Optimal Investment Strategies for Gold Bitcoin Portfolio Based on the LSTM and AHP Models

Zihao Zhu, Bozhong Zheng and Ruoxi Li*
University of Shanghai for Science and Technology, Shanghai, China

Keywords: Gold, Bitcoin, AHP, LSTM, Transaction Costs.

Abstract: Bitcoin is the most innovative digital cryptocurrency, but its price is highly volatile and cannot maintain price stability in the event of market shocks. It is regarded as a long-term safe haven from the natural value of gold. Therefore, this research selects gold and bitcoin as a portfolio investment product to explore the optimal portfolio investment problem with given investment period, initial assets and transaction costs. This research uses the hierarchical analysis process (AHP) model to solve for the optimal investment ratio. After obtaining the investment ratio, a gold bitcoin investment decision model is established to determine the daily trading operation. And using the simulated ups and downs analysis, it is concluded that the investment proportion solved by AHP model has better investment efficiency. Later, the long-short-term memory (LSTM) model is constructed to predict the rise and fall , and the annualized interest rate is 158.85%. Finally, the sensitivity analysis of the transaction cost to the model is also carried out, and the results show that the investment ratio derived from the AHP method has some value in practical applications. The research in this paper provides some guidence for the optimal investment strategies for gold bitcoin portfolio.

1 INTRODUCTION

In recent years, in the context of economic globalization and financial crisis, experts and scholars not only focus on stock market research, but also gradually pay attention to the research of gold futures prices, bitcoin and other digital currencies (Atamian, 2022). For a long time in the past, gold has been regarded as a natural currency and a safe-haven store due to its stable properties over a long period of time (Dongfang Y, 2021). Using a classical Markov transition model, gold prices continue to increase once they have risen (Qilian Z, 2011). With the popularity of digital currencies, a cashless society is expected to be nurtured (Kabaklarlı E, 2022), eventually replacing paper money and coins. From this perspective, cryptocurrencies can be seen as environmentally friendly (Sagheer, 2022). Bitcoin has been one of the most innovative digital cryptocurrencies since its inception (Zheshi C, 2021), and its price has often risen and fallen dramatically, providing investors with high returns but also possessing significant risks.

Using a T-GARCH model, it was found that bitcoin has similarities to gold and can be a hedge against stock market risk in times of stock turbulence (Anne Haubo Dyhrberg, 2016). In addition, the varying-coefficient quantile approach was applied and it was found bitcoin to be a safe haven (Ren, 2022) and good speculative asset (Baek, 2015). However, several researches indicate that bitcoin is a risky investment with limited hedging ability. For example, Bouri built the DCC dynamic conditional model and found that it is less suitable as a hedging instrument but can be used for diversification (Elie, 2017). Nonetheless, investment strategies in gold, stocks, oil and bitcoin can be implemented to reduce investment risk to a large extent (Khaled, 2019). In order to investigate the relationship between the prices of the gold and bitcoin, Yechen explored the impact of dollar index, gold price on bitcoin price and found that the correlation between the prices of the gold and bitcoin is relative weak (Yechen Z, 2017). Besides, the newly proposed Wavelet Quantile Correlation (WQC) implemented by Kumar AS *et al.* concluded that gold has a safe hedge in all markets, while bitcoin has mixed results (Kumar, 2022). By applying the (vine) copula, it was found that gold substantially reduces the downside risk of a portfolio containing gold and energy commodity allocations, in contrast to the inconsistent hedge function of bitcoin (Syuhada, 2022). The DCC-FIGARCH model was

Zhu, Z., Zheng, B. and Li, R.
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DOI: 10.5220/0012030300003620
In *Proceedings of the 4th International Conference on Economic Management and Model Engineering (ICEMME 2022)*, pages 328-337
ISBN: 978-989-758-636-1

constructed to show that bitcoin qualifies as a hedge against Islamic stock market declines (Chkili, 2021). Furthermore, Chemkha *et al.* show the effectiveness of bitcoin and gold as hedging assets in reducing international portfolio risk based on a multivariate asymmetric dynamic conditional correlation model (Chemkha, 2021).

Due to the diverse nature of investment products, the portfolio investment model was conceived as a way for rational investors to allocate their funds to several different assets to obtain more return than a single investment while taking as little risk as possible (Xue, 2022). In addition, Ma Y *et al.* show that diversification increases returns, reduces portfolio volatility across all portfolios, and diversification also provides higher returns (Ma, 2020). In order to make the investment strategies, the multi-factor stock selection model with better accuracy and stability can be developed an entropy-weighted method combined with Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) model for comprehensive evaluation of each stock (Yeong Z, 2021). Moreover, the portfolio investment model based on Long Short-Term Memory (LSTM) neural network can also be built to construct different portfolios for different risk aversion coefficients (Yue, 2021). The combined machine learning methods and portfolio models can also be implemented to build stock portfolios by using support vector machine (SVM) model and LSTM model to predict stock ups and downs (Jun, 2019). Besides, Fang *et al.* uses conditional value-at-risk (CVaR) model to measure risk while considering investment transaction costs (Chengde F, 2015). Furthermore, Wang *et al.* constructed a mean-variance-down-run variance portfolio model with transaction costs, introduced risk aversion coefficients, and solved the model using a teach-and-learn algorithm to obtain optimal portfolios with different returns (Xiaoqin W, 2020). Later, they also considered the case of multi-stage portfolios and built a mean-VaR multi-stage portfolio optimization model to obtain the optimal investment strategies for each stage under different paths (Xiaoqin W, 2020). Zhu *et al.* applied particle swarm optimization (PSO) method combined with the metaheuristic approach on solving the intractability of portfolios (Zhu, 2011). Also, Butler *et al.* pointed out that PSO approach can offer better results (Butler, 2010). Consequently, it is possible to predict the future price or rise and fall of investment products, determine their fluctuation status, and select the combination. It is particularly important to carry out portfolio investment with appropriate investment products, which can enable investors to obtain greater returns on the basis of taking smaller risks.

In this research, we choose gold and bitcoin as portfolio investment products. We build the EEMD-PCA-LSTM model to predict the price, and construct analytic hierarchy process (AHP) model to solve the portfolio investment ratio and get the daily recommended investment weight ratio of gold and bitcoin. Finally, it is solved by the gold bitcoin investment decision model.

2 THEORY AND METHODS

2.1 Data Collection and Pre-processing

The data samples in this research include the daily prices of gold and bitcoin from 2016 to 2021, mainly from the London gold market and NASDAQ.(Mathematical modeling competition for American College Students) On this basis, the date with incomplete data is eliminated and the date with abnormal data is handled. We selected five indicators: daily yield R_t, daily average yield $E(R_t)$, volatility σ_R (standard deviation of R_t), the maximum retracement rate MaxDawndown, and the longest consecutive rising days MaxDay.

We build the Ensemble Empirical Mode Decomposition (EEMD) (Dongfang Y, 2021) model to decompose signals (Xiwen S, 2022). In order to increase the number of features and improve the efficiency of training neural networks. We input the original data into EEMD model and decompose nine signals. The results are shown in the Fig.S1 of the supporting information (SI).

Then, the Principal Component Analysis (PCA) (Chen, 2022) is used to analyze on the nine signals output by EEMD model, calculate their eigenvectors and eigenvalues, respectively (Xu, 2022), and sort them from large to small. The results are shown in the Fig.S2 of the supporting information (SI). The eigenvalues of the last four principal components are too small, so they are discarded as noise. Only the first five features are retained and used as input variables for subsequent prediction models.

2.2 Long Short-Term Memory (LSTM) Model

The Long Short-Term Memory (LSTM) model is an improved version of Recurrent Neural Network (RNN). The structure of LSTM model is demonstrated in the Fig.1. The “Gate” structure of

LSTM model is used to alleviate the gradient disappearance and gradient explosion of RNN (Sako, 2022). This "Gate" structure makes the gradient not disappear completely no matter how long the time series propagates. The hidden state of the LSTM model is called cells. These cells are repetitive structures. Each cell receives the cell state of the previous step and the current input, and determines which information to retain and which to forget (Li, 2022). This can effectively preserve long-term information.

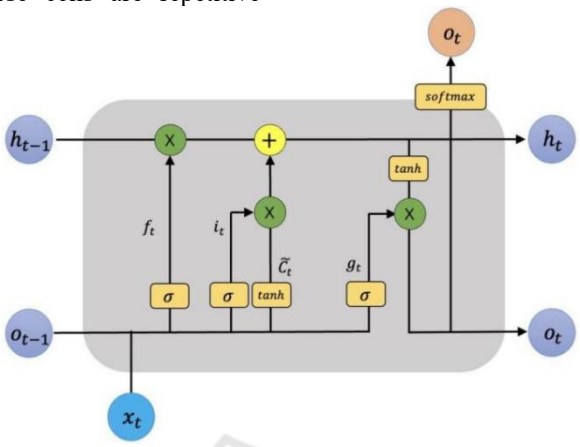

Figure 1: Single cell structure of LSTM model.

In each day's investment, we need to choose the length of the training set. We take the bitcoin market price from May 1st to May 20th, 2019 as the test set, and set the length of the training set as [30,60,90,120,150]. The R^2 on test set is shown in the Fig.2. It is concluded that the best training window length is 60 days.

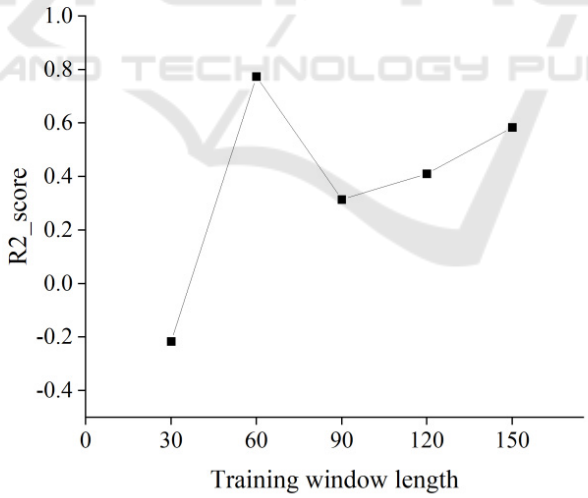

Figure 2: R2_Score on test set predicted by LSTM model on training sets of different lengths.

In this research, the LSTM model is built by using the Keras framework in Python 3.8 Environment. After training and parameter adjustment, the hyper parameters of the model are finally determined as shown in Table I.

Table1: The final hyper parameters of the LSTM model.

Epochs	Batch size	Verbose	Loss	optimizer
200	1	1	MAE	Adam

2.3 Modern Portfolio Theory (MPT)

Modern portfolio theory (MPT), or mean-variance analysis, is a mathematical framemwork for assembling a portoflio of assets such that the expected return is maximized for a given level of risk. (Zola, 2021). It measures Portfolio Utility from the perspective of investment risk and return, and puts forward a quantitative method to measure financial risk for the first time. The establishment of mean-variance analysis is based on the following three assumptions: investors are risk averse; each asset has risks; the distribution of return on assets is normal. Mean-Variance analysis is the process of weighting risk, expressed as variance, against expected return. Investors use mean-variance analysis to make decisions about which financial instruments to invest in, based on how much risk they are willing to take on in exchange for different levels of reward. (Zhang, 2022). The covariance between gold and bitcoin yields σ_{gb} can be calculated as:

$$\sigma_{gb} = \frac{\sum_{i=1}^{n}(r_{(g:t)}-\overline{r_g})(r_{(b:t)}-\overline{r_b})}{n-1} \quad (1)$$

where $r_{(g:t)}$ and $r_{(b:t)}$ are the yield of gold and bitcoin on that day in the time period T, respectively. T indicates the statistical period, which is set to 30 in this research. $\overline{r_g}$ and $\overline{r_b}$ denote the average yield of gold and bitcoin, respectively. n is the number of observations of the sample.

The standard deviation of the portfolio STD is calculated to represent the overall risk and the weighted return rate r_p is calculated to represent the overall portfolio return efficiency.

$$r_p = w_g r_g + w_b r_b \quad (2)$$

$$\sigma_p^2 = \omega_g^2\sigma_g^2 + \omega_b^2\sigma_b^2 + 2\omega_g\omega_b\sigma_{gb} \quad (3)$$

$$STD = \sqrt{\sigma_p^2} \quad (4)$$

where r_p is the weighted yield of portfolio; σ_p^2 denotes ortfolio risk; σ_g, σ_b are standard deviation of gold and bitcoin yields, respectively ; ω_g, ω_b are the percentages of gold and bitcoin in the portfolio, respectively.

2.4 Analytic Hierarchy Process (AHP) Model

Analytic hierarchy process, or AHP for short, is a multi-objective decision analysis methodology proposed by Satty in the 1970s (Huo, 2021). Its principle is to decompose the factors related to decision-making into several levels, such as target level, criterion level and scheme level. Through the calculation and comparison of each factor, the weights of different factors are obtained, which provides a reference basis for decision-makers to select the optimal scheme (Yadav P, 2022). The specific operation steps are as follows:

•***Step 1:*** *Establish a hierarchical structure.*

According to the analysis of the problem, the decision-making problem is divided into three levels (Himanshu, 2021). The top level is the target layer M which selects the most appropriate investment weight for the balance of income and risk. The lowest layer is the scheme layer, including gold and bitcoin. The middle layer is the standard layer, including average daily yield, volatility, commission, maxdawndown and maxday.

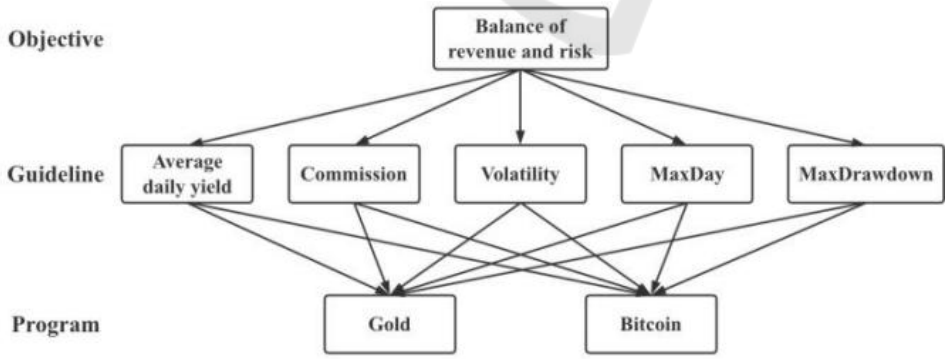

Figure 3: Analytic hierarchy process three-tier structure model diagram.

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•***Step 2:*** *Solve the hierarchical structure.*

Firstly, the judgment matrix is constructed according to the investment principles to determine the weight distribution value of each criterion as shown in Table II. By comparing the values of the five criteria of gold and bitcoin, the judgment matrix is set to determine how much weight should be allocated to gold and bitcoin every day under each criterion.

Table 2: Guideline Layer Judgment Matrix.

	Volatility	MaxDawndown	Average daily yield	Commission	MaxDay
MaxDawndown	1	2	1/5	2	1
Average daily yield	1/2	1	1/7	1	1/2
Average daily yield	5	7	1	7	5
Average daily yield	1/2	1	1/7	1	1/2
MaxDay	1	2	1/5	2	1

•***Step 3:*** The eigenvector of the previously obtained eigenmatrix is obtained by the eigenvalue method, and then normalized to obtain the weight matrix as shown in Table III. (The data on December 10th, 2016 is taken as an example).

Table 3: Weight Matrix of Gold and Bitcoin under Each Criterion.

	Volatility	MaxDawndown	Average daily yield	Commission	MaxDay
Weights of Criterion	0.134	0.072	0.586	0.072	0.134
Gold	0.855	0.092	0.1	0.869	0.152
Bitcoin	0.144	0.907	0.9	0.130	0.847

•***Step 4:*** Calculate the score according to the weight matrix and determine the weight of gold bitcoin investment.

$$\omega_g = 0.134 \times 0.855 + 0.072 \times 0.092 + 0.586 \times 0.1 + 0.072 \times 0.869 + 0.134 \times 0.152 = 0.263 \quad (5)$$

$$\omega_b = 0.134 \times 0.144 + 0.072 \times 0.907 + 0.586 \times 0.9 + 0.072 \times 0.130 + 0.134 \times 0.847 = 0.736 \quad (6)$$

Then, the proportion of gold bitcoin investment k is calculated according to the weight of gold bitcoin investment.

$$k = \frac{\omega_g}{\omega_b} = 0.358 \quad (7)$$

Through the above steps of the AHP model, we obtained the gold bitcoin investment ratio K on December 10th, 2016. In addition, we use the data of the same day, set the change range of investment weight distribution as 0.05 and the change range as 0 ~ 1, and then we can get 21 investment ratios plus the investment ratio K obtained by AHP model. A total of 22 groups of investment ratios have solved their weighted yield and standard deviation, respectively, and drawn the return / risk curve as shown in the Fig.4.

This curve is called the Markowitz bullet. In the efficient frontier, there is a position where the volatility of yield is the smallest, that is, point Q in the graph. Point P in the figure is the highest income point, and the investment proportion closer to point Q represents the lower risk.

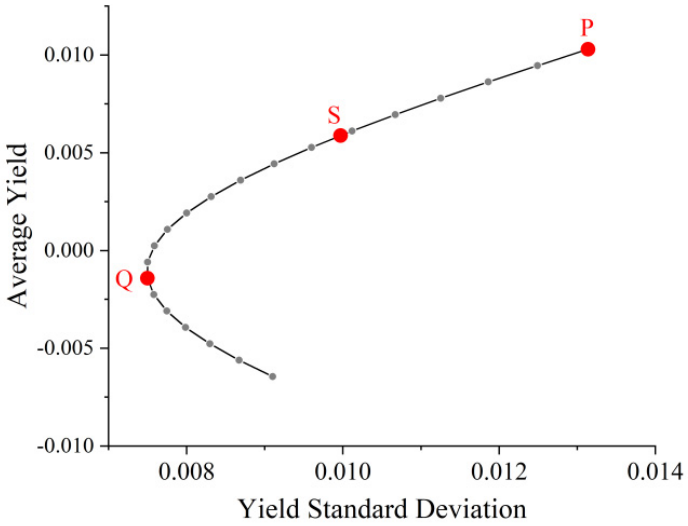

Figure 4: Return / Risk Curve of 22 Investment Proportions.

The investment proportion closer to point *P* represents a higher yield, but also bears greater risks. It can be concluded that the point S obtained by AHP is just between the lowest risk point *Q* and the highest return point *P*, which is a proportion that balances the return and risk. Therefore, in the following investment, we implement AHP model to determine the daily investment proportion of gold and bitcoin.

2.5 Gold Bitcoin Investment Decision Model (GBIDM)

1) Optimal Strategies

a) Principle 1: Risk and return are positively correlated (Tu, 2018).

- **Strategy I:** For gold and bitcoin, the investment ratio can converge to the *K* value or remain as constant as possible.

b) Principle 2: Gold has value preservation and safe-haven properties (Yang, 2019); Bitcoin has the characteristics of high profit and high risk.

- **Strategy II:** Bitcoin is allowed to be bought or sold frequently, and then, gold is not, instead.

c) Principle 3: Long-term assets are often accompanied by value growth.

- **Strategy III:** Do not sell assets easily unless it is estimated that the assets sold can produce higher returns.
- **Strategy IV:** Hold as little cash as possible.

2) Model Building

f is a standard used to judge the relationship between Commission and expected return.

$$f = T \times E(R_t) - (1-\alpha_g)(1-\alpha_b) \quad (8)$$

α_g, α_b are the commissions for gold and bitcoin, respectively. The condition that f is positive means that the expected revenue from a trading operation is greater than the deducted commission. Therefore, trading operations between gold and bitcoin can be performed when f is positive, and not when f is negative.

The GBIDM model is based on three basic principles of investment and four basic strategies. For each trading day, 14 scenarios are classified and the corresponding decisions are made based on P_g, P_b, $\frac{G}{B}$, k and f, as shown in Fig. 5. $\frac{G}{B}$ means the current holding ratio of gold to bitcoin. Since gold can only be traded on weekdays, trading decisions are not the same on weekdays and weekends. Taking for example, when it is a weekday and P_g is positive, P_b is positive, $\frac{G}{B}$ is greater than k, and f is negative, then buy gold and bitcoin in a ratio of cash so that $\frac{G}{B}$ is closer to k. However, if it is a weekend and P_b is negative, then no trade operation is carried out. The other detailed process is shown in the Fig. S3. of supporting information (SI).

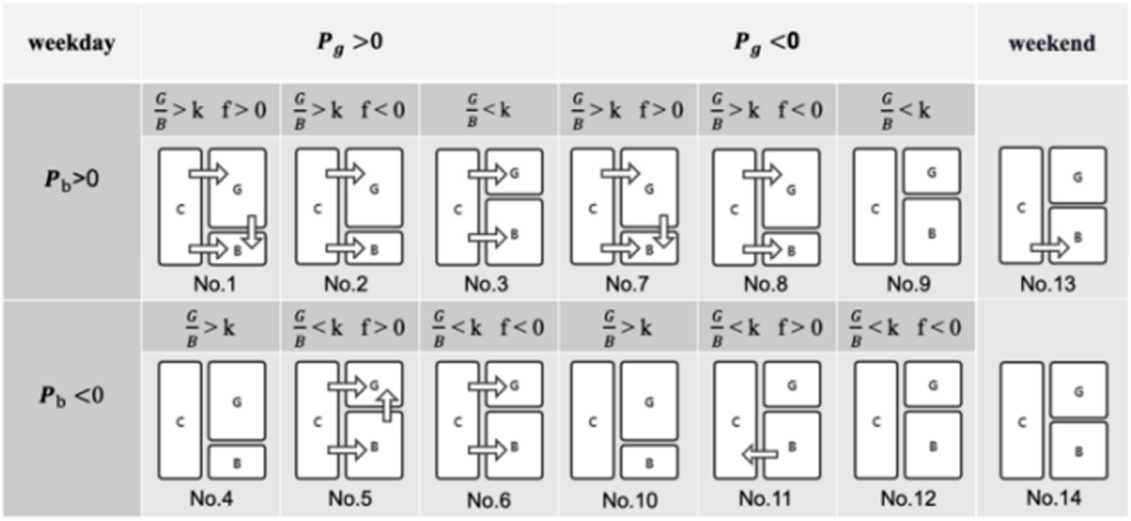

Figure 5: Decision diagram of GBIDM model.

2.6 Model Connection

With the above-mentioned models and data processing methods, they are fused and combined into the GBIDM-AHP-LSTM model, and the framework of the model is shown in Fig. 6 First, the LSTM model will predict the value of P_g, P_b by learning the data of bitcoin and gold for the previous 60 days. The detailed data is shown in Fig. S3 and Fig. S4 of supporting information (SI). After that, various quantitative indicators including R_t, $E(R_t)$, σ_R, $Maxday$, $MaxDawndown$ are calculated. These indicators are then brought into the AHP model to find k. Finally, the GBIDM model combines the results of the above models with its own characteristics to derive recommended trading operations.

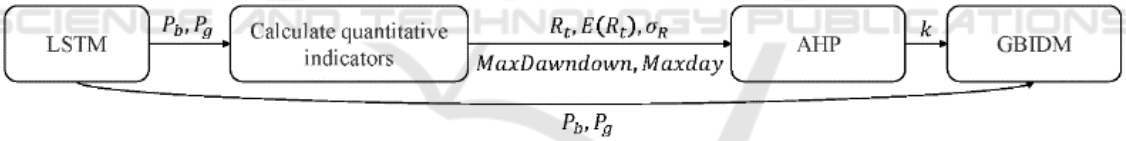

Figure 6: Framework of GBIDM-AHP-LSTM model.

3 RESULTS AND DISCUSSION

3.1 Case Analysis

To verify the validity of the GBIDM-AHP-LSTM model we proposed, the price data of gold and bitcoin from December 6th, 2016, to September 10th, 2021, were chosen as the back testing target. The data source for gold is London Bullion Market Association, and the data source for bitcoin is NASDAQ, with a total of 1739 data items and the date and closing price are included in each data item.

C_i、G_i、B_i are the holdings of USD, gold, and bitcoin on the *i-th* day.v_{gi}, v_{bi} are the values of gold and bitcoin on that day, respectively. a_g, a_b are the commissions for gold and bitcoin, respectively. There is no limit to the number of trades per day and the commissions are deducted for all trades. The initial asset C_0 is given on the first day, after that no new money is provided daily, so the daily available assets are $W_i = C_i + G_i + B_i$.

Let $a_g = 0.01$, $a_b = 0.02$, the quantitative investment model is constructed (Liu, 2020) and back-tested according to the above method. Daily trading operations are conducted according to the GBIDM-AHP-LSTM model, and detailed daily operations are shown in the TABLE SI. of the supporting information (SI). The return is evaluated as shown in Table Ⅳ. From Table Ⅳ, we can conclude that the GBIDM-AHP-LSTM model has a

high rate of return. The final total return in the case analysis is 756.80%, the average annual return is 158.85%, the average monthly return is 13.24%, and the average daily return is 4.41‰. So the model is helpful for traders' investment decisions.

Table 4: Return on investment using the GBIDM-AHP-LSTM model.

	GBIDM-AHP-LSTM Model
Total Return (%)	756.80
Average Annual Return (%)	158.85
Average Monthly Return (%)	13.24
Average Daily Return (‰)	4.41

3.2 Sensitivity Analysis

If the operation of transactions is too frequent, too much money will be spent on commissions. To further verify the stability and reasonableness of the GBIDM-AHP-LSTM model we proposed, the commissions of gold and bitcoin were set from zero to three percent, and the trend of the average annual return of the model was observed. The specific results are shown in Fig. 6.

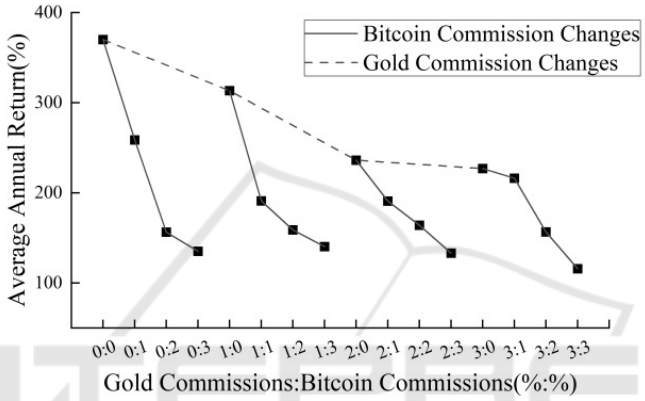

Figure 7: Change in average annual return when gold commissions and bitcoin commissions change.

As shown in Fig. 6, the average annual return is maintained above 100% for different combinations of gold and bitcoin commissions. With a_g being held constant, the average annual return decreases significantly as a_b increases (as shown in the solid line in Fig. 7); with a_b held constant, the average annual return decreases flatly as a_g increases (as shown in the dashed line in Fig. 7).

When the overall commission increases, the commission-limiting mechanism in the GBIDM model increases the trading threshold and discourages investment operations when the expected return may be less than the transaction cost (Sun, 2019). AHP model can balance risk and return and select the appropriate portfolio investment ratio, so although the return decreases as the transaction cost increases, the decrease is not significant and tends to be flat overall. Moreover, it can be concluded that can be seen is that the stability of GBIDM-AHP-LSTM model is stronger on the changes of a_g than on the changes of a_b.

4 CONCLUSION

In conclusion, we build the LSTM model to predict the price of gold and bitcoin at first, then various quantitative indicators are calculated from the predicted prices. In addition, these quantitative indicators are brought into the AHP model to solve for the recommended investment ratio of gold to bitcoin, and the GBIDM model is constructed to solve for the daily investment trading operations to bring the current gold and bitcoin holding ratio close to the recommended investment ratio. Next, the GBIDM-AHP-LSTM model is used for investment case analysis, and the annualized interest rate is 158.85%. Finally, a sensitivity analysis of commissions is performed on the GBIDM-AHP-LSTM model, and it is concluded that bitcoin commission has a certain impact on the model, but gold commission has a slight impact on the model. Through the result of this model, we have formulated a strategy, which can help investors get greater profits within the range of acceptable risks.

ACKNOWLEDGMENT

The authors would like to express our heartfelt thanks to the professors of Shanghai University of technology for their enlightening discussion. The authors would also like to thank our supervisors for their friendly help and timely guidance, which helped us to complete the writing of the whole paper.

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