A Novel Dual-Role Two Pursuers and Two Evaders Simple Motion Game

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Abstract: Multiple players pursuit evasion differential games are significant for the development of unmanned aircraft. In this study, a two pursuers and two evaders game is to be investigated, of which the payoff function is the instant when both the evaders are first time captured by the pursuers. The pursuers wish to minimize the capture instant while the evaders have the opposite purpose. In this game, the positional coincidence of any two players implies that the two players are destroyed. Different from previous literature, the players are considered to have two roles, that is the evader has the role of escaping from the pursuer as well as the role of pursuing the pursuer. To solve the problem, we first present the state feedback strategies of the pursuers. Secondly, the payoff function and the best strategies of the evaders are deduced under the condition that the evader does not intercept the pursuers. Thirdly, by contradiction, it is proved that the capture instant will be decreased when the evaders try to intercept one of the pursuers. As a result, the underlying strategies are equilibrium strategies.

1 INTRODUCTION

The topic of multi-player pursuit evasion games becomes popular due to the development of unmanned artificial vehicles (Exarchos et al., 2015; Zhang et al., 2022). In a multi-player pursuit evasion differential game, the pursuers try to capture all the evaders while the evaders act against being captured. Compared with the one-one, multi-one and one-multi differential games, the underlying problem is more complicated. The major reason is that the players should be allocated so as to make it clear that which pursuer pursuit which evader. Moreover, the allocation might not be fixed in the whole game interval, for instance, one pursuer might change the pursuing target. In real combats, the evader can also act as the "pursuer" to attack the pursuers, thus forming a more intricate problem, which is of concern to this paper.

In regard to the simple motion differential games, there have been extensive studies (Yan et al., 2019; Garcia et al., 2017; Makkapati et al., 2018; Pachter et al., 2020; Pachter et al., 2019; Sun et al., 2017; Ibragimov et al., 2018). Issacs' two cutters and fugitive ship differential game where two faster pursuers cooperate to capture a slower evader in minimum time is a typical simple motion differential game (Isaacs, 1965). The Appolonius circle is often adopted to analyze the simple motion differential games. In the simple motion games, most of the optimal strategies are moving straight. Garcia et.al (Garcia et al., 2017) proposed a geometric approach for the Issacs' two cutters and fugitive ship game to obtain the solution. In another publication (Garcia et al., 2019), Garcia et.al studied the goal line two pursuers one evader game, in which the evader aims at reaching a goal line which is protected by the pursuers, showing that the optimal strategies of the players are moving straight. Pachter et.al (Pachter et al., 2019) also studied the issue of two pursuers and one evader game. Moll et.al (Von Moll et al., 2020) studied the mutiple-pursuer and one evader border defense differential game, with a geometric property-based approach. Pachter et.al (Pachter et al., 2020) studied the multiple-pursuer and one evader differential game. Sun et.al (Sun et al., 2017) studied the multiple pursuers and one evader game in dynamic flowfields. Chen et.al (Chen et al., 2016) studied the multi-player game with one superior evader, providing a cooperative scheme for the pursuers to shrink the encirclement. Alias et.al (Alias et al., 2017) studied the simple motion pursuit evasion game with many evaders and many pursuers subject to integral constraints, obtaining the conclusion that the evasion is possible when the total resource of the evaders is greater than that of the pursuers. Ibragimov et.al (Ibragimov et al., 2018) studied the simple motion differential game with many pursuers

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and many evaders with integral constraints and explicitly constructed the evasion strategies. Makkapati et.al (Makkapati and Tsiotras, 2019) studied the multi-player pursuit evasion problem with a dynamic divide and conquer approach. In regard to the active target defense games, Garcia et.al (García et al., 2015) studied the simple motion game with a fast defender, and the result showed that the optimal trajectories are straight lines. For the same problem, Garcia et.al (Garcia et al., 2018) developed the state-feedback optimal strategies. It has been proved that it is possible for several pursuers capture one evader with zero miss, in which the control bounds of the players are identical (Kumkov et al., 2017).

The aforementioned studies have investigated different kinds of simple motion games, including the life line game, multiple pursuers one evader game, multiple pursuers multiple evaders game and integral constraint game. Rare works have considered the many pursuers and many evaders game in which the evader could attack the pursuer. For this sake, this paper focuses on studying the underlying problem. The approach is by providing the best strategies in advance, and checking whether intercepting the pursuer by the evader could bring in benefit or not.

2 MATHEMATICAL FORMULATION

Four players move on a plane, marked by A, B, C and D. Player A and C are the pursuers, player B and D are the evaders, As depicted in Fig. 1.

The initial positions of the four players are denoted by $\mathbf{r}_{A0}, \mathbf{r}_{B0}, \mathbf{r}_{C0}, \mathbf{r}_{D0}$, respectively. The four players have simple motion dynamics, the state equation are written as:

$$\begin{array}{l} \dot{x}_A = v_A \cos\theta_A \\ \dot{y}_A = v_A \sin\theta_A \\ \dot{x}_B = v_B \cos\theta_B \\ \dot{y}_B = v_B \sin\theta_B \\ \dot{x}_C = v_C \cos\theta_C \\ \dot{y}_C = v_C \sin\theta_C \\ \dot{x}_D = v_D \cos\theta_D \\ \dot{y}_D = v_D \sin\theta_D \end{array} . \tag{1}$$

where θ_A, θ_B, θ_C, and θ_D are the control variables belong to the interval $[0, 2\pi)$.

For simplification, some new variables are introduced:

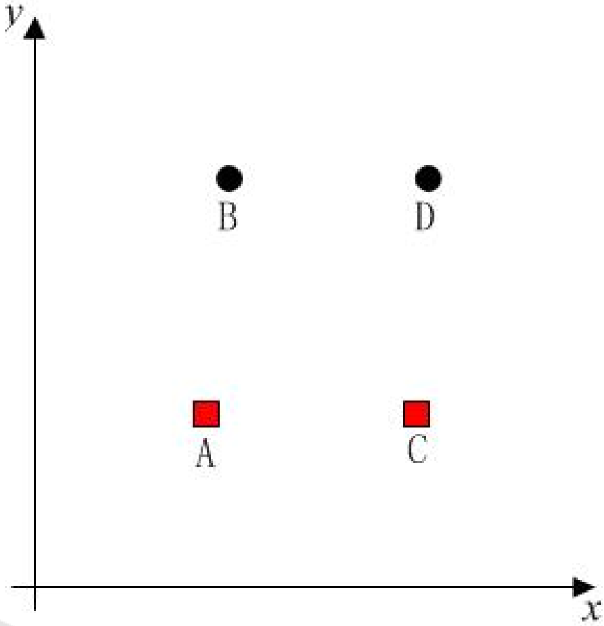

Figure 1: Sketch of the two pursuers and the two evaders.

$$\begin{array}{l} \mathbf{r}_A = \begin{bmatrix} x_A \\ y_A \end{bmatrix}, \mathbf{v}_A = \begin{bmatrix} v_A \cos\theta_A \\ v_A \sin\theta_A \end{bmatrix} \\ \mathbf{r}_B = \begin{bmatrix} x_B \\ y_B \end{bmatrix}, \mathbf{v}_B = \begin{bmatrix} v_B \cos\theta_B \\ v_B \sin\theta_B \end{bmatrix} \\ \mathbf{r}_C = \begin{bmatrix} x_C \\ y_C \end{bmatrix}, \mathbf{v}_C = \begin{bmatrix} v_C \cos\theta_C \\ v_C \sin\theta_C \end{bmatrix} \\ \mathbf{r}_D = \begin{bmatrix} x_D \\ y_D \end{bmatrix}, \mathbf{v}_D = \begin{bmatrix} v_D \cos\theta_D \\ v_D \sin\theta_D \end{bmatrix} \end{array} \tag{2}$$

Based on Eq. (1) and Eq. (2), the state equation is rewritten as:

$$\begin{array}{l} \dot{\mathbf{r}}_A = \mathbf{v}_A \\ \dot{\mathbf{r}}_B = \mathbf{v}_B \\ \dot{\mathbf{r}}_C = \mathbf{v}_C \\ \dot{\mathbf{r}}_D = \mathbf{v}_D \end{array} \tag{3}$$

In this problem, any two players are destroyed when their positional coordinates coincides. Different from the previous pursuit evasion games, the evaders B and D in this game can also pursuit the pursuers A and C. In other words, the pursuers and the evaders have two roles. The evader is called captured when its positional coordinate coincides with a pursuer. The game ends when all the evaders are captured by the pursuers. The purpose of the players A and C is to capture the players B and D as soon as possible, while the purpose of B and D is the opposite. We assume that the magnitudes of the velocity vectors v_A, v_B, v_C and v_D satisfy $v_A > v_B > v_C > v_D$. By now, the game is established and to be analyzed.

3 GAME ANALYSIS WHEN THE EVADERS DO NOT INTERCEPT THE PURSUERS

In this section, the evaders do not intercept the pursuers, only playing the role of escaping from the pursuers. Since $v_A > v_B > v_C > v_D$, A has to capture B while C has to capture D. The one-one game of A and B is called sub-game 1; the one-one game of C and D is called sub-game 2. Evidently, the equilibrium strategies in the sub-game 1 and sub-game 2 are written as:

$$\begin{cases} \mathbf{v}_A^* = v_A \frac{\mathbf{r}_B - \mathbf{r}_A}{\|\mathbf{r}_B - \mathbf{r}_A\|_2} \\ \mathbf{v}_B^* = v_B \frac{\mathbf{r}_B - \mathbf{r}_A}{\|\mathbf{r}_B - \mathbf{r}_A\|_2} \end{cases}, \begin{cases} \mathbf{v}_C^* = v_C \frac{\mathbf{r}_D - \mathbf{r}_C}{\|\mathbf{r}_D - \mathbf{r}_C\|_2} \\ \mathbf{v}_D^* = v_D \frac{\mathbf{r}_D - \mathbf{r}_C}{\|\mathbf{r}_D - \mathbf{r}_C\|_2} \end{cases} \quad (4)$$

The payoff functions, the capture instants, of sub-game 1 and sub-game 2 are denoted as J_1 and J_2, respectively. Under the the equilibrium strategies listed in Eq. (4), the capture instants of B and D are $J_1(\mathbf{v}_A^*, \mathbf{v}_B^*)$ and $J_2(\mathbf{v}_C^*, \mathbf{v}_D^*)$, respectively. The payoff function of the two-two game is denoted as J. Obviously, $J = \max(J_1, J_2)$ when the evaders do not intercept the pursuers.

Lemma 1: If the payoff function J of the underlying two-two game is bigger than $\max(J_1(\mathbf{v}_A^*, \mathbf{v}_B^*), J_2(\mathbf{v}_C^*, \mathbf{v}_D^*))$, then player B must intercept player C.

*P*roof: Based on the law of contrapositive, the proposition is equivalent with "Suppose that the player B does not intercept the player C in the underlying two-two game, then the equilibrium strategies are the same with sub-game 1 and sub-game 2, while the $J = \max(J_1(\mathbf{v}_A^*, \mathbf{v}_B^*), J_2(\mathbf{v}_C^*, \mathbf{v}_D^*))$."

Since $v_A > v_B > v_C > v_D$, C cannot capture B for B has a bigger velocity. Thereby, C has to capture D and A has to capture B. In the case A and C take the best strategy of sub-game 1 and-sub game 2, B and D will make the payoff functions of sub-game 1 and sub-game 2 decrease when taking non-optimal strategies, implying that:

$$J(\mathbf{v}_A^*, \mathbf{v}_B, \mathbf{v}_C^*, \mathbf{v}_D) \leq J(\mathbf{v}_A^*, \mathbf{v}_B^*, \mathbf{v}_C^*, \mathbf{v}_D^*) \quad (5)$$

In the case B and D take the best strategies of sub-game 1 and sub-game 2, A and C will make the the payoff functions of sub-game 1 and sub-game 2 increase when taking non-optimal strategies, implying that:

$$J(\mathbf{v}_A^*, \mathbf{v}_B^*, \mathbf{v}_C^*, \mathbf{v}_D^*) \leq J(\mathbf{v}_A, \mathbf{v}_B^*, \mathbf{v}_C, \mathbf{v}_D^*) \quad (6)$$

From Eq. (5) and Eq. (6), it is concluded that the best strategies of sub-game 1 and sub-game 2 are the equilibrium strategies while the game value $J = \max(J_1(\mathbf{v}_A^*, \mathbf{v}_B^*), J_2(\mathbf{v}_C^*, \mathbf{v}_D^*))$. This completes the proof. ■

Another lemma about the time moment when B intercepts C is given below.

Lemma 2: Under the best strategies of A and C in sub-game 1 and sub-game 2, if the payoff function J is bigger than $\max(J_1(\mathbf{v}_A^*, \mathbf{v}_B^*), J_2(\mathbf{v}_C^*, \mathbf{v}_D^*))$, then the player B must intercept the player C before C captures D and A captures B.

Proof: The proposition can be proved by contradiction. Suppose the player B intercepts C after C captures D, the capture time of B by A must be less than or equal to $J_1(\mathbf{v}_A^*, \mathbf{v}_B^*)$, indicating that $J \leq \max(J_1(\mathbf{v}_A^*, \mathbf{v}_B^*), J_2(\mathbf{v}_C^*, \mathbf{v}_D^*))$; Suppose the player B intercepts C after A captures B, B is destroyed and can not intercept C. Therefore, the player B must intercept the player C before C captures D and A captures B, from which the proof is completed. ■

In summary, it is concluded that when A and C adopt the best sub-game strategies, there is no possibility for B and D to obtain a better result except that B intercepts C earlier than C captures D and A captures B.

4 GAME ANALYSIS WHEN THE EVADER INTERCEPT THE PURSUER AT A TIME

The time moment when B captures C is denoted as t_{BC}, as depicted in Fig. 2. The positions of the four players at time t_{BC} are denoted as $\mathbf{r}_A(t_{BC}), \mathbf{r}_B(t_{BC}), \mathbf{r}_C(t_{BC}), \mathbf{r}_D(t_{BC})$.

Lemma 3: If A and C adopt the best strategies of the two sub-games, the following inequalities hold:

$$\begin{aligned} t_{BC} &\leq \frac{\|\mathbf{r}_{A0} - \mathbf{r}_{B0}\|_2}{v_A - v_B} - \frac{\|\mathbf{r}_A(t_{BC}) - \mathbf{r}_B(t_{BC})\|_2}{v_A - v_B} \\ t_{BC} &\leq \frac{\|\mathbf{r}_{C0} - \mathbf{r}_{D0}\|_2}{v_C - v_D} - \frac{\|\mathbf{r}_C(t_{BC}) - \mathbf{r}_D(t_{BC})\|_2}{v_C - v_D} \end{aligned} \quad (7)$$

Proof: We only prove the inequality in the first row, the second row can be proved by a similar argument.

Let:

$$\rho^2 = \langle \mathbf{r}_A - \mathbf{r}_B, \mathbf{r}_A - \mathbf{r}_B \rangle \quad (8)$$

Taking derivative of the two sides in Eq. (8):

$$\dot{\rho} = \langle \frac{\mathbf{r}_A - \mathbf{r}_B}{\rho}, \mathbf{v}_A \rangle - \langle \frac{\mathbf{r}_A - \mathbf{r}_B}{\rho}, \mathbf{v}_B \rangle \quad (9)$$

By integrating Eq. (9):

$$\begin{aligned} &\rho(t) - \rho(0) = \int_0^t \langle \frac{\mathbf{r}_A - \mathbf{r}_B}{\rho}, \mathbf{v}_A \rangle dt - \int_0^t \langle \frac{\mathbf{r}_A - \mathbf{r}_B}{\rho}, \mathbf{v}_B \rangle dt \\ &\leftrightarrow \\ &\rho(t) = \rho(0) + \int_0^t \langle \frac{\mathbf{r}_A - \mathbf{r}_B}{\rho}, \mathbf{v}_A \rangle dt - \int_0^t \langle \frac{\mathbf{r}_A - \mathbf{r}_B}{\rho}, \mathbf{v}_B \rangle dt \end{aligned} \quad (10)$$

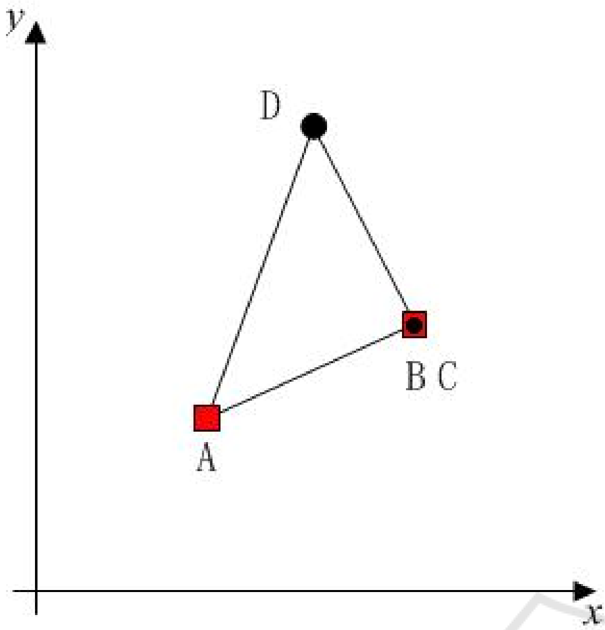

Figure 2: The player B captures player C at t_{BC}.

The purpose of A in sub-game 1 is to reduce ρ as quick as possible with a strategy of A:

$$\rho(t) = \rho(0) - v_A - \int_0^t \langle \frac{\mathbf{r}_A - \mathbf{r}_B}{\rho}, \mathbf{v}_B \rangle dt \tag{11}$$

Based on Eq. (11):

$$\begin{aligned} &\rho(t) \leq \rho(0) - v_A t + v_B t \\ &\leftrightarrow \\ &t \leq \frac{\rho(0)-\rho(t)}{v_A - v_B} \end{aligned} \tag{12}$$

Substituting $t = t_{BC}$ into Eq. (12):

$$\begin{aligned} &t_{BC} \leq \frac{\rho(0)-\rho(t_{BC})}{v_A - v_B} \\ &\leftrightarrow \\ &t_{BC} \leq \frac{\|\mathbf{r}_{A0}-\mathbf{r}_{B0}\|_2}{v_A - v_B} - \frac{\|\mathbf{r}_A(t_{BC})-\mathbf{r}_B(t_{BC})\|_2}{v_A - v_B} \end{aligned} \tag{13}$$

The second row can be proved by a similar argument. The proof is completed. ■

The relations between the payoff function J and $J_2(\mathbf{v}_C^*, \mathbf{v}_D^*)$, $J_1(\mathbf{v}_A^*, \mathbf{v}_B^*)$ are presented in lemma 4 and lemma 5.

Lemma 4: The inequality

$$\begin{aligned} J = t_{BC} + \frac{\|\mathbf{r}_A(t_{BC})-\mathbf{r}_D(t_{BC})\|_2}{v_A - v_D} \leq \\ \frac{\|\mathbf{r}_{C0}-\mathbf{r}_{D0}\|_2}{v_C - v_D} = J_2(\mathbf{v}_C^*, \mathbf{v}_D^*) \end{aligned} \tag{14}$$

holds if

$$\frac{\|\mathbf{r}_A(t_{BC}) - \mathbf{r}_D(t_{BC})\|_2}{v_A - v_D} \leq \frac{\|\mathbf{r}_C(t_{BC}) - \mathbf{r}_D(t_{BC})\|_2}{v_C - v_D} \tag{15}$$

holds at time t_{BC}.

Proof: After time t_{BC}, only A and D participates in the game. The total time B and D captured by C and A is derived as:

$$J = t_{BC} + \frac{\|\mathbf{r}_A(t_{BC}) - \mathbf{r}_D(t_{BC})\|_2}{v_A - v_D} \tag{16}$$

Based on Eq. (15) and Eq. (16):

$$J \leq t_{BC} + \frac{\|\mathbf{r}_C(t_{BC}) - \mathbf{r}_D(t_{BC})\|_2}{v_C - v_D} \tag{17}$$

The inequality of the second row of Eq. (7) in lemma 3 and Eq. (17) yield:

$$\begin{aligned} J \leq &\frac{\|\mathbf{r}_{C0}-\mathbf{r}_{D0}\|_2}{v_C - v_D} - \frac{\|\mathbf{r}_C(t_{BC})-\mathbf{r}_D(t_{BC})\|_2}{v_C - v_D} + \frac{\|\mathbf{r}_C(t_{BC})-\mathbf{r}_D(t_{BC})\|_2}{v_C - v_D} \\ = &J_2(\mathbf{v}_C^*, \mathbf{v}_D^*) \end{aligned} \tag{18}$$

Lemma 5: The inequality

$$\begin{aligned} J = t_{BC} + \frac{\|\mathbf{r}_A(t_{BC})-\mathbf{r}_D(t_{BC})\|_2}{v_A - v_D} \leq \frac{\|\mathbf{r}_{A0}-\mathbf{r}_{B0}\|_2}{v_A - v_B} \\ = J_1(\mathbf{v}_A^*, \mathbf{v}_B^*) \end{aligned} \tag{19}$$

holds if

$$\frac{\|\mathbf{r}_A(t_{BC}) - \mathbf{r}_D(t_{BC})\|_2}{v_A - v_D} > \frac{\|\mathbf{r}_C(t_{BC}) - \mathbf{r}_D(t_{BC})\|_2}{v_C - v_D} \tag{20}$$

holds at time t_{BC}.

Proof: Based on Eq. (20) and the sum of two sides of a triangle is greater than the third side:

$$\begin{aligned} &\frac{\|\mathbf{r}_A(t_{BC})-\mathbf{r}_D(t_{BC})\|_2}{v_A - v_D} > \frac{\|\mathbf{r}_C(t_{BC})-\mathbf{r}_D(t_{BC})\|_2}{v_C - v_D} \\ &\rightarrow \\ &\frac{\|\mathbf{r}_A(t_{BC})-\mathbf{r}_D(t_{BC})\|_2}{v_A - v_D} > \frac{\|\mathbf{r}_A(t_{BC})-\mathbf{r}_D(t_{BC})\|_2 - \|\mathbf{r}_A(t_{BC})-\mathbf{r}_B(t_{BC})\|_2}{v_C - v_D} \\ &\leftrightarrow \\ &\frac{\|\mathbf{r}_A(t_{BC})-\mathbf{r}_D(t_{BC})\|_2}{v_C - v_D} - \frac{\|\mathbf{r}_A(t_{BC})-\mathbf{r}_D(t_{BC})\|_2}{v_A - v_D} < \frac{\|\mathbf{r}_A(t_{BC})-\mathbf{r}_B(t_{BC})\|_2}{v_C - v_D} \\ &\leftrightarrow \\ &\frac{\|\mathbf{r}_A(t_{BC})-\mathbf{r}_D(t_{BC})\|_2}{v_A - v_D} < \frac{\|\mathbf{r}_A(t_{BC})-\mathbf{r}_B(t_{BC})\|_2}{v_A - v_C} < \frac{\|\mathbf{r}_A(t_{BC})-\mathbf{r}_B(t_{BC})\|_2}{v_A - v_B} \end{aligned} \tag{21}$$

Based on lemma 3 and Eq. (16):

$$\begin{aligned} J \leq &\frac{\|\mathbf{r}_{A0}-\mathbf{r}_{B0}\|_2}{v_A - v_B} - \frac{\|\mathbf{r}_A(t_{BC})-\mathbf{r}_B(t_{BC})\|_2}{v_A - v_B} + \frac{\|\mathbf{r}_A(t_{BC})-\mathbf{r}_B(t_{BC})\|_2}{v_A - v_B} \\ = &J_1(\mathbf{v}_A^*, \mathbf{v}_B^*) \end{aligned} \tag{22}$$

This completes the proof. ■

We end this section with a theorem to show that the equilibrium strategies of the two-two game is consistent with the equilibrium strategies of the sub-game 1 and sub-game 2.

Thereom 1: The value of the dual-role game of two pursuers and two evaders with simple motion, where $v_A > v_B > v_C > v_D$, is written as:

$$J = \max\left(\frac{\|\mathbf{r}_{A0} - \mathbf{r}_{B0}\|_2}{v_A - v_B}, \frac{\|\mathbf{r}_{C0} - \mathbf{r}_{D0}\|_2}{v_C - v_D}\right) \tag{23}$$

The corresponding equilibrium strategies are listed in Eq. (4).

Proof: For the evaders B and D, they have two choices. One is escaping from A and B without intercepting them; the other one is escaping as well as intercepting the pursuers.

In the first choice, the game is decomposed of sub-game 1 and sub-game 2. The game value and equilibrium strategies are the same with the sub-game 1 and sub-game 2, which completes the proof in this case.

In the second choice, B and D will not use the strategies $\mathbf{v}_B^*$ and $\mathbf{v}_C^*$. From lemma 4 and lemma 5, it can be inferred that the $J \leq J_1(\mathbf{v}_A^*, \mathbf{v}_B^*)$ or $J \leq J_2(\mathbf{v}_C^*, \mathbf{v}_D^*)$, yielding that $J \leq \max(J_1(\mathbf{v}_A^*, \mathbf{v}_B^*), J_2(\mathbf{v}_C^*, \mathbf{v}_D^*))$. It turns out that the capture time is less than or equal to Eq. (23) no matter what the evaders act. Since the evaders wish to maximize the capture time, there's no need for them to intercept the pursuers, which completes the proof in this case. ■

5 SIMULATION

Two simulation cases are developed. One is that the players adopt the equilibrium strategies of the sub-game 1 and sub-game 2; the other one is that the players A and C still adopt the best strategies in sub-game 1 and sub-game 2, while the player B tries his best to intercept the player C and the player D tries his best to escape from the player A. In both cases, $v_A = 5(m/s), v_B = 4(m/s), v_C = 3(m/s), v_D = 2(m/s)$.

a) Case 1

Under the given strategies, the coordinates of the four players with respect to time are shown in Fig. 3 and Fig. 4.

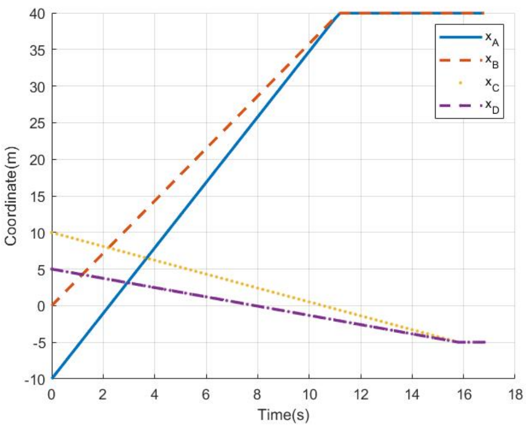

Figure 3: x coordinate of the four players.

The distances of (A, B) and (C, D) with respect to time are shown in Fig. 5. The captures of B and D occur at time 11.171 s and 15.802 s, respectively. Thus, the capture time is 15.802 s.

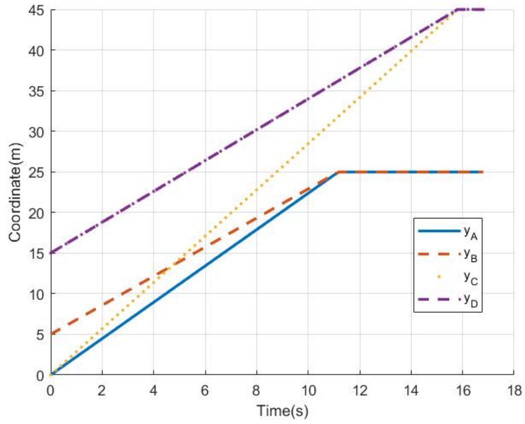

Figure 4: y coordinate of the four players.

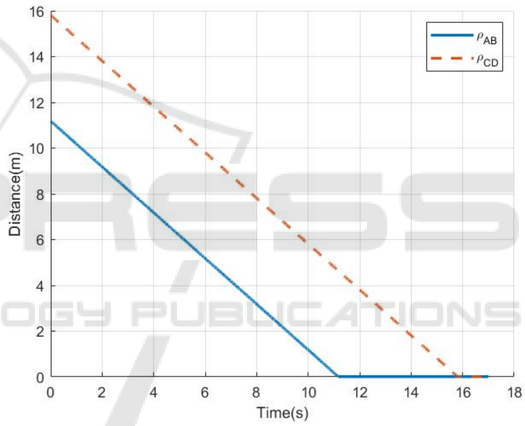

Figure 5: The distances of (A, B) and (C, D) with respect to time.

The trajectories of the four players is shown in Fig. 6.

b) Case 2

Under the given strategies, the coordinates of the four players with respect to time are shown in Fig. 7 and Fig. 8.

The distances of (A, D) and (C, B) with respect to time are shown in Fig. 9. The captures of B and D occur at time 3.472 s and 8.381 s, respectively. Thus, the capture time is 8.381 s.

The trajectories of the four players is shown in Fig. 10.

The capture time in case 1 is greater than case 2, which is consistent with the theoretical results in the previous section.

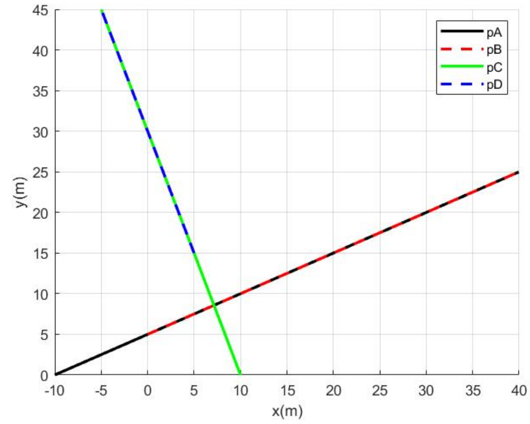

Figure 6: The trajectories of the four players.

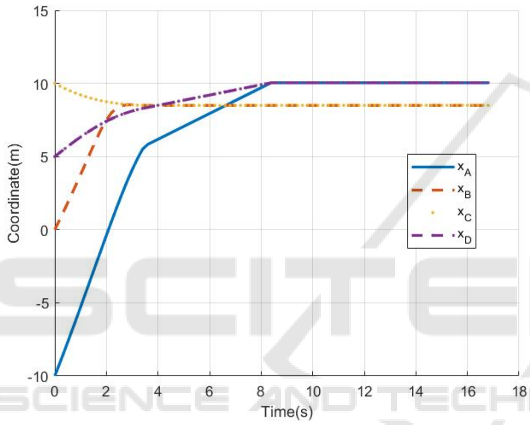

Figure 7: *x* coordinate of the four players.

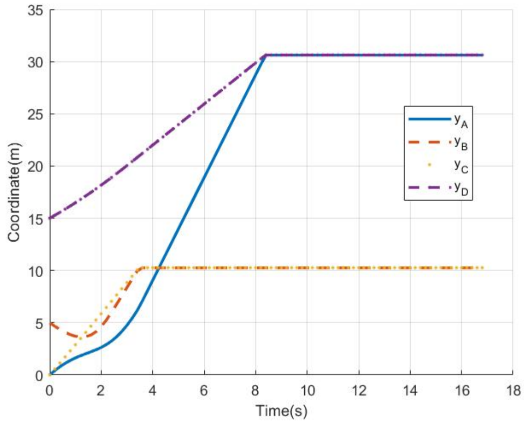

Figure 8: *y* coordinate of the four players.

6 CONCLUSIONS

This paper has investigated a novel two pursuers and two evaders game, considering the dual-role of the players. The evaders act as the evader as well as the pursuer, that is the evader can choose to hit the pursuer to protect his teammate. Accordingly, the pursuers also have two roles, for they might be captured and destroyed by the evader. The state feedback strategies of all the players are presented and proved. In particular, it is rigorously demonstrated that the interception by the evader will not increase the capture time. Therefore, the evaders' best strategies are consistent with the two one-one games. For validation, two simulation cases have been developed and compared. The results have a good agreement with the theoretical analysis.

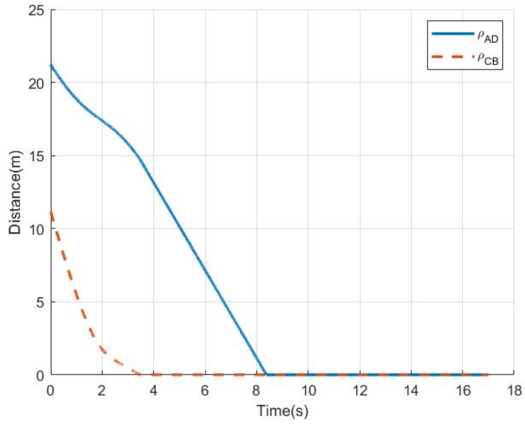

Figure 9: The distances of (A, D) and (C, B) with respect to time.

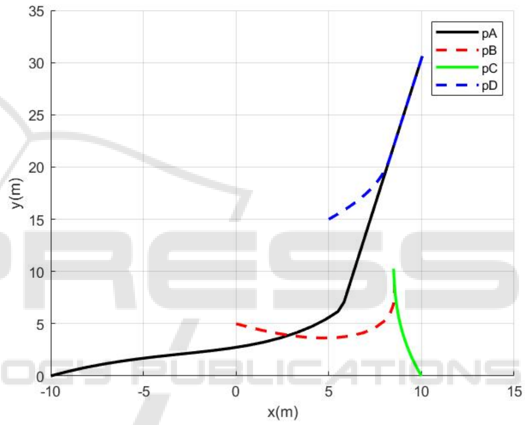

Figure 10: The trajectories of the four players.

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