Performance of Delta-Hedging on Black-Scholes Model and Heston Model

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Abstract: This paper mainly studies the Black-Scholes model, Heston model and their delta hedge, using the same data about stock and option price of Ford Motor, which is useful for investors and company managers to build their portfolios. In this study, unknow volatility is firstly assumed for the sake of simplicity. Then through calculating, the theoretical volatility is estimated for establishing the model. Finally, after two models having been established, the delta-hedging performances on them are compared in terms of the total gain or loss. Relatively, the profit Heston model brings to the company is higher than the Black-Scholes model and the error sum of squares of Heston model is lower than the Black-Scholes model. The results confirm that choosing Heston model is more beneficial than BS model to investors for processing the data of Ford Motor and gets a more accurate prediction.

1 INTRODUCTION

Black-Scholes model and Heston model are both the most important achievements in modern economics which mainly discuss the method of determining option value (Wu, 2004). In addition, Venture capital is a comprehensive investment system, closely related to high-tech industry (Liu, 2022).

Hence, In the fierce stock and option market, the study of establishing models and delta hedging is significant. To illustrate, A large number of scholars have done profound research in this aspect in these years. Shiyu Wang studies and analyzes the pricing of European option in risk averse market (Wang, 2022), and Yanming Liu discusses the effect of option pricing theory on venture capital (Liu, 2022). Meanwhile, On the basis of option pricing model, some scholars have extended their research to other fields. For example, Wanshan Xie makes a task about the time fractional Asian option pricing problem based on high precision finite difference method and in another subfield (Xie, 2022), Yan Qing doed some numerical simulations and empirical analysis on Pricing model of European option based on radial function (Qing, 2022). What's more, Ziqi Lei and Qing Zhong provide an explanation about the option pricing based on uncertain fractional differential equation in the floating rate case (Lei, 2022).

Nevertheless, the research of Specific implementation of the company and reality to deal with the problem is rare, most of them focusing on the theoretical data construction. This paper based on the stock data of Ford Motor, compare the earnings of Black-Scholes model, and Heston model using delta hedging. To begin with, based on the logical structure and formulation of models, the parameters can be calculated and adjusted which refers to the establishment of models. After two models are both founded, the theoretical and practical value are compared to calculate the error sum of squares.Finally,delta hedging is used to calculate the gain or loss and analyze which model brings more profit to the company. The result of the study proves that the delta hedging strategy performs well on ford motor's stock options and Heston model relatively is more accurate and profitable.

This paper contains the following: Section 2 shows data and methods. Section 3 describes the results and discussion, and Section 4 gives the conclusion.

2 DATA AND METHODS

2.1 Data

Ford Motor is an enterprise with a long history and

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reputation. According to the 2021 annual financial results released by Ford China, the development of Ford in the past two years is quite good, with a total of 624,802 vehicles delivered in the whole year, with a year-on-year growth of 3.7%. Hence, the future growth trend of Ford stock is promising. Thus, in this paper, the Fort Motor is selected as the research target. The data selected is from wind (https://www.wind.com.cn) and Yahoo Finance (https://ca.finance.yahoo.com). The stock of Ford motor is collected from June.22th, 2021 to June.22th,2022 and five call options and five put options on Ford motor are used to adjust parameters to build models, namely to estimate an annual volatility for two different option pricing models, which are from June.1th, 2022 to June.22th, 2022.After establishing models, a hedging strategy is constructed for the Black-Scholes model and Heston model from June.2th, 2022 to June.22th, 2022. In general, the collected data is shown in the table below.

Table 1: Partial screenshot of stock data of Ford Motor.

Date	Open	High	Low	Close	Adj Close	Volume
2022/6/1	13.88	13.97	13.4	13.55	13.55	50726200
2022/6/2	13.64	13.96	13.6	13.89	13.89	42979700
2022/6/3	13.63	13.78	13.36	13.5	13.5	43574400
2022/6/6	13.74	13.74	13.38	13.46	13.46	37711100
2022/6/7	13.26	13.77	13.19	13.74	13.74	38940300
2022/6/8	13.63	13.85	13.44	13.53	13.53	39441900
2022/6/9	13.51	13.59	13.28	13.28	13.28	30468000
2022/6/10	13	13.21	12.63	12.75	12.75	55644400
2022/6/13	12.3	12.38	11.74	11.81	11.81	80676300
2022/6/14	11.99	12.42	11.91	12.2	12.2	82369300
2022/6/15	12.22	12.42	12	12.27	12.27	70393200
2022/6/16	11.8	11.91	11.12	11.25	11.25	80380100
2022/6/17	11.24	11.44	10.9	11.23	11.23	80166800
2022/6/21	11.55	11.66	11.35	11.46	11.46	65671600
2022/6/22	11.55	11.42	11.21	11.375	11.375	3182685

This a partial screenshot of stock data of Ford Motor from June.1st,2022 to June.22nd, 2022. As shown above, the stock price fluctuated between 10 and 14 during this period and the overall trend is downward. And to calculate the parameters of Black-Scholes model and Heston model, making sure the error is minimal, the data of options on June.1th,2022 is chosen for numerical modeling. The following table shows the options collected for calculating volatility.

Table 2: The 10 options used for calculating volatility.

Call option		
2022/6/1	strike price	option price
	12	1.96
	13	1.49
	13.5	0.86
	14	0.58
	14.5	0.47
	Put option	
2022/6/1	strike price	option price
	11.5	0.1
	12	0.13
	13	0.32
	13.5	0.49
	14	0.67

As shown above, the strike prices set for two different options respectively are 12-14.5, decreasing by 0.5 for call option and 11.5-14 increasing by 0.5 for put option. Moreover, according to the model built on data on June.1st,2022, the delta hedge of two options is calculated for the profit and loss. The parameters in hedging strategy are shown below.

Table 3: Parameters in hedging strategy.

date	strike price(K)	option price	type
2022/6/2	12.5	1.48	call
2022/6/3	12.5	1.32	call
2022/6/6	12.5	1.34	call
2022/6/7	12.5	0.97	call
2022/6/8	12.5	1.19	call

2022/6/9	12.5	1.17	call
2022/6/13	12.5	0.41	call
2022/6/14	12.5	0.28	call
2022/6/15	12.5	0.34	call
2022/6/16	12.5	0.17	call
2022/6/17	12.5	0.08	call
2022/6/21	12.5	0.03	call
2022/6/22	12.5	0.02	call
date	strike price(K)	option price	type
2022/6/2	12.5	0.19	put
2022/6/3	12.5	0.24	put
2022/6/6	12.5	0.23	put
2022/6/7	12.5	0.25	put
2022/6/8	12.5	0.13	put
2022/6/9	12.5	0.16	put
2022/6/10	12.5	0.26	put
2022/6/13	12.5	0.57	put
2022/6/14	12.5	0.71	put
2022/6/15	12.5	0.5	put
2022/6/16	12.5	0.9	put
2022/6/17	12.5	1.33	put
2022/6/21	12.5	0.9	put
2022/6/22	12.5	1.09	put

As shown above, strike price is set to 12.5 and option data is form June.2nd,2022 to June.22nd,2022.

2.2 Methods

To begin with, Black-Scholes model and Heston model are both one of the most popular mathematical models which are used for the option pricing in contracts. (MacBeth, 1979; Backus, 2004; Bohner, 2009).

In this study, to compare the advantages and disadvantages of the two models, the profit and loss by delta hedging of these two models are calculated and analyzed. There are three steps which are essential process the study needs.

Firstly, the Black-Scholes model is established according to the data of option price on June.1st, 2022.It was proposed by Black and Scholes in the 1970s. According to this model, only the current value of the stock price is related to the future forecast. The past history and evolution of variables are not correlated with future predictions. And there are a couple of assumptions if the model is set up. It assumes no arbitrage pricing and the volatility of the stock is constant and follows normal distribution. Using Black-Scholes model to price an option has to do with five parameters--current price, strike price, volatility, risk-free return and time to maturity. Volatility means the annualized standard deviation of stock prices. Risk-free return means the rate of return you can get from investing money in a risk-free investment. The formulas of call and put option are here.

$$Call: S(t)N(d_1) - Ke^{\wedge}(-r(T - t))\, N(d_2) \quad (1)$$

$$Put: -Se^{\wedge}(-D(T-t))\, N(-d_1) + Ke^{\wedge}(-r(T - t))\, N(d_2) \quad (2)$$

$$d_1 = 1/(\sigma\sqrt{}(T-t))[\log(S(t)/k) + (r + 1/2\,\sigma^{\wedge}2)(T-t) \quad (3)$$

$$d_2 = 1/(\sigma\sqrt{}(T-t))[\log(S(t)/k) + (r - \sigma^{\wedge}2/2\,(T-t)]] = d_1 - \sigma\sqrt{}(T-t) \quad (4)$$

As the formulas shown above, S means current price of the stock, K refers to the strike price assumed, r means risk-free return, σ refers to the volatility.

After plugging in the data and recalculating the theoretical price, the sum of squares due to error can be got by using formulas below.

$$\sigma(sigma) = \sum_{I=1}^{M}(Pi(a) - 〖Pi〗^{\wedge}m)^{\wedge}2 / 〖Pi〗^{\wedge}m \quad (5)$$

Pi(a) means the theoretical price, and Pi(m) means the actual price. The steps above is all the process for establishing the Black-Scholes model.

Secondly, Heston model need to be established using the same data calculated. Due to the deficiency of the Balck-Scholes model in assumptions, subsequent scholars have continuously modified and improved the Balck-Scholes model. Heston model is an extension of Black-Scholes model.

Heston model is a model that introduces stochastic volatility on the traditional BS model. It assumes that the fluctuation of underlying is not a constant, but a random process of reverting to the mean. This process involves a long-run average of volatility and a rate of recurrence of this volatility. If the previous volatility is below the long-run average, then the model can adjust upward at a certain rate.

In general, Heston model assumes that the underlying asset price follows a Brownian motion, and the volatility is regarded as a stochastic process. Assume that the price and variance of the underlying asset follow the following diffusion process formula:

$$dS = \mu Sdt + \sqrt{}(vSdW\_t^{\wedge}(1)) \quad (6)$$

$$dv\_t = k(\theta - v\_(t))\, dt + \delta\sqrt{v\_t}\, dW\_t\hat{}2 \quad (7)$$

As the formulas shown above, S means current price of the stock, K refers to the adjusting speed, θmeans a long-run mean level, σ refers to the fluctuations in volatility.

And according to ITO's lemma, the variance of the option value at time t is C(S,v,t) as the formula below:

$$\frac{vS^2\partial^2 C}{2\partial S^2} + \frac{\rho\delta vS\partial^2 C}{\partial S\partial v} + \frac{\delta^2 v\partial^2}{2\partial v^2} + \frac{rS\partial c}{\partial S} - rC + \frac{[k(\theta - v) - \lambda]\,\partial \mathrm{C}}{\partial \mathrm{v}} + \frac{\partial C}{\partial t} = 0 \quad (8)$$

Analyzing this formula above, the model satisfy some other formulas below:

$$\lambda = \lambda(\mathrm{S}, \mathrm{v}, \mathrm{t}) = \mathrm{k}\sqrt{\mathrm{v}} \quad (9)$$

$$C(S, \infty, t) = S \quad (10)$$

$$\frac{rS\partial c}{\partial s}(S, 0, t) + \frac{k\theta\partial c}{\partial v(S, 0, t)} - rC(S, 0, t) + C_t(S, 0, t) = 0 \quad (11)$$

Considering that volatility risk is only related to volatility, the option price calculated in the risk-neutral case can be applied in practice, that is, ρ= 0, where:

$$dv_t = k^*(\theta^* - v_t)dt + \delta\sqrt{v_t dW_t^2} \quad (12)$$

$$Cov(W_t^1, W_t^2) = \rho dt \quad (13)$$

As the formulas shown above, k*= k + λ, θ*=kθ/k+1, ρ means the correlation between Wt1 and Wt2.

So, the partial differential equation solution of Heston model and part of the relationship between variables are got, then it's time to adjust the parameters. According to the partial differential equation of Heston's model, calculating the European option price needs to know the rate of adjustment, the mean of the long run level, fluctuations in volatility, variance, correlation, volatility risk, but due to the assumption that it is a risk-neutral world ,the volatility risk namely ρ is equal to 0.Hence,only five parameters are required.

Table 4: Stock data in past 14 day.

Date	Open	High	Low	Close	Adj Close	Volume
2022/5/17	13.34	13.53	13.16	13.53	13.53	50891400
2022/5/18	13.25	13.36	12.71	12.78	12.78	68362500
2022/5/19	12.64	13.12	12.63	12.85	12.85	58459600
2022/5/20	13.05	13.12	12.07	12.5	12.5	78183400
2022/5/23	12.64	12.95	12.5	12.83	12.83	51929600
2022/5/24	12.6	12.68	12.27	12.42	12.42	51082800
2022/5/25	12.33	12.81	12.32	12.71	12.71	41193100
2022/5/26	12.8	13.2	12.79	13.12	13.12	45709200
2022/5/27	13.26	13.63	13.24	13.63	13.63	54195700
2022/5/31	13.68	13.82	13.35	13.68	13.68	79689900

As the table shown above, the variance of open price in past 14 days calculated equals to 19.3%.

$$F(x) = \frac{1}{2} - (\frac{1}{\Pi}) \int_0^\infty Re[\frac{e^{iu(InK)}\varphi(u)}{iu}]du \quad (14)$$

$$k\theta > \frac{1}{2}\sigma^2 \quad (15)$$

Plugging in the data, the parameters rho, sigma, theta, kappa, and v can be estimated through formulas (8), (11), (12), (14), (15).

Thirdly, after establishing Black-Scholes model and Heston model, delta hedge is carried out on the basis of the two models, and based on the data of stock and option, the returns of the two models are estimated respectively. Delta hedging is the operation of keeping the Delta value of an asset portfolio close to zero. The value of delta is equal to the ratio of the change in the option price to the change in the underlying asset price, which aims to reduce the impact of the change in the underlying asset price on the asset portfolio.

$$delta(\Delta) = \frac{\partial \Pi}{\partial S} \quad (16)$$

As the formulas shown above, Π means the option price, S means the stock price.

To start with, for the delta hedge of Black-Scholes model: Through the establishment of Black-Scholes model, the formulas (1), (2) of call and put options are got. Hence, N'(d1) and N'(d2) can be got by calculating.

$$N'(d_1) = \frac{\partial N(d_1)}{\partial d_1} = \frac{1}{\sqrt{2\pi}} e^{-\frac{{d_1}^2}{2}} \quad (17)$$

$$N'(d_2) = \frac{1}{\sqrt{2\pi}} e^{-\frac{{d_2}^2}{2}} \quad (18)$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{(d_1-\sigma\sqrt{T-t})^2}{2}}$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{{d_1}^2}{2}} . \frac{S_t}{X} e^{r(T-t)}$$

As a result, the delta formula for non-dividend stock options can be derived based on the Black-Scholes model: Δ (call option)= N(d1), Δ (put option)= N(d1) - 1.

Since in the establishment of Black-Scholes model, σ has been got which is equal to 43.5%, setting k is 12.5 and r is 0.0325, the gain or loss in Black-Scholes model based on the Ford Motor data can be calculated. Then do the delta hedge of Heston model, using definition of differential to estimate the delta. The formulas are as follows:

$$Delta_{heston} = \frac{C(s+\Delta s) - C(s)}{\Delta s} \quad (19)$$

Plugging in the data, the gain or loss of Heston model can be calculated.

After getting the profit of the stock and option data of Ford Motor through two different models, the data obtained from the two groups can be compared to analyze the advantages and disadvantages of the two models.

3 RESULTS AND DISCUSSION

3.1 Results

Based on the logical structure and formulation of the model, Black-Scholes model can be established by excel like the table below.

Table 5: Black-Scholes model (sigma assumed as 0.2).

Current price	Strike price	Risk-free return	Time to maturity	Volatility	Dividend	Type
13.88	14.00	3.25%	0.07	20.00%	0.00%	Call option
d1	d2	N(d1)	N(d2)	Exp (-r T)	Exp (-q T)	
-0.0909	-0.1443	0.4638	0.4426	0.9977	1.0000	
Option price						
0.26						
Current price	Strike price	Risk-free return	Time to maturity	Volatility	Dividend	Type
13.88	14.00	3.25%	0.07	20.00%	0.00%	Put option
d1	d2	N(d1)	N(d2)	Exp (-r T)	Exp (-q T)	
0.0909	0.1443	0.5362	0.5574	0.9977	1.0000	
Option price						
0.34						

As the table shown above, time to maturity is 0.07. Due to the model and the research content of this study, the dividend is set to 0, namely there does not exist dividend. And the yield on the 10-year Treasury note is chosen to be the risk-free return, which is 3.25%. Since the volatility is an indeterminate value, it was set to 20% in the beginning. After Plugging the option data into the model, the theoretical value of options can be obtained when volatility is 20%. By Analyzing and calculating the theoretical and actual value of the option, the volatility namely sigma can be fitted out which is 0.424. The unknown parameters of Black-Scholes model are all calculated which are in the table below:

Table 6: Parameters of Black-Scholes model.

σ	T	r
0.424	0.07	0.0325

According to the parameters estimated, the Black-Scholes model is established resetting the volatility to 42.4.

Table 7: Theoretical value (sigma is 0.424).

	N(d1)	N(d2)	Theoretical value
Call	0.91	0.89	1.97
	0.74	0.71	1.17
	0.63	0.58	0.85
	0.50	0.46	0.59
	0.38	0.34	0.39
Put	0.62	0.66	0.03
	0.09	0.11	0.07
	0.26	0.29	0.26
	0.37	0.42	0.43
	0.50	0.54	0.67

As shown in the table above, the theoretical price of option can be calculated from June.2,2022 to June.22,2022 through Black-Scholes model.

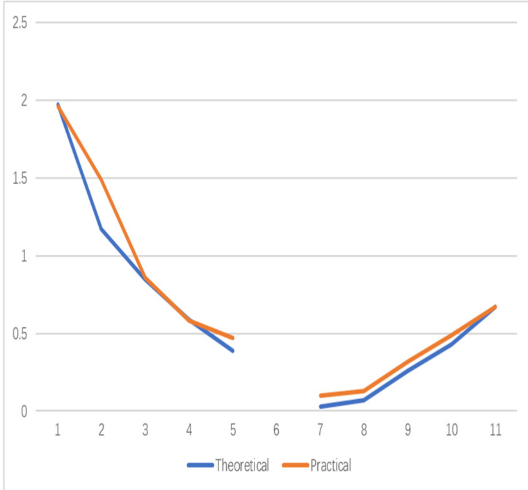

Figure 1: Comparison between theoretical and practical value.

Table 8: Error sum of squares of Black-Scholes model.

Black-Scholes model						
6.1	sigma=0.424	Call	K	theoretical value	practical value	(P-Pm)^2/Pm
			12	1.97	1.96	5.10204E-05
			13	1.17	1.49	0.068724832
			13.5	0.85	0.86	0.000116279
			14	0.59	0.58	0.000172414
			14.5	0.39	0.47	0.013617021
		SSE				0.082681567
		Put	11.5	0.03	0.1	0.049
			12	0.07	0.13	0.027692308
			13	0.26	0.32	0.01125
			13.5	0.43	0.49	0.007346939
			14	0.67	0.67	0
		SSE				0.095289246
		Total SSE				0.177970813

As shown in the figure above, for both call and put options, the actual Ford Motor option prices are generally higher than the theoretical values estimated by the Black-Scholes model.

As shown in the table above, the sum of squares of call option is 0.083 and put option is 0.095, the total sum of squares of Black-Scholes model is 0.178. The fitting result is valid. Based on the logical structure and formulation of the model, the parameters of Heston model can be calculated.

Table 9: Parameters of Heston model.

	ρ	σ	θ	k	v0
	rho	sigma	theta	kappa	v0
value	0	0.4326	0.6454	0.7376	0.193

As the table above shows, the volatility risk is 0, the fluctuation in volatility is 0.4326, the mean of the long run level is 0.6454, the rate of adjustment is 0.7376, and the variance of open price is 0.193. Hence, after adjusting parameters, the Heston model can be established.

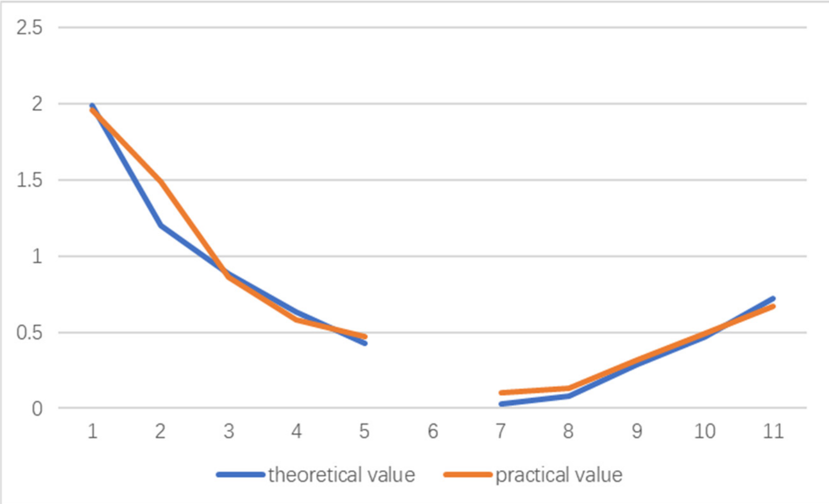

Figure 2: Comparisons between theoretical and practical value.

As shown in the figure above, this is a statistical processing about the option price of Heston model. For put option (in the right), most conditions practical value is higher than theoretical value. While for call option (in the left), both theoretical and practical value have higher points.

Table 10: Error sum of squares of Heston model.

Heston model						
6.1	sigma=0.433	Call	K	theoretical value	practical value	(P-Pm)^2/Pm
			12	1.99	1.96	0.000459184
			13	1.2	1.49	0.056442953
			13.5	0.88	0.86	0.000465116
			14	0.63	0.58	0.004310345
			14.5	0.43	0.47	0.003404255
		SSE				0.065081853
		Put	11.5	0.03	0.1	0.049
			12	0.08	0.13	0.019230769
			13	0.29	0.32	0.0028125
			13.5	0.47	0.49	0.000816327
			14	0.72	0.67	0.003731343
		SSE				0.075590939
		Total SSE				0.140672792

As shown in the table above, the sum of squares of call option is 0.065 and put option is 0.049, the total sum of squares of Heston model is 0.141. The fitting result is valid.

After establishing Black-Scholes model and Heston model, do delta hedging based on the two models and calculate the gain or loss from June.2,2022 to June.22,2022.

Table 11: Delta hedge of call option in Black-Scholes model.

date	stock price(open)	d1	N(d1)	actual call price	gain/loss	total gain/loss
2022/6/2	13.64	2.05	0.98	1.48	0	-1.50
2022/6/3	13.63	2.12	0.98	1.32	-0.009796422	
2022/6/6	13.74	2.64	1.00	1.34	0.108126984	

2022/6/7	13.26	1.88	0.97	0.97	-0.477996657	
2022/6/8	13.63	2.74	1.00	1.19	0.358942818	
2022/6/9	13.51	2.66	1.00	1.17	-0.119632467	
2022/6/10	13	1.63	0.95	0.89	-0.508023709	
2022/6/13	12.3	-0.31	0.38	0.41	-0.6637545	
2022/6/14	11.99	-1.61	0.05	0.28	-0.117774181	
2022/6/15	12.22	-0.86	0.20	0.34	0.012339877	
2022/6/16	11.8	-3.29	0.00	0.17	-0.082143867	
2022/6/17	11.24	-7.74	0.00	0.08	-0.000279742	
2022/6/21	11.55	-29.82	0.00	0.03	0	
2022/6/22						

As the table above shows, the total loss of call option in Black-Scholes model is 1.5, basically most days in the red. On June.13,2022, it losses the most which reaches 0.66 and on June,8,2022, it gains the most which reaches 0.36.

Table 12: Delta hedge of put option in Black-Scholes model.

date	stock price (open)	d1	N(d1)	actual put price	gain/loss	total gain/loss
2022/6/2	13.64	2.05	-0.02	0.19	0	0.59
2022/6/3	13.63	2.12	-0.02	0.24	0.000203578	
2022/6/6	13.74	2.64	0.00	0.23	-0.001873016	
2022/6/7	13.26	1.88	-0.03	0.25	0.002003343	
2022/6/8	13.63	2.74	0.00	0.13	-0.011057182	
2022/6/9	13.51	2.66	0.00	0.16	0.000367533	
2022/6/10	13	1.63	-0.05	0.26	0.001976291	
2022/6/13	12.3	-0.31	-0.62	0.57	0.0362455	
2022/6/14	11.99	-1.61	-0.95	0.71	0.192225819	
2022/6/15	12.22	-0.86	-0.80	0.5	-0.217660123	
2022/6/16	11.8	-3.29	-1.00	0.9	0.337856133	
2022/6/17	11.24	-7.74	-1.00	1.33	0.559720258	
2022/6/21	11.55	-29.82	-1.00	0.9	-0.31	
2022/6/22				1.09		

As the table above shows, the total gain of put option in Black-Scholes model is 0.59, basically most days are profitable. On June.21,2022, it losses the most which reaches 0.31 and on June,17,2022, it gains the most which reaches 0.56.

Table 13: Delta hedge of put option in Heston model.

date	stock price(open)	delta	actual put price	gain/loss	total gain/loss
2022/6/2	13.64	-0.20451	0.19	0	0.71153653
2022/6/3	13.63	-0.200254	0.24	0.0020451	
2022/6/6	13.74	-0.1742165	0.23	-0.02202794	
2022/6/7	13.26	-0.264333	0.25	0.08362392	
2022/6/8	13.63	-0.178801	0.13	-0.09780321	
2022/6/9	13.51	-0.194189	0.16	0.02145612	

2022/6/10	13	-0.3144005	0.26	0.09903639	
2022/6/13	12.3	-0.5489845	0.57	0.22008035	
2022/6/14	11.99	-0.670097	0.71	0.170185195	
2022/6/15	12.22	-0.592458	0.5	-0.15412231	
2022/6/16	11.8	-0.767487	0.9	0.24883236	
2022/6/17	11.24	-0.9340715	1.33	0.42979272	
2022/6/21	11.55	-0.91652	0.9	-0.289562165	
2022/6/22		-0.94613	1.09		

As the table above shows, the total gain of put option in Heston model is 0.71, basically most days are profitable. On June.21,2022, it losses the most which reaches 0.29 and on June,17,2022, it gains the most which reaches 0.43.

3.2 Discussion

As calculated and discussed above in the section about result, the total sum of squares of Black-Scholes model is 0.178 and the total sum of squares of Heston model is 0.141. And the total gain of put option in Heston model is 0.71 and the total gain of put option in Black-Scholes model is 0.59. Therefore, the quasi value of Heston's model is relatively more accurate and profitable. The reason is that The Black-Scholes model is an idealized model that does not perform so well in practice since it has some flaws in its assumptions. To illustrate, the model assumes that stock prices follow a continuous geometric Brownian motion, whereas in reality stock prices may jump like the stock data from June.14.2022 to June.16,2022. Moreover, if the stock price volatility specified by the Black-Scholes model is constant, then the implied volatility surface should be smooth which is impossible. Nevertheless, relatively speaking, the Heston model ensures the randomness of volatility. As a result, the Heston model performs better than the Black-Scholes model for the stock data of Ford Motor.

4 CONCLUSION

This paper mainly studies the effect of the same delta hedging on Black-Scholes model and Heston model using the same data about stock and option price of Ford Motor. Although there are some researchers have studied the difference and the pros and cons of the two models, the topic which discuss the option price model about Ford Motor has not been studied before. In this paper, Black-Scholes model and Heston model are established and be compared in terms of error sum of squares and the total gain or loss. Firstly, an assumed value of sigma is set and based on the logical structure and formulation of the model, the theoretical value of sigma is calculated. Then according to the model established, the theoretical price of options are calculated, Finally, using delta hedging, the total gain or loss of the two models can be worked out and be compared to analyses which model is more profitable and suitable for Ford Motor.

However, since the status of dividends is not taken into account, and the Black-Scholes model ignores the transaction costs in real market, The results can be significantly biased. In addition, apart from these two models, there are many other famous option pricing models like Cox-Ross-Rubinstein model, which deserve more investigation in the future.

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