Low-Orbit Satellite Orbit Prediction Algorithm Based on Near-Polar Circular Orbit

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Abstract: Satellite orbit prediction is a significant research problem in visualizing low-orbit giant constellation networks. However, due to the limited resources of the onboard network equipment, the existing satellite orbit prediction methods are challenging to balance the accuracy and rapidity of the prediction. These traditional forecasting methods tend to construct perturbation models and obtain accurate orbital dynamics equations for calculation. Due to the complicated establishment of the perturbation model and the tedious calculation process, the prediction accuracy of the low-order analytical solution is relatively low, and the calculation efficiency of the high-order analytical solution is not high, which is not suitable for the orbit prediction of large-scale long-period low earth orbit (LEO) satellites. This paper proposes an orbit prediction algorithm for LEO satellites based on near-polar circular orbits. By simplifying the satellite motion model and using the least squares method to fit the data errors, we finally obtain the position information of the LEO satellite constellation network. Experimental results show that the method can perform orbit forecasting of large-scale LEO satellite constellation networks while ensuring accuracy and rapidity compared with satellite tool kit (STK) software.

1 INTRODUCTION

In developing low-orbit giant constellation networking visualization systems, satellite orbit prediction technology is one of the key research directions (Ren et al., 2019; Deng et al., 2021). The prediction speed and accuracy play a vital role in the design and optimization of inter-satellite link visualization, which also directly reflects the authenticity and reliability of the system. Currently, most the satellite orbit prediction is based on the classical mechanics model, which defines the perturbation model by analyzing the satellites' forces, and then establishes the satellite orbit dynamics equations and performs the orbit calculation through the numerical integration method. For LEO satellites, the main regenerative forces affecting the satellite orbit calculation are the second-order band harmonic terms of the earth's non-spherical gravitational field, atmospheric drag, other harmonic terms of the earth's non-spherical gravitational field, and the gravitational force of the Sun and Moon. Since the accuracy of the perturbation model is low, it can cause errors in the traditional calculation methods. The other parameters introduced by the perturbation model can lead to the low prediction accuracy of the low-order analytical solution and the cumbersome process of the high-order analytical solution. (Wang et al., 2018).

This paper proposes a near-polar circular orbit-based orbit prediction algorithm for LEO satellites. The satellite motion model is simplified to a uniform circular motion model with only the force provided by gravity, and the computational errors caused by other perturbations are fitted to the time-dependent primary term by the least squares method and added to the simplified satellite orbital dynamics equation to form a new satellite orbital dynamics equation for orbit calculation. By comparing the data obtained from the high-precision satellite orbit prediction model in STK software with a large amount of data obtained through several experiments, the accuracy and feasibility of the algorithm are verified. The fast and accurate prediction of satellite orbit positions in the LEO giant constellation network visualization system shown in Fig. 1 is achieved with a limited computer system.

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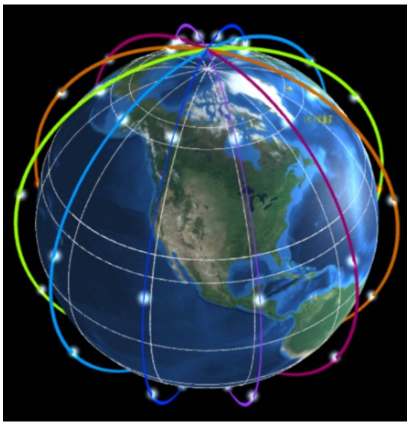


Figure 1: Visualization system diagram of constellation networking.

2 SATELLITE CONSTELLATION MOTION MODEL

2.1 The Law of Motion of the Satellite

Taking the near-polar circular orbit as an example, the motion law of the satellite in an ideal state can be simplified to a two-body problem (Ge et al., 2020). According to the law of gravity, the linear and angular velocities of the satellite orbiting at a given orbital altitude h are calculated as:

$$v = \sqrt{\frac{GM}{r+h}} \quad (1)$$

$$\omega = \sqrt{\frac{GM}{r+h}} \quad (2)$$

here G is the universal gravitational constant in $m^3/(kg \cdot s^2)$, M is the total mass of the earth in kg, r is the mean radius of the earth in m.

2.2 Position Description of the Satellite

The position description of a satellite in a LEO constellation network includes how to represent the position of a point in the orbit and then consider how to represent a point in the constellation network. There are two main methods for describing the position of a point in orbit: namely, phase representation, and latitude and longitude representation (Rabjerg et al., 2020).

In the phase representation, the orbit ascending intersection point is specified as the 0 phase point. As the satellite moves from south to north, the phase varies between $(0, 2\pi)$. It increases linearly along the direction of satellite motion, with the points on the orbit corresponding to the phase values. In order to extend the phase description method to the satellite constellation, it is sufficient to add the orbit number S_i again, denoted by (S_i, φ)).

In the latitude-longitude representation, at any moment, the subsatellite points of the satellite uniquely map a latitude-longitude coordinate. The satellite's orbit and the earth's rotation will cause the mapping relationship to change with time. The latitude and longitude description method can be directly extended to the constellation.

In order to illustrate the calculation process of the longitude and latitude coordinates of the satellite subsatellite, a typical satellite orbit model is introduced as shown in Fig. 2. OA represents the rotation axis of the earth, the orbital inclination can be expressed as i, and the current phase angle of satellite M is φ. Under the premise that the current phase of the satellite is known, the latitude and longitude of the satellite can be obtained according to the geometric relationship in the Fig. 2 as follows:

$$\theta_{lat} = \sin^{-1}(\sin\varphi \sin i) \quad (3)$$

$$\theta_{\text{lng}} = \tan^{-1}(\tan\varphi \cos i) \quad (4)$$

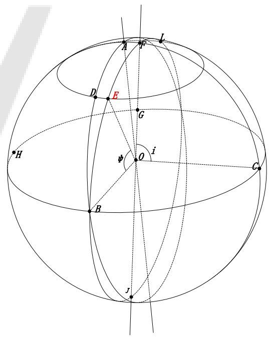


Figure 2: Satellite orbit model.

2.3 Location Distribution Strategy of Satellites

Consider the problem of uniform coverage of the satellite constellation network, the location distribution of satellites can be carried out according to the following strategy. Assume that a satellite constellation network contains M orbits and N satellites on each satellite orbit, and the total number of satellites in the constellation is S. The first orbit is selected as the reference orbit, and the first satellite on the first orbit is the reference satellite. The longitude of the ascending node of the reference orbit is 0°, and the initial phase of the reference satellite is 0 phase point. The satellite orbit starts from the ascending intersection point of the reference orbit and is evenly distributed from west to east. The longitude difference between adjacent orbits ($\Delta\varphi_f$) is π / M; the satellites in the same orbit start from the 0 phase point and are evenly distributed along the direction of the increasing phase. The phase difference between adjacent satellites in the same orbit ($\Delta\varphi_s$) is $2\pi / N$.

From the above, for any satellite in the satellite constellation network S_{mn} (denoting the *n*-th satellite in the *m*-th orbit, where $m = 1, 2, \cdots, M$, $n = 1, 2, \cdots, N$) the latitude and longitude of the initial state can be expressed as:

$$\theta_{lat}(t_0) = \sin^{-1}(\sin\varphi \sin i) = \sin^{-1}(\sin((j-1)\frac{2\pi}{N})\sin i) \tag{5}$$

$$\theta_{\ln g}(t_0) = \theta_{js0} + \tan^{-1}(\tan\varphi \cos i) = (i-1)\Delta\varphi_f + \tan^{-1}(\tan\varphi \cos i) \tag{6}$$

Since the satellite constellation network is in constant motion, the positions of the satellites in the constellation are also changing. The satellite orbit forecast is to calculate the position information of the satellite at t based on the initial position of the satellite and then to obtain the position information of all the satellites in the entire constellation network at t based on the distribution law of the satellites in the constellation network. Considering the time factor, the calculation formula of latitude and longitude is rewritten as

$$\theta_{lat}(t) = \sin^{-1}(\sin\varphi(t)\sin i) \tag{7}$$

$$\theta_{\ln g}(t) = \theta_{jst} + \tan^{-1}(\tan\varphi(t)\cos i) \tag{8}$$

3 POSITION DATA ERROR CORRECTION METHOD OF SATELLITE

When using the satellite orbit motion model and method composed of formula (1) to formula (8), which is called the MNC model algorithm, the obtained satellite position data has a significant error compared with the data obtained by STK high-precision orbit prediction model (MSGP4). The error increases with the increase in prediction time.

The MNC calculation model simplifies the satellite motion problem into a two-body problem. This model does not consider the perturbation caused by the second harmonic term of the earth's aspheric gravitational field, atmospheric resistance, and other disturbances. It results in a significant error in the calculation results.

Because of the difficulty of establishing an accurate perturbation model and the complexity of calculating the acceleration caused by each regression force on satellite motion. The more orders of spherical harmonic coefficients are considered in the integration, the more time consumed. It is not conducive to large-scale orbit position prediction and visualization of LEO satellites with limited system resources.

Therefore, in the subsequent, the least-squares method is used for fitting. We fit the data errors caused by various disturbances into polynomials related to the running time and add them to the original latitude and longitude calculation motion model, which is called the MAC model. It solves the problem that the data obtained by the MNC model and the data obtained by the MSGP4 model have a significant error. Under the condition of real-time and accuracy, the position prediction and visualization of large-scale LEO satellite constellation networks are realized.

4 EXPERIMENTAL RESULTS

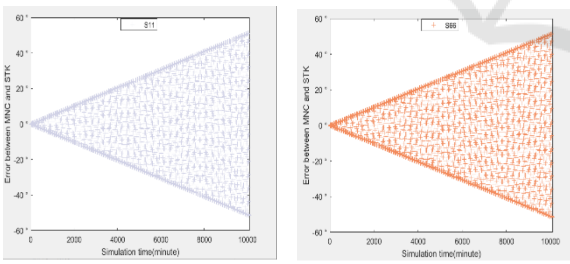
The target satellites in our experiments are low orbit satellites in a near-polar circular orbit. A constellation network of 6 orbits with 9 satellites in each orbit is used as an example, and the satellites are distributed according to the strategy described in subsection 2.3, with an orbit inclination of 90°, and the experiments are conducted under the settings of orbital altitudes of 500 km, 800 km, and 1000 km, respectively.

The experimental environment was configured with a processor AMD Ryzen 7 5800H with Radeon Graphics 3.20 GHz, 16 GB of RAM, and an operating system of 64-bit Windows 10 version 19044.1706.

4.1 Accuracy of Orbit Prediction Algorithms for LEO Satellites in near Polar Circular Orbit

Based on the given orbits, the satellite positions are distributed according to the strategy proposed in the previous section, and a satellite constellation network is established. The initial latitude and longitude of all satellites in the constellation network are obtained by traversing the number of satellite orbits, the number of satellites in orbit, and the latitude and longitude position of any satellite at a given observation moment is obtained by the latitude and longitude calculation algorithm (MNC).

Without loss of generality, we selected any two satellites among 54 stars with an orbital altitude of 500 km and observation time of 7 days for position data calculation and then compared them with the position data calculated by the identical numbered satellites in the same scale constellation network of STK 11.6 software under the MSGP4 model, and the error of the satellite latitude data obtained from the MNC model experiment and the latitude data obtained from the MSGP4 model in STK. The distribution of error with time is shown in Figure 3.



(a) Error of S11 (b) Error of S66

Figure 3: Error distribution plots.

Since our experiment is for a constellation of satellites in a near-polar circular orbit, it can be seen in the longitude calculation equation that when the orbital inclination i is 90, the longitude variation depends only on the mean angular velocity of the earth's rotation ω_e. The value of the mean angular velocity of the earth selected in the experiment is consistent with the value set in the MSGP4 of STK software. For the calculation of longitude, the errors in the data obtained from the two models are minimal and negligible (10^{-3}). Therefore, only the errors in the calculation of latitude are considered in this paper.

Figure 3 shows that the error in the latitude direction is approximately linear for time. The positive and negative error is that the latitude takes a range of (-90°, 90°), and there are negative values. Observing a large amount of data, it is found that the error is positive when the satellite moves from south to north and negative when the satellite moves from north to south. Therefore, we set d as the direction factor and find a primary polynomial by least squares fitting $y = at + b$ to fit the error curve to a time-dependent primary term and add it to the latitude calculation expression as an error correction term to improve the accuracy of satellite position data forecasting. The correction terms at three different altitudes are shown in Table 1.

Table 1 Expressions of correction terms at different orbital heights.

Track height (km)	Correction function	R^2 (fitting degree)	RMSE
500	$y = 0.005094t + 0.00671$	0.9996	0.294
800	$y = 0.004378t + 0.005281$	0.9997	0.212
1000	$y = 0.003973t + 0.00312$	0.9997	0.204

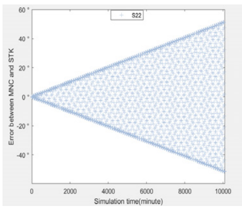
The correction term is added to the satellite latitude calculation model to form a new calculation model, which can be expressed as:

$$\theta_{wt} = \theta_{lat}(t) + dy(t) \tag{9}$$

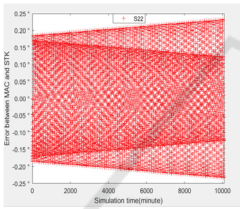
here d is the orientation factor. d is -1 when the satellite is moving from south to north, d is 1 when the satellite is moving from north to south, and $y(t)$ represents the expression of the correction term at the corresponding orbital altitude.

We refer to the satellite orbit prediction model after adding the correction term as the MAC model. By experimenting with the MAC model, the latitude data of the satellite are calculated at three different orbital altitudes. The latitude data of a satellite at three different orbital altitudes are randomly selected and compared with the latitude data of the corresponding satellite in the MSGP4 model of STK. The error of the data obtained from the MAC model compared with the MSGP4 model of STK and the error of the data obtained from the MNC model compared with the MSGP4 model of STK are shown in Fig. 4. From the figure, it can be seen that the data calculated by the MAC model is closer to the data calculated by the MSGP4 model in STK.

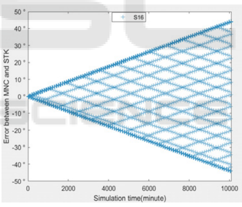
We experimentally tested the accuracy of the satellite orbit prediction models for satellite constellation networks with orbital altitudes of 500 km operating for 7, 15, and 30 days, and calculated the correlation degree of the data obtained from the MAC model proposed in this paper and the GSRPS model in STK, calculated the MSE (mean square error) of the data obtained from the two models, and compared them with the MNC model. The experimental results are shown in Table 2.



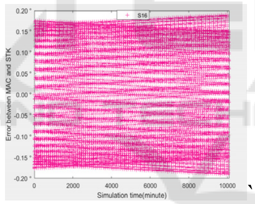
(a)Error of S22 on MNC



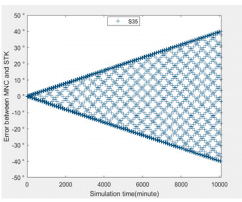
(b)Error of S22 on MAC



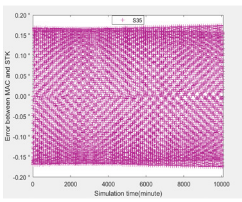
(c)Error of S16 on MNC



(d)Error of S16 on MAC



(c)Error of S35 on MNC



(d)Error of S35 on MAC

Figure 4: Error comparison graph. (a) (b) with h = 500 km, (c) (d) with h = 800km, (e) (f) with h = 1000km.

Table 2 Comparison of the accuracy of satellite orbit prediction models before and after adding correction terms.

Satellite constellation network operation time	MSE for MNC and STK	MSE for MAC and STK	Order of magnitude improvement in data accuracy
7 days	754.41	0.0170	4.438×10^4
15 days	2810.45	0.0201	1.398×10^5
30 days	6309.11	0.0315	2.003×10^5

4.2 Superiority of Orbit Prediction Algorithms for LEO Satellites in near Polar Circular Orbit

The superiority of the near-polar circular orbit LEO satellite forecasting algorithm in terms of execution time is given in this subsection. The calculation method proposed in this paper is applied to a network of LEO satellites with different sizes of a near-polar circular orbit, the orbital altitude is set to 500 km. The operation time is set to 7 days, and the step size is 60 s. The position data of all satellites in the whole constellation are calculated and output as a .txt file, and the execution time of the MAC model algorithm is obtained through experiments, and then compared with the algorithm used by the MSGP4 model in STK software, the experimental results are shown in Table 3.

Table 3 Comparison of the time consumption of the two algorithms under different satellite constellation size.

Number of satellite orbits	Number of satellites per orbit	STK execution time (unit: s)	MAC execution time (unit: s)
6	9	269.98	1.005
10	9	701.11	2.190
12	15	975.43	3.309
188	40	4263.61	13.278

With the increase of satellite constellation network scale, the algorithm execution time gradually increases. In the calculation of the orbit forecast of the same scale satellite constellation network, the speed of the algorithm used in this paper is nearly 300 times higher than that of STK software, which can calculate the position data of the large-scale LEO satellite constellation network more quickly.

5 CONCLUSION

This paper proposes a prediction algorithm for LEO satellite orbits based on near-polar circular orbits. Based on the prediction above model and algorithm, the position data of the satellite at a given observation moment and the time consumed by the algorithm execution are obtained. In the experimental validation, the predicted longitude and latitude position errors are within the acceptable range compared to the STK high-precision orbit prediction model. Four orders of magnitude improve the accuracy compared to before the correction term is added, and the predicted position errors tend to be stable for a time. The computation speed of the algorithm proposed in this paper is nearly 300 times higher than that of the STK software in the same scale LEO satellite constellation, which is more suitable for the long-period orbit prediction of large-scale LEO satellite constellation networks.

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