up and down coefficients of each level of the binomial tree according to equations (10) and (11). Then bring each coefficient into equation (12) to get the prices of the option. Using equation (13), the calculated option price and the corresponding stock price are brought in to calculate the SSE error, when the SSE value is the smallest, the desired σ value can be obtained (Muroi, 2022).

$$u = e^{\sigma\sqrt{t}} \tag{10}$$

$$d = \frac{1}{u} \tag{11}$$

$$p = \frac{e^{rt} - d}{u - d} \tag{12}$$

$$SSE = \frac{(P_1 - P_1^m)^2}{P_1^m} + \frac{(P_2 - P_2^m)^2}{P_2^m} \tag{13}$$

After the above calculation process, the obtained $\boldsymbol{\sigma}$ value can be the variable value used after the model has been calibrated. After the model has been calibrated, it can be combined with Option A for hedging. All three models use the delta hedging method. In this step, the hedging process of the historical data model and the BS model is the same, and the BT model is slightly different. For BS model, p(t) represents the portfolio value, and using equation (14) to get each p(t), then using equation (15) to finish the hedging process. The BT model will additionally use equation (16) in the hedging phase, where S represents the option price. Collect the results of the three model hedges, compare them, and draw a conclusion.

$$p(t) = p(t-1) + N(d_1)(S(t) - S(t-1)) \tag{14}$$

$$Loss\ with\ hedging(t) = S(t) - K - p(t) \tag{15}$$

$$Hedge\ ratio = \frac{D_u - D_d}{S_u - S_d} \tag{16}$$

4 RESULTS

According to the numerical results, the three models have played a practical role in the manifestation of the hedging effect, and the historical data model has the best final effect in the presentation of the results.

Table 3: The gain/loss of holding one unit of the option on Pfizer Inc.'s stock with hedging by historical data model.

	Days	T-t	T_A	Stock	A(put)	Delta_A(put)	Sell/Buy stock	Cash
2022/6/6	1	39	0.1553785	53.19	1.338987	-0.285192551	0.285192551	16.508378
2022/6/7	2	38	0.1513944	53.28	1.284773	-0.279170686	-0.006021865	16.189375
2022/6/8	3	37	0.1474104	54.06	1.053795	-0.24084287	-0.038327816	14.119179
2022/6/9	4	36	0.1434263	53.27	1.229061	-0.276060889	0.035218019	15.996818
2022/6/10	5	35	0.1394422	51.31	1.83916	-0.381873758	0.105812868	21.427861
2022/6/13	6	34	0.1354582	48.82	2.950776	-0.540372814	0.158499056	29.168176
2022/6/14	7	33	0.1314741	47.75	3.536934	-0.613388331	0.073015517	32.65792
2022/6/15	8	32	0.12749	47.88	3.427221	-0.607455173	-0.005933157	32.377484
2022/6/16	9	31	0.123506	47.69	3.5134	-0.623120722	0.015665548	33.128186
2022/6/17	10	30	0.1195219	47.38	3.679893	-0.647319738	0.024199016	34.278431

According to the table above, the result of hedging using the historical data model shows that the final loss is $0.071472 per unit.

Table 4: The gain/loss of holding one unit of the option on Pfizer Inc.'s stock with hedging by BS model.

	Days	T-t	T_A	Stock	A(put)	Delta_A(put)	Sell/Buy stock	Cash
2022/6/6	1	39	0.1553785	53.19	1.233434	-0.278295356	0.278295356	16.035964
2022/6/7	2	38	0.1513944	53.28	1.181519	-0.272043762	-0.006251594	15.704668
2022/6/8	3	37	0.1474104	54.06	0.958312	-0.232535959	-0.039507803	13.570628
2022/6/9	4	36	0.1434263	53.27	1.129156	-0.268778694	0.036242735	15.502792
2022/6/10	5	35	0.1394422	51.31	1.730362	-0.378709777	0.109931083	21.145086
2022/6/13	6	34	0.1354582	48.82	2.844506	-0.544474042	0.165764266	29.240056
2022/6/14	7	33	0.1314741	47.75	3.438411	-0.620624141	0.076150098	32.879485
2022/6/15	8	32	0.12749	47.88	3.329489	-0.614415597	-0.006208543	32.585888
2022/6/16	9	31	0.123506	47.69	3.418796	-0.630684062	0.016268465	33.365367
2022/6/17	10	30	0.1195219	47.38	3.589578	-0.655785007	0.025100944	34.558372

According to the table above, the result of hedging using the BS model shows that the final loss is $0.102301 per unit.

Table 5: The gain/loss of holding one unit of the option on Pfizer Inc.'s stock with hedging by BT model.

	Days	T-t	T_A	Stock	A(put)	Delta_A(put)	Sell/Buy stock	Cash
2022/6/6	1	39	0.1553785	53.19	0.733978	-0.232385794	0.232385794	13.094578
2022/6/7	2	38	0.1513944	53.28	0.694675	-0.22488392	-0.007501873	12.696339
2022/6/8	3	37	0.1474104	54.06	0.519476	-0.1794846	-0.04539932	10.243468
2022/6/9	4	36	0.1434263	53.27	0.659125	-0.220793512	0.041308912	12.445137
2022/6/10	5	35	0.1394422	51.31	1.19646	-0.355724826	0.134931314	19.369851
2022/6/13	6	34	0.1354582	48.82	2.320051	-0.570073709	0.214348883	29.836524
2022/6/14	7	33	0.1314741	47.75	2.960814	-0.66647809	0.096404381	34.443162
2022/6/15	8	32	0.12749	47.88	2.854744	-0.658574284	-0.007903806	34.06857
2022/6/16	9	31	0.123506	47.69	2.961679	-0.678597143	0.020022859	35.027261
2022/6/17	10	30	0.1195219	47.38	3.157355	-0.709127711	0.030530568	36.477707

According to the table above, the result of hedging using the BT model shows that the final loss is \$0.278274 per unit.

5 DISCUSSION

As shown in the results section, the results of the hedging strategy based on the historical data option pricing model are the best, the results of the hedging strategy based on the BS model are second only to the former, and the results of the hedging strategy based on the BT model are not satisfactory. The methods of historical data and BS models in the hedging stage are delta hedging, so the factor that affects the hedging result is the definition of delta value. In the evaluation method of delta, the historical data model is obviously more accurate. The values of the parameters of this model come from the actual stock price and option data. The BS model is subject to its formula principle, and the calculation steps of many parameters are more responsible, and the occurrence of errors can also be attributed to this. When the BT model is established in the option pricing model, the value of $\boldsymbol{\sigma}$ after calibration is different from the value of the historical data model and the BS model. At the same time, the hedging process of the BT model is slightly different from the above two models in steps, and the calculation steps are slightly more. As the calculation steps increase, the possibility of the existence of errors also increases. The comprehensive analysis of the BT model is unsatisfactory in the final hedging results because of the large error of the $\boldsymbol{\sigma}$ value and the added hedging step also increases the error.

6 CONCLUSION

This paper studies the use of different methods to establish option pricing models for a single company's stock, and conducts hedging, and analyzes and discusses the different performances of different pricing models in hedging. This paper selects the options of Pfizer Inc. stock and uses historical data, BS model, and BT model to establish option pricing models. Use the Historical Volatility method to calibrate the historical data model and use the Implied Volatility method to calibrate the BS model and BT model. After calibration, the historical data model, the BS model, and the BT model use delta hedging as the hedging method to complete the hedging process. When comparing the results, the hedging results of the historical data model performed the best, with the smallest amount of loss per unit. This study emphasizes that when hedging stock options of the same company, hedging strategies based on different option pricing models perform differently, which is beneficial for investors to flexibly use different hedging strategies and option pricing models for portfolio investment.

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