Performance of the Delta-Hedging Strategies Based on the Heston Model and the Black Scholes Model on Meta Platforms

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Abstract: This paper researches the performance of the Delta-Hedging strategies and compares the different results between the Black Scholes (BSM) model and the Heston model. In this study, parameters in two models are calibrated separately by utilizing data on ten options on the stock of Meta Platforms. Then, after the two models have been built, the price of the options and options' delta values can be calculated. The delta hedging portfolios are composed of one share of a call option and delta shares of the underlying stock. Finally, the results of different strategies are compared. The sum of squared errors (SSE) of the BSM model is higher than the SSE of the Heston model. It means that the Heston model prices the options more precisely than the Black Scholes model. In addition, both two models can transfer the risk of fluctuating prices by implementing delta-hedging strategies. This paper is helpful for investors to build their portfolios by using different models in option pricing.

1 INTRODUCTION

Risk management is a significant topic of companies' investment. Many companies utilize derivatives instruments to transfer the risks and fix the profit they could make. For the commodity suppliers, implementing hedging strategies can avoid the risk of fluctuating prices, and then the suppliers produce and sell their products for making a relatively fixed profit. Hedging strategy is also a popular way in the field of investment. Investors can compose a portfolio by purchasing a set of stocks and shorting index futures to gain excess profits. When the hedging strategies are implemented, using different models to price the options may have different results. Fischer Black derived a theoretical formula to price the options in 1973, and this Black-Scholes model has become one of the most important models in option pricing. In the BSM model, the volatility is assumed to be a constant number, but the volatility in the real options market is arbitrary (Black, 1973). Then, stochastic volatility models were researched. Heston built up a stochastic volatility model and derived a formula for calculating a call option price. Since then, option pricing and risk hedging become interesting topics in the financial field (Heston, 1993).

To illustrate, Xiong used three delta hedging strategies on Shanghai Stock Exchange 50 ETF and its options, and they proved that three strategies can significantly reduce investment losses (Xiong, 2017). And Jerbi, Y used neural networks (NNs) to model a European call option price and found that the NNs method performed fairly well in option pricing (Jerbi, 2020). Similarly, JAG Cervera used artificial neural networks to calibrate a call option price efficiently (Cervera, 2019). Furthermore, D Lamberton researched some properties of the American option price and extended some results in the BS model to the Heston model (Lamberton, 2019). Euch, O. E. composed the portfolios by the forward variance curve and the asset, which is theoretically perfect hedging (Euch, 2018). In addition, L Goudenège utilized the Heston model and the BSM model in valuing Guaranteed Minimum Withdrawal Benefit and presented the numerical results (Goudenège, 2019). With the rapid development of the metaverse, the stock of Meta Platforms has caused many investors' concerns, since Meta Platforms is one of the biggest companies in the field of the metaverse. However, the performance of pricing options on the Meta Platforms' stock is rarely discussed. Therefore, this paper focuses on this matter.

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This paper is structured as follows: Section 2 shows the data and methods. Section 3 shows the results of delta-hedging strategies in different models, and also discusses the results. Section 4 concludes this paper.

2 DATA AND METHODS

2.1 Data

The data is all from Yahoo Finance (https://finance.yahoo.com). Meta Platforms' stock is chosen as the underlying stock in this paper since its' options have a huge trading volume, so enough trading data can be used to calibrate the parameters. The Data of the underlying stock is collected from June 8th, 2022 to July 11th, 2022. The data of the options are collected from June 24th, 2022 to July 11th, 2022. In the BSM model, these data are used for calibrating implied volatility for the underlying stocks. In the Heston model, these data are used for calibrating three parameters. Then, the option prices and delta values can be calculated by using the two models. Finally, two delta hedging strategies are constructed for a call option of Meta Platforms' stock from June. 27th, 2022, to July. 11th, 2022.

Table 1: The 10 options of the meta platforms

	Call options chosen for calibration				
Meta Platforms	META 220722C00170000	META 220722C00175000	META 220722C00180000	META 220722C00185000	META 220722C00190000
	Put options chosen for calibration				
Meta Platforms	META 220722P00165000	META 220722P00155000	META 220722P00150000	META 220722P00145000	META 220722P00140000

Table 2: The option of the Meta Platforms' stock for hedging.

	Call option selected for hedging
Meta Platforms	META220722C00165000

In detail, the options contracts that are collected to calibrate parameters in different models for Meta Platforms' stock are shown in the table I. And the contract of the American call option selected for Meta Platforms for delta hedging is shown in the table II. Also, the stock price trend for Meta Platforms is shown in the Figure 1.

As shown above, the stock price of Meta Platforms has been volatile from June 24th, 2022, to July 11th, 2022. In detail, Meta Platforms' stock price experienced a slip from June 27th, 2022, to July 5th, 2022, but it bounced back to the previous range on July 8th.

The descriptive statistics of the rate of return of Meta Platforms' stock from June 24th, 2022, to July 11th, 2022, are shown below. As can be seen, Meta Platforms' stock has a large size range.

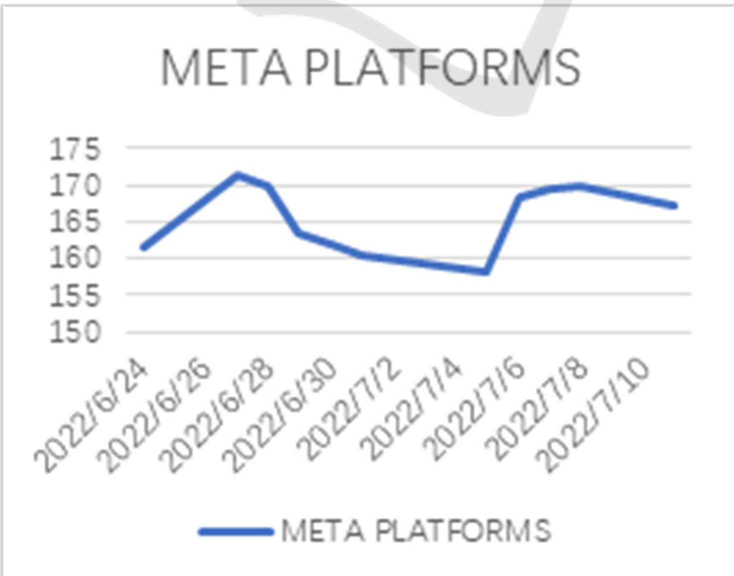

Figure 1: The price trend of Meta.

Table 3: Descriptive Statistics.

	Mean	Median	Variance	Max	Mini
value	0.007	-0.008	0.001	0.064	-0.037

2.2 Methods

In this study, two models are used for options pricing. There are three steps in researching this project. The first step is to calibrate the parameters in two models. For the BSM model, Meta Platforms' volatility is calibrated by utilizing the open price of Meta Platforms' stock and the open price of ten options on the underlying stock. For the Heston model, three parameters are calibrated. Secondly, after the parameters in models are calculated, the theoretical price of call options and options' delta values can be calibrated in both models separately. Finally, two delta hedging strategies are composed of one share of a call option on Meta Platforms' stock and delta shares of the underlying stock. In comparison, one trading strategy without hedging is also composed. The results are shown and discussed in Section 3.

Some assumptions are proposed as follows: 1. The dividend rate of underlying stock D=0; 2. The options are American options, but they are treated as European Options. Therefore, the options theoretical price can be calibrated by the formula in the Heston model and the BS formula; 3. In the Heston model, ρ equals 0, which is the correlation between two Brownian Motions.

In the BSM model, the stock price is described by a geometric Brownian motion, which is shown in the formula (1), where σ stands for the implied volatility, W_t stands for a Brownian motion. Furthermore, the call and put options prices are calibrated by the Black-Scholes formula (2) and (3), where $N(d_1)$ stands for the normal distribution of d_1. d_1 and d_2 can be calculated by the formula (4) and formula (5) (Black, 1973), where T-t is the time to maturity, S_t is the open price of the stock at the time t, K is the strike price of the option, and r is the interest rate, which is assumed to be 3.13% in these models since the USA ten-years Treasury bond yield on June 24th, 2022, equals to 3.13%.

$$dS_t = \mu S_t dt + \sigma S_t dW_t \quad (1)$$

$$C(t,S) = S_t N(d_1) - Ke^{-r(T-t)} N(d_2) \quad (2)$$

$$P(t,S) = -S_t N(-d_1) + Ke^{-r(T-t)} N(-d_2) \quad (3)$$

$$d_1 = \frac{\log(S_t/K) + (r + 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \quad (4)$$

$$d_2 = \frac{\log(S_t/K) + (r - 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t} \quad (5)$$

In the Heston model, the stock price is also described by a geometric Brownian motion, but the volatility is modelled by the Cox-Ingersoll-Ross (CIR) process (Heston, 1993). The volatility can be described by the formula (6) (Heston, 1993).

$$dV_t = \kappa(\theta - V_t)d_t + \sigma\sqrt{V_t}dZ_t^* \quad (6)$$

Where Z_t^* is another Brownian motion, and ρ is the correlation between Z_t^* and W_t, which is assumed to be 0 in this model. Kappa κ, theta θ, and sigma σ are three parameters that need to be calibrated. Some different methods are used for calculating the option price by constructing different characteristic function. One of the methods is shown in the formula below. Because the correlation is assumed to be 0, the characteristic function is given by formula (7) (Simonsy):

$$\phi(u,t) = E[\exp(iu\log(S_t)\,|S_0,\sigma_0^2] = \exp(iu(logS_0 + rt) \times \exp(\eta\kappa\theta^{-2}((\kappa-d)t - 2\log\left(\frac{1-ge^{-dt}}{1-g}\right)) \times \exp(\sigma_0^2\theta^{-2}(\kappa-d)\frac{1-e^{-dt}}{1-ge^{-dt}}) \quad (7)$$

Where

$$d = \kappa^2 - \theta^2(-iu-u^2)^{0.5} \quad (8)$$

$$g = \frac{\kappa-d}{\kappa+d} \quad (9)$$

Carr, P utilized the fast Fourier transform in option valuation. Referring to the method mentioned above, a European call option price is calculated by the formula (10) as below (Hunter, 2000):

$$C_0 = S_0 - \frac{\sqrt{S_0K}e^{-rT}}{\pi}Re[\int_0^{+\infty}\exp(iuK)\emptyset_T(u - \frac{i}{2})\frac{1}{u^2+0.25}du] \quad (10)$$

After the option prices are calculated, the sum of squared errors is calculated for options on June 24th, 2022, by using the formula (11) below.

$$SSE = \sum_{i=1}^{M}\frac{(P_i(a) - P_i^m)^2}{P_i^m} \quad (11)$$

Where $P_i(a)$ represents the theoretical option price in the BSM model or in the Heston model, P_i^m represents options' market price, and M is the number of options, which equals to 10 in these models. In the

Black-Scholes model, the SSE is minimized to calibrate the implied volatility. In the Heston model, the calibration of the SSE is different. Since the volatility is modelled by the CIR process, the SSE is minimized to calibrate three parameters, which are kappa κ, theta θ and sigma σ.

The data of the stock price from June 8th, 2022, to June 23th, 2022 is utilized to calculate the daily yields of the stock. Then the initial volatility of the underlying stock v_0 can be estimated, which is assumed to be the standard deviation of the daily yields of the stock. v_0 equals to 0.2161 after calibration. After the calibration of the parameters in different models. Two delta hedging strategies are composed by one share of a call option on Meta Platforms' stock and delta shares of the underlying stock. Firstly, the stock price data from June 24th, 2022, to July 11th, 2022, and also the market price of the option contracts chosen are collected. Secondly, the theoretical prices of the call option can be calculated on a daily basis by using the BS model and the Heston model. Thirdly, the hedging portfolios are composed by one unit of the call option, and delta shares of the stocks. The delta value equals to $N(d_1)$ for the BSM model. For the Heston model, the delta value can be estimated by formula (12).

$$delta(s) = \frac{C(s+\Delta s) - C(s-\Delta s)}{2\Delta s} \tag{12}$$

Finally, the data of profit or loss is calculated by implementing the delta-hedging strategies every day from June 24th, 2022, to July 11th, 2022. The profit or loss without hedging is also calculated. In detail, to calculate the implied volatility in the Black Scholes model, the Solver function in the Microsoft Excel software is utilized by minimizing the sum of squared errors. Also, a Python programme is cited (Jace, 2021). The function named optimize in the Python module is used to minimize the SSE and calibrates the parameters in the Heston model.

3 RESULTS AND DISCUSSION

3.1 Results

By minimizing the SSE, the parameters in two models are calibrated. In the Black Scholes model, the optimize sigma σ=0.498. The SSE equals to 0.799. In the Heston model, the parameters are shown in the table IV. The SSE equals to 0.772.

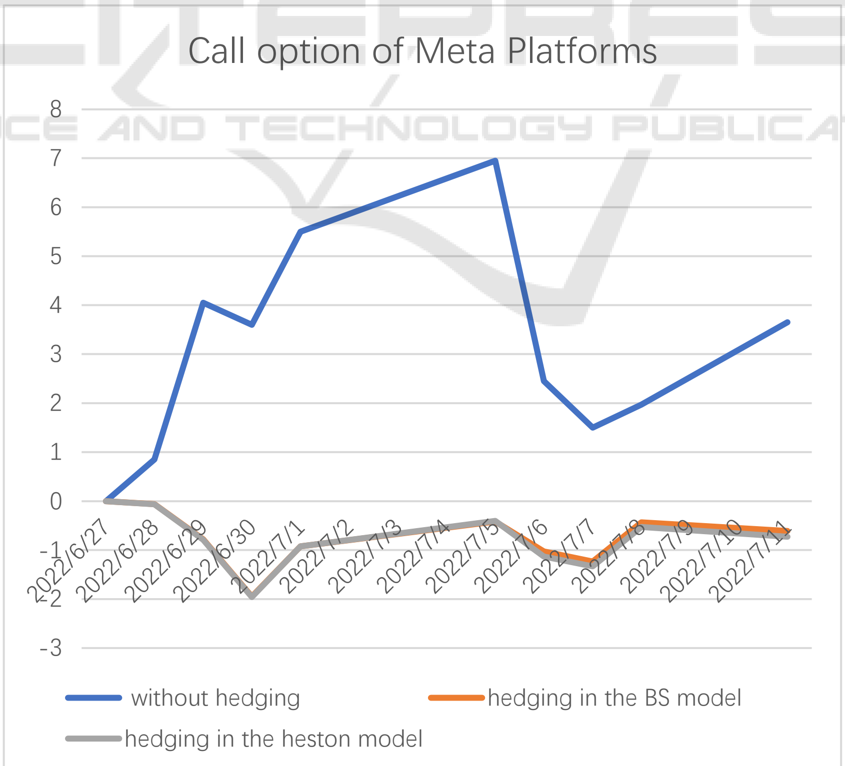

Figure 2: Comparision of profit/loss holding.

Table 4: The parameters of Heston Model.

	ρ	σ	θ	κ	V_0
value	0	0.0248	0.487	0.6839	0.2161

As shown in the Figure 2, the profit without hedging always exceeds the profit with hedging in two models. The trend of the profit with hedging in the Black Scholes model is similar to that in the Heston model. From June 27th, 2022 to July 11th, 2022, the profit without hedging is \$3.65, while the loss with hedging by using the Heston model is \$0.726. By using the Black Scholes model, the loss with hedging is \$0.609.

3.2 Discussion

As shown in the Result section, the strategy of selling the call option without hedging performs very well. The reason is that the stock price of the Meta Platforms decreased during that period. The market price of the underlying stock fluctuates a lot. It decreased from 171.32 to 167.07, from June 27th, 2022 to July 11th, 2022. Although this strategy makes some profits, it does not mean that it has effectively reduced the risk. In comparison, the performance of hedging strategies performs well. The profit or loss changes slightly on a daily basis.

In addition, the SSE in the BSM model is higher than that in the Heston model. It shows that the Heston model performs better than the BSM model in the accuracy of option pricing.

There are some shortages in this paper. Firstly, when calibrating the parameters in the Heston model and the BSM model, the open price of the stock and its options on June 24th, 2022 is used. These data are discrete. If the continuous time series data is used, the parameters in two models are revised in a period. The theoretical options' price and the delta values can also be calculated with higher accuracy. Secondly, the method that calculated the delta values in the Heston model is not the precise way. In the Black Scholes Model, the delta value equals to $N(d_1)$ and the delta values are estimated by the formula (11) above. If the formula of the delta values in the Heston model is known, the delta value can be calculated in a more precise way. Thirdly, some error in calculating the options price is caused, because the American options are considered the European Options. The American option is a kind of path-dependent option. If the formula to calculate an American option price is known, the theoretical price and delta values of the options can be calculated more accurate.

Finally, the correlation between the two Brownian motions is assumed to be 0 in the Heston model. If the correlation ρ is not 0, the formula for calibrating the option price may be more complicated. However, the option price can be calculated with higher accuracy.

4 CONCLUSION

This paper studies the performance of the delta-hedging strategies on Meta Platforms by utilizing the Heston model and the BSM model. Although researchers have studied hedging strategies in the Chinese option market. The topic this paper researched has not been discussed before. Firstly, the implied volatility is calibrated using the information from five call options and five put options on the shares of Meta Platforms in the BSM model. Three parameters in the Heston model are also calibrated by utilizing the same data. Then, after the Black Scholes model and the Heston model are built, the prices of an option and delta values of the option are calibrated every day. Two strategies with hedging are composed, including one share of a call option and delta shares of the Meta Platforms' stock. Finally, the performances of the hedging strategies are compared. Compared with the strategy without hedging, the hedging strategies effectively avoid the risk of fluctuating prices. Furthermore, when the Heston model is used to calculate the option price and its delta values, the sum of squared errors is lower than that when the Black Scholes model is used. This paper proved that the BSM model and the Heston model perform similarly on Meta Platforms options. This result may help the investors choose a proper delta hedging strategy to avoid the risk and gain profits in the options market.

However, some assumptions are assumed to be correct in Section 2, while these assumptions may not perfectly accurate in the real financial market. Therefore, the results of these two models deserve more research and discussions in the future.

REFERENCES

Black, F. , & Scholes, M. S. . (1973). The pricing of options and corporate liabilities. Journal of Political Economy, 81(3), 637-654.

Cervera, J. . (2019). Solution of the black-scholes equation using artificial neural networks. Journal of Physics: Conference Series, 1221, 012044-.

Euch, O. E. , & Rosenbaum, M. . (2018). Perfect hedging in rough heston models. Annals of Applied Probability, 28(6), 3813-3856.

Heston, S. L. . (1993). A closed-form solution for options with stochastic volatility with applications to bond and currency options. Review of Financial Studies(2), 2.

Hunter, D. . (2000). Option valuation using the fast Fourier transform.

Jace, Y. . (2021). Heston Model: pricing formulas and calibration. https://github.com/Jace-Yang/heston-model_pricing-formulas-and-calibration/

Jerbi, Y. , & Chaabene, S. . (2020). European call price modelling using neural networks in considering volatility as stochastic with comparison to the heston model. Journal of Statistical Computation and Simulation, 90(10), 1793-1810.

Lamberton, D. , & Terenzi, G. . (2019). Properties of the american price function in the heston-type models. Working Papers.

L Goudenège, Molent, A. , & Zanette, A. . (2019). Pricing and hedging gmwb in the heston and in the black-scholes with stochastic interest rate models. Computational Management Science, 16(269).

Simonsy, J. E. . A Perfect Calibration! Now What? Wim Schoutens.

Xiong, X. , & Liu, Y. . (2017). The dynamic hedging strategy of sse 50etf options and empirical test. Journal of Chongqing University of Technology (Natural Science).