


The Time-Varying of Topological Characteristics: Analysis Based on the Temporal Network on Public Bikes

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Abstract: Most networks evolve in time. We study the structure of interaction with time. Compared with the traditional public transport, the flexibility of public bikes and the randomness of users' riding behaviours make the riding route and riding time full of uncertainty. It is the task of scientific research to explore the regularities behind these uncertainties. By mining the data of user's riding trajectories; we construct the temporal network and the 24-layer multilayer network respectively. The topological characteristics of network presents double peak. There is a strong correlation between the topological parameters, including positive and negative correlations. Furthermore, bike ridings among stations distribute heterogeneously and the hourly flow of stations distributes heterogeneously. Transport system is a typical complex system. This research provides new evidence for empirical research on temporal network, multilayer network and transport network.

1 INTRODUCTION

In recent years, researches on transport networks have been receiving close attention by the physics community. Transport system is a typical complex system. Scientists use the thoughts and methods of network to study traffic problems. For example, traffic congestion problems (Jang et al., 2019), invulnerability research (Zhang et al., 2018; Cats et al., 2020) and key road identification (Feng et al., 2019).

Traditional public transport systems, composed of buses and subways, have fixed routes, fixed mileage, fixed running time, and fixed running interval. We study the public transport system composed of bikes in this paper. Compared with traditional public transport systems, the randomness of individual users' riding behavior makes the riding routes, riding mileage and riding durations full of uncertainty. It is the task of scientific research to explore the regularities behind these uncertainties.


Traditional studies on complex network consider time-independent structures, but most networks evolve in time (Porter, 2020). In this paper, we study the temporal network, structure of interaction with time. The time-dependent nature of

the network reflects the nature of system, and these time-dependent behaviors are manifestations of human behavior. Citi Bike of New York is a public bikes system. Based on user's riding data, the flow information among stations and the time information of user's riding are excavated. We analyze the flow-weighted temporal network and the 24-layer multilayer network respectively, to capture the unobservable characteristics of networks under the static model.

The paper is arranged as follows. We introduce related works on transport networks in the second part. In the third part, we introduce the public bike system of Citi Bike, and the modeling method of the temporal network and the definition of the 24-layer multilayer network are given. Then the topological characteristics of the networks are analyzed in the fourth part. Finally, we summarize the conclusion.

2 RELATED WORKS

Recent years, the application of network ideas to the study of public transport has become a research

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hotspot. Sienkiewicz et al. established the public transport network of Polish cities, and found the small world and hierarchical characteristics (Sienkiewicz et al., 2005). Ferber et al. studied the public transport systems in major cities such as Los Angeles, and found that: the networks show the characteristics of small world or scale free (Ferber et al., 2009). Derrible et al. analyzed the urban subway system and found that most of the subway networks are scale free (Derrible et al., 2010). Taking streets of London and highways of American for instance, Viana et al. applied multidimensional scaling methods to visualize the small world characteristics of the network (Viana et al., 2011). Xu et al. analyzed the public transport of Chinese cities and found that these public transport networks have small world features (Xu et al., 2013). Gallotti et al. studied the public transport in Britain from the perspective of multi-layer network (Gallotti et al., 2015). Using methods of network, Bona et al. analyzed the public transport of Curitiba in Brazil, and found the characteristics of small world and scale free (Bona et al., 2016). Using the network approach, Ren et al. analyzed the public transport of Shenyang and found that the network is scale free (Ren et al., 2016). Candelleri et al. analyzed the public transport of Florence in Italy and Attica in Greece, and the networks were found to be potentially vulnerable (Candelleri et al., 2019). Yang et al. developed a network analysis model to study the accessibility of public transport (Yang et al., 2019). Using smart card data, they examined the association between public transport in Wuhan and urban accessibility. Ma et al. established a bus-subway network, constructed a vulnerability operator, and studied the impact of rainstorm on urban public transport (Ma et al., 2019). Based on the network model, Yu et al. studied the spatial and temporal distribution of the metro passenger flow in Nanjing (Yu et al., 2020). Wang et al. analyzed the bus systems in Hohhot by using network modelling, and the network has the characteristics of small world and robustness (Wang et al., 2020). Cao et al. analyzed the public transport in Changsha and found that the network has scale-free characteristics but does not satisfy the small world characteristics (Cao et al., 2020).

The researches on the above mentioned public transport networks are mainly focused on the bus and subway systems. These traditional public transport systems have fixed routes, fixed mileage, fixed running time, and fixed running interval. Unlike them, bikes are more convenient. Public bikes have recently entered the field of public transport as a new sharing tool. The public bikes take on the

functions of public transport, enrich the types of public transport, and enhance the efficiency of public transport.

In the past years, scientists have used thoughts of networks to study the public bike system in London and some cities of China. Munoz et al. studied the London public bike system from the perspective of network (Munoz et al., 2018). Communities were regarded as nodes. If there were public bike riding trajectories between communities, the corresponding nodes were connected. Saberi et al. regarded public bike stations as nodes of the network. If there was riding behaviour between stations, the nodes were connected. They found that the cumulative degree distribution of London public bike network is power law (Saberi et al., 2018). Using the same modelling method, Wei et al. studied public bike system of Yixing city, and found that the degree distribution and strength distribution of the network obeys normal distribution (Wei et al., 2019). Yao et al. constructed a public bike network of Nanjing, with stations as nodes and the number of rides between stations as the edge weight. They found that the degree distribution of the network is power law (Yao et al., 2019). In addition, Shi et al. used the same method to build the public bike network of Hangzhou. They divided the network community by different modular algorithm (Shi et al., 2019).

Citi Bike is the largest bike sharing program in the United States. The existing literature on the Citi Bike system is mainly on traffic flow prediction. Based on clustering and geographically weighted regression, Bao et al. constructed the relationship between traffic flow and various factors. They found that the split riding model gave a better prediction (Bao et al., 2018). Wang et al. predicted bike demands based on the feature model with contextual, correlation and user features (Wang et al., 2018). Using graph convolution neural network, Lin et al. (Lin et al., 2018) and Yang et al. (Yang et al., 2018) predicted the bike demands per hour.

3 DATA AND METHODS

3.1 Data

The Citi Bike is designed for quick trip, and it is a fun and affordable way. From the Citi Bike official website (<https://www.citibikenyc.com>), download the user's riding trajectory data of October 3, 2017. The format of the initial data in the Citi Bike system is shown in Figure 1. Each column represents a complete riding trajectory, including the riding duration,

the start time and start location, and the end time and location. The time information is accurate to seconds, and the location information contains the longitude and latitude of the station. In addition, user types include subscriber and non-subscriber. The subscriber records gender characteristic, and gender tags include 1 (male) and 2 (female). While the non-subscriber does not record gender characteristic and tag 0 is used to indicate.

For instance, the second column of Figure 1 shows a riding trajectory of a male subscriber. At 00:00:00 on October 1, 2017, he rented a public bike with ID 30951 at station "9 Ave & W 45 St". After riding for 457 seconds, he returned to the station "11 Ave & W 41 St". Delete trajectories with too short riding durations, considering the abnormal ridings caused by vehicle failure or other reasons. Delete the trajectories with riding duration less than one minute. Delete trajectories with too long riding durations, considering the abnormal ridings caused by vehicle theft, user forgetting or other reasons. Delete the trajectories with riding duration longer than six hours. After data preprocessing, 69066 valid data of the riding trajectories are retained.

Attributes	Trip1	Trip2	...
Trip duration (Seconds)	457	6462	...
Start time	2017-10-01 00:00:00	2017-10-01 00:00:20	...
Stop time	2017-10-01 00:07:38	2017-10-01 01:48:03	...
Start station ID	479	279	...
Start station name	9 Ave & W 45 St	Peck Slip & Front St	...
Start station latitude	40.76019252	40.707873	...
Start station longitude	-73.9912551	-74.00167	...
End station ID	478	307	...
End station name	11 Ave & W 41 St	Canal St & Rutgers St	...
End station latitude	40.76030096	40.71427487	...
End station longitude	-73.99884222	-73.98990025	...
Bike ID	30951	14809	...
User type	Subscriber	Customer	...
Birth year	1985	NULL	...
Gender	1	0	...

Figure 1: Examples for the data of user's riding trajectories in Citi Bike.

3.2 Methods

Based on user's riding data, the flow information of users among stations and the time information of user's riding are excavated. The flow information will be used to build the weight of the edge. The time information of user's riding will be the basis for constructing the time layer. Taking the riding time as the hierarchical label, the flow-weighted temporal network and the 24-layer multilayer network are constructed respectively.

The network modelling method is illustrated by taking the data of four riding trajectories as an example. Assume that there are four riding trajectories, as shown in Figure 2(a). There are two riding trajectories from station A to station B. One riding trajec-

tory from station A to station C, and the other one from station B to station C. The bike station is regarded as the node of the network. An edge will be linked between the nodes if the riding behaviour occurs between the stations. The direction of the edge is from the start station to the end station. The traffic flow between the stations is taken as the weight of the edge. A flow-weighted directed network is constructed. According to the time information of riding behaviour, the temporal network is established, as shown in Figure 2(b).

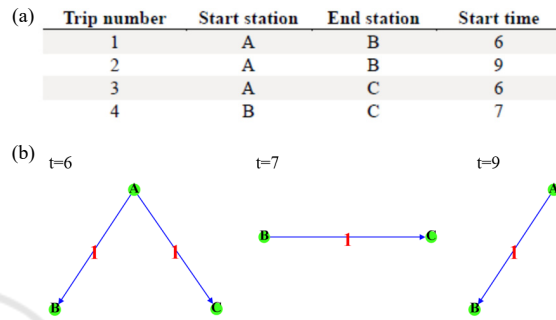


Figure 2: The modelling diagram of temporal network.

When $t = 6$, there are two riding trajectories generated from station A to station B and to station C. Thus, the network at $t = 6$ is composed of three nodes A, B and C. While, at time $t = 7$, there is only one riding trajectory generated from station B to station C, thus the network at $t = 7$ consists of node B and node C. Compared with the static network, nodes in the temporal network change dynamically with time. In addition, edges between nodes in the temporal network are not persistent. For example, there is no edge between nodes B and C at $t = 6$, but directed edges exist between nodes B and C when $t = 7$. The temporal network enables understanding of network changes over time.

Through the above modelling method, each time layer network can be obtained, and we can study changes of the network over time. In addition, all time layers are coupled into a network without considering the inter-layer links. That is, the adjacency matrix of each layer is coupled in a hyper-adjacency matrix. Suppose that the adjacency matrix of each layer network is $A_{(t)}$, $t = 0, 1, \dots, 23$. $A_{(t)}$ is an n_t -order square matrix. Where n_t represents the number of nodes per time layer network. They are different from layer to layer. Without considering the inter-layer links, the hyper-adjacency matrix is defined as

$$A = \begin{bmatrix} A_{(0)} & 0 & \dots & 0 \\ 0 & A_{(1)} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & A_{(23)} \end{bmatrix} \quad (1)$$

Hyper-adjacency matrix A is an $\sum n_t$ -order square matrix. It corresponds to a 24-layer multilayer network.

4 NETWORKS

4.1 Temporal Networks

The temporal network dynamically displays the changes in the system twenty-four hours a day. According to the above method, the temporal network is established, as shown in Figure 3. Obviously, the network is time-varying. At $t = 3$, the network is sparse. However, at $t = 8$ and $t = 17$, the network is obviously dense, and the naked eye can no longer count the number of nodes and edges. In addition, the hub nodes (red colour) have also changed with time. Through the visualization of these 24 networks, we can intuitively find that the network is denser during the day and sparse at night. The dynamic change of the network with time is related to the law of human behaviour. During rush hours on and off work, a large number of riding behaviours occur, which change the topology of the network.

In temporal network, the number of nodes and the number of edges are time-varying. The time series of changes is shown in Figure 4. On the left and right sides of the ordinate, the number of nodes and the number of edges are identified respectively. The number of edges shows bimodal feature. The first peak appeared at 8 o'clock and the second peak appeared at 17 o'clock, which indicate that a large number of riding trajectories occurred during these two periods. This is related to the law of human behaviour. More users use public bikes during rush hours. During the period from $t = 6$ to $t = 20$, the number of nodes in the network is stable. In the system, there are about 600 stations with bike rental. When $t = 3$, the number of nodes and the number of edges are the least. At this time, there are only less than one hundred stations with bike rental.

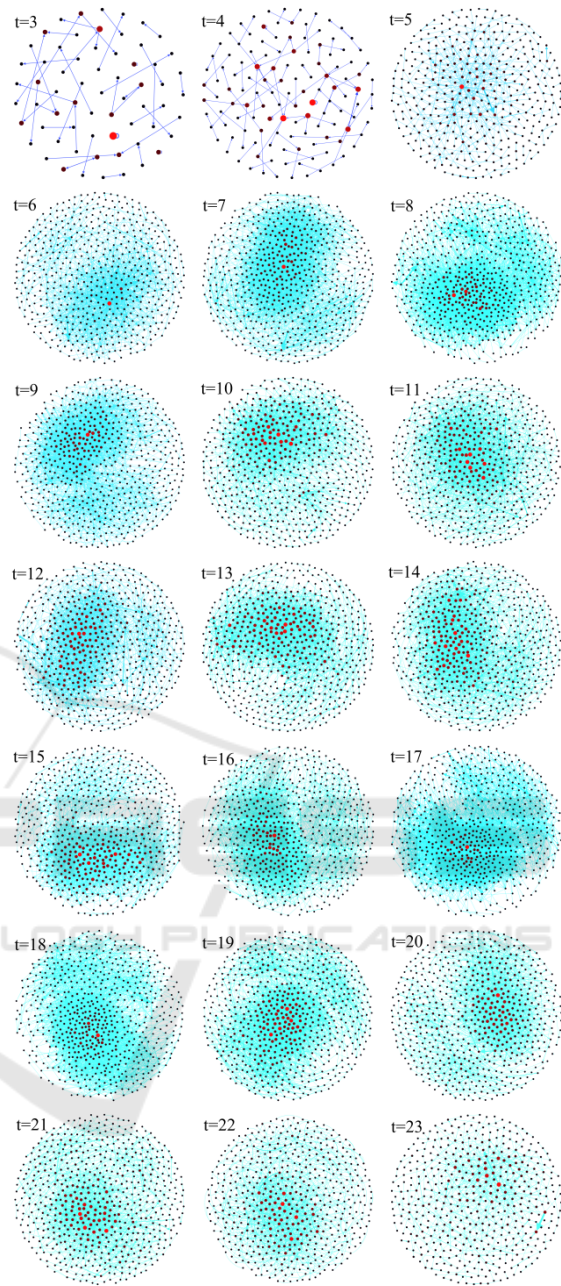
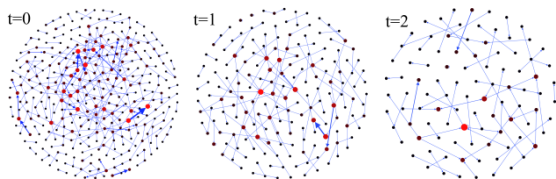


Figure 3: The temporal network.

In a network, the number of connected edges is called the degree of the node; in a weighted network, the sum of the edges weights is called the strength of the nodes. In our flow-weighted temporal network, the degree k_i of node i indicates that there are riding trajectories between the station i and other k_i stations, and the strength s_i represents the total traffic flow between station i and other k_i stations. The time-series changes of the topological characteristic parameters of the temporal network are shown in Figure 5. The

average degree $\langle k \rangle$, the average strength $\langle s \rangle$, and the clustering coefficient C show obvious bimodal characteristics. The two peaks appeared at 8 a.m. and 17 p.m. respectively. This is consistent with the bimodal feature of the number of edges in the temporal network. When studying the time-varying nature of temporal network, we find that some topological parameters are strongly correlated.

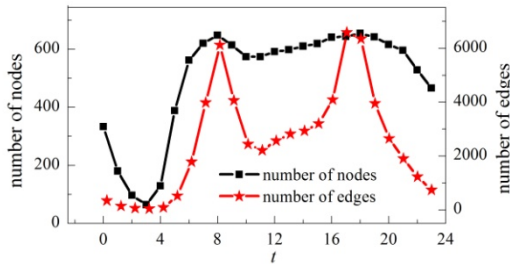


Figure 4: The number of nodes and edges of the temporal network.

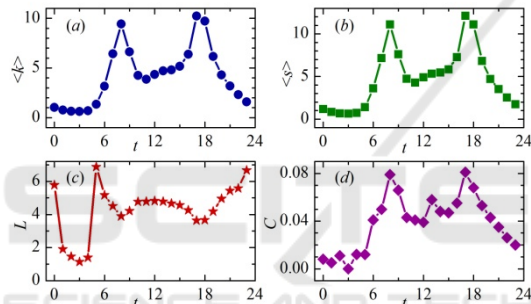


Figure 5: The topological characteristics of temporal networks: average degree, average strength, average smallest path length and average clustering coefficient.

In fact, by analyzing correlations of topological parameter time series, it can be found that: the number of edges, average degree, average strength and clustering coefficient show strong positive correlation (as shown in Figure 6); when the number of nodes is stable (from $t = 6$ to $t = 20$), the average shortest path length shows a strong negative correlation with other topological parameters (as shown in Figure 6(b)). When the number of nodes is fixed, the average degree of the network will be larger if there are more connected edges. More edges will bring more total edge weights, so the average strength will be greater. The existence of more edges will make it easier to form triangles, and then the average clustering coefficient will be larger. In addition, more edges increase the accessibility of the network, which results in a smaller average shortest path length. It can be found that the average shortest path length of the network is smaller at 8 a.m. and 17 p.m. Therefore,

from 6:00 to 20:00, the number of nodes is stable, and the number of edges, the average degree, the average strength and the clustering coefficient show a highly similar time-series trends. While the average shortest path length exhibits the opposite trends.

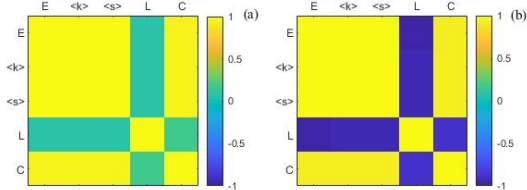


Figure 6: Diagram for the correlation matrix of topological parameters. The analysis object of (a) is the whole time series of five topological parameters; and (b) is for fragments of time series, from $t = 6$ to $t = 20$.

4.2 The 24-Layer Multilayer Network

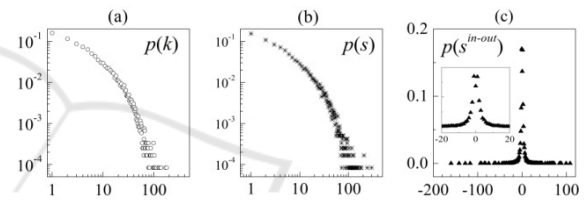


Figure 7: The degree distribution $p(k)$, the strength distribution $p(s)$ and the strength-difference distribution $p(s^{in-out})$ of the 24-layer multilayer network.

The time-varying characteristics of network topology are found through the temporal network. On the other hand, according to Equation (1), all the time layers are coupled, and the 24-layer multilayer network is used to analyze the Citi Bike system as a whole. The network has 11988 nodes and 60786 edges. The degree distribution and the strength distribution are all power law distribution, as shown in Figure 7(a) and Figure 7(b). Eighty percent of nodes have a degree value less than 15. Only one thousandth of the nodes has a degree value greater than 100. Bike ridings among rental stations distribute heterogeneously. Furthermore, ninety percent of nodes have a strength value less than 30. Nodes with strength value greater than 100 account for three thousandth. At bike rental stations, the hourly bike flow distributes heterogeneously.

Strength-difference of a node is equal to the in-strength minus the out-strength, which can measure the net flow of information at the node. In the 24-layer multilayer network, the strength-difference s_i^{in-out} of node i represents the net bike flow at the rental station i . When $s_i^{in-out} \approx 0$, the inflow and outflow of bikes are in balance and the site is called a balanced

station. When $s_i^{in-out} > 0$, the inflow is large and the outflow is small. For this type of site, we need to consider the transfer out of bikes when scheduling. When $s_i^{in-out} < 0$, the inflow is small and the outflow is large. Such site needs to transfer in bikes to meet larger rental demands of bikes. The strength-difference distribution $p(s^{in-out})$ of the 24-layer multilayer network obeys Gaussian distribution, as shown in Figure 7(c). The mean of the distribution is 0.045 and the standard deviation is equal to 2. In general, at most sites, the number of rental bikes and the number of returned bikes can maintain a balance.

5 CONCLUSIONS

Transport system is a typical complex system. We study the structure of networks evolving with time. In the temporal network, the number of edges, the average degree, the average strength, and the clustering coefficient present obvious bimodal characteristics. The two peaks appeared at 8 a.m. and 17 p.m., which is consistent with rush hours on and off work. The time-dependent nature of the network reflects the nature of system, and these time-dependent nature are manifestations of human behaviour. In the 24-layer multilayer network, the degree distribution is power law, the strength distribution is power law, and the strength-difference distribution obeys Gaussian. In the system, bike ridings among stations distribute heterogeneously and the hourly flow of the station distributes heterogeneously. In most stations, the number of rental bikes and returned bikes maintain balance. Furthermore, in temporal network, we found strong correlations of topology parameters. The research provides evidence for empirical researches on temporal network, multilayer network and transport network.

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