Stability of Circular Toroidal Shell Subjected to Uniform External Pressure

Chengyuan Zhou and Shijie Su
Jiangsu University of Science and Technology, Zhenjiang, Jiangsu 212003, China

Keywords: Circular Toroidal Shell; Stability; Numerical Calculations; Shell Unit; Solid Unit.

Abstract: In this paper, the theoretical and numerical analysis methods for the stability of circular torodial shells under external pressure are studied. Effects of unit type, unit density and boundary condition on the stability analysis of circular toroidal shell are discussed. A reasonable method for establishing analysis model is proposed. The theoretical and numerical solution are compared with the experimental value. The results show that the numerical solution is consistent with the experimental result, while the theoretical solution calculated by the Jordan's formula has a large deviation.

1 INTRODUCTION

The circular toroida shell structure is widely used in various industrial fields, such as underwater pressure shells, reservoirs, tokamak devices, etc. The circular toroida shell solved the problem of the overall arrangement of space and personnel connection, thus it became the main structural form in the underwater space station.

Since the 1960s, the problem of the bowing of the circular toroidal shell has begun to receive attention. Machnig first researched the buckling problem of a circular toroidal shell under hydrostatic pressure in 1963. L.H. Sobel by expanding the buckling displacement component along the direction of the ring and meridian direction as a double triangle number (Flügge W, and Sobel, L. H，1965), the stability equation of the circular toroidal shell under uniform external pressure is processed, the stress state before buckling is obtained by the no moment solution. Fishlowitz, et al., proved that for less thin ring shells, the buckling mode is rotationally symmetric and antisymmetric to the equatorial plane (Fishlowitz,E.G,1972). Jordan derived the formula for calculating the critical pressure based on the DMV equation of the shallow shell (Jordan,P.F,1973). Cui and Du et al researched the stability of circular toroidal shells by means of theoretical analysis, numerical simulation and experimental verification (Du , 2015; Du, 2010).

Due to the complexity of the circular toroidal shell structure itself, for its buckling problem, there is no comprehensive and uniform standard. With the development of simulation technology, scientists have begun to use more and more methods of numerical analysis and experimental verification to conduct study. However, due to different constraints, unit type, unit density, mesh generation and solution methods are chosen by operators in numerical analysis, it will lead to the results of numerical analysis far apart.

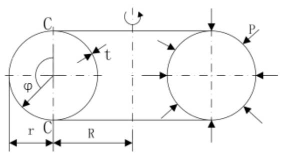
In this paper, the stability of the circular toroidal shell under external pressure was studied from theoretical analysis and numerical calculation. The buckling load prediction formula (Jordan formula) of circular toroidal shell was analyzed and its characteristics and applicability were studied; numerical models of shell unit and solid unit of toroidal shells were established to research the effects of unit type, unit density and boundary conditions on the stability of circular toroidal shell; different thickness ratio t/r the circular toroidal shells were established to analyze and compare the numerical calculation results of the buckling load with the calculation results of the Jordan formula.

Zhou, C. and Su, S.
Stability of Circular Toroidal Shell Subjected to Uniform External Pressure.
DOI: 10.5220/0011925300003536
In *Proceedings of the 3rd International Symposium on Water, Ecology and Environment (ISWEE 2022)*, pages 117-122
ISBN: 978-989-758-639-2; ISSN: 2975-9439

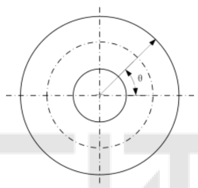
2 THEORETICAL STUDY ON STABILITY OF CIRCULAR TOROIDAL SHELL

2.1 Structural Parameter of Circular Toroidal Shell

A diagram of the circular toroidal shell structure is shown in Fig. 1, P represents the static external pressure, φ represents the meridian direction coordinate, r is the radius of the circle midsection of the shell, R is the distance from the center of the circle to the axis of rotation, t is the thickness of the circular toroidal shell, and θ is the ring direction.



(a) C-C cross-sectional view of toroidal pressure shell



(b) Top view of toroidal pressure shell

Figure 1: Diagram of the circular toroidal shell structure.

2.2 Prediction Formula of Buckling Load of Circular Toroidal Shell

Jordan proposed the following formula to predict the buckling load of a circular shell（Jordan, 1973）:

$$P_{0a} \approx 0.1738\mathrm{E}\left\{\frac{(t/r)^7}{(R/r)^2\left(1-\nu^2\right)^2}\right\}^{1/3} \quad (1)$$

Where E is the modulus of elasticity and ν is Poisson's ratio.

The eight test models in reference were calculated using Eq. (1) (Fishlowitz, 1972), the results of which are shown in Table 1. Comparing the test results of P_e, the errors ((P_e - P_{0a}) / P_{0a} *100%) are -16%, 2%, -5%, 4%, 4%, 12%, 32%, and 9%, respectively. It can be seen from Table 1 that the results of the P_e and Jordan formulas of test models 1 and 7 are quite different because model 1 experienced a premature partial failure due to the existence of the exhaust pipe; while model 7 experienced a stable post-buckling deformation due to the small value of Rt/r^2. Experiments show that the Jordan formula can effectively predict the buckling load of the circular toroidal shell under external pressure within a certain parameter range (Jordan，1973)

Table 1: Comparison of theoretical value P_{0a} and Fishlowitz test value P_e (Jordan, 1973).

	R/r	r/t	t	P_{0a}	P_e	$P_{e/}$ P_{0a}
1	7.94	23.3	1.092	0.068	0.057	0.84
2	3.57	24.7	2.032	0.102	0.104	1.02
3	2.34	47.7	1.461	0.029	0.028	0.95
4	2.35	23.4	2.946	0.153	0.159	1.04
5	1.37	92.7	1.092	0.009	0.009	1.04
6	1.36	48.9	2.083	0.039	0.044	1.12
7	1.19	105.4	0.965	0.007	0.010	1.32
8	1.19	47.5	2.134	0.046	0.050	1.09

It is worth noting that Jordan formula is sensitive to thickness to diameter ratio of circular shells, which is t/r, however, if the change in thickness t occurs away from the Gaussian curvature change point (point C in Figure 1 (a)), the fluctuation of t will have little effect on the buckling load (Jordan，1973). And considering the Poisson's ratio ν is in the range [0, 0.5], and the Poisson's ratio of common metal materials is around 0.3. Therefore, the Jordan formula is not very sensitive to Poisson's ratio.

However, Jordan's prediction formula can't be generalized to the scope of thick shell, because it is based on thin shell theory (Galletly,1995; Jordan, 1965; Jordan, 1966); even in the scope of thin shells, the scope of the Jordan formula is also limited, when Rt/r^2 is small, the calculation result is necessarily conservative (Jordan, 1973). Therefore, it is still necessary to analyse the stability of the circular toroidal shell under external pressure by using numerical analysis.

3 NUMERICAL ANALYSIS OF STABILITY OF CIRCULAR TOROIDAL SHELL

Considering the comparison between Fishlowitz's experimental and existing analysis and the "perfectness" of the test model, the numerical analysis model uses the parameters of the model 8 of the Fishlowitz test. (The test value is P_e=0.0504MPa), the specific parameters are as follows: R=120.6mm，r=101.3mm，t=2.134 mm，E=2240.8MPa，ν=0.4.

3.1 Type of Shell Unit

Creo was used to establish a three-dimensional model of the surface structure. When modelling, the circular toroidal shell was artificially divided into two parts from the Gaussian curvature point, then it was imported into ANSA for mesh division to generate an INP file, which was finally imported into ABAQUS (K.Hibbitt, 2006). In addition, the calculated load was applied to the outer surface of the circular toroidal shell with a uniform pressure. Three-dimensional model and meshing are shown in Figure 2.

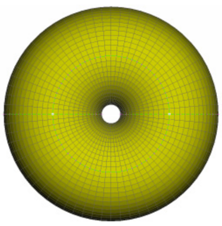


Figure 2: Three-dimensional model and meshing.

In theory, the circular toroidal shell is unconstrained under external pressure, in order to eliminate the rigid displacement of the model without hindering the relative deformation (Jian Zhang, 2015), this paper referred to the Chinese ship classification society's constraint on the spherical shell and the reference the suggestion for the constraint position and the suggestion of applying symmetric boundary condition or antisymmetric boundary condition in the analysis of symmetric structures (Blachut,2000). The three-point constraint and the four-point constraint with 90° symmetry of the spherical shell were set, and the linear buckling analysis was carried out in ABAQUS, namely eigenvalue buckling prediction analysis. Single factor control variable method was used to analyse the influence of unit type, unit density, boundary condition on the stability of the circular toroidal shell.

The traditional method that seeds were arranged along *R* and *r* directions were used for meshing, this method is the same as the random division method. Unit type was selected as 4-node fully integrated linear universal shell unit (S4), and the number of mesh units was 53424, and the boundary conditions of three-point constraint and four-point constraint were set, corresponding to plan 1 and plan 2.

Unit type was set as 4-node fully integrated linear universal shell unit (S4), 4-node reduced integral linear universal shell unit (S4R), 4-node degree reduced integral linear thin shell unit with 5 degrees of freedom per node unit (S4R5), 8-node reduced integral linear thick shell unit (S8R), 8-node degree reduced integral linear thin shell unit with 5 degrees of freedom per node (S8R5), the number of units is 53424, the boundary condition are all four-point constraint, corresponding to plan 2, plan 3, plan 4, plan 5 and plan 6.

Meshes with average sizes of 3mm, 5mm, 7mm, 9mm, 11mm, 13mm and 5mm were set to research mesh convergence. Unit types were all S8R, The boundary condition were all four-point constraint, corresponding to plan 5, plan 6, plan 7, plan 8, plan9, plan10, plan11, plan12. The above shell unit plan and numerical analysis results are shown in Table 2.

Table 2: Shell unit plan information and numerical analysis results.

Plan	Boundary condition	Number of units	Unit type	P_c (MPa)	$(P_c-P_e)/P_e$	$(P_c-P_{0a})/P_{0a}$
1	three-point constraint	53424（3）	S4	0.050601	0.4%	6.14%
2	four-point constraint	53424（3）	S4	0.050601	0.4%	6.14%
3	four-point constraint	53424（3）	S4R	0.050549	0.3%	6.03%
4	four-point constraint	53424（3）	S4R5	0.050515	0.2%	5.96%
5	four-point constraint	53424（3）	S8R	0.050445	0.09%	5.81%
6	four-point constraint	53424（3）	S8R5	0.050466	0.13%	5.86%
7	four-point constraint	19456（5）	S8R	0.050473	0.15%(0.84%)	5.87%
8	four-point constraint	9720（7）	S8R	0.050517	0.23%(1.55%)	5.96%
9	four-point constraint	5880（9）	S8R	0.050576	0.35%(2.46%)	6.09%
10	four-point constraint	3944（11）	S8R	0.050652	0.50%(3.53%)	6.25%
11	four-point constraint	2784（13）	S8R	0.050749	0.69%(5.11%)	6.45%
12	four-point constraint	2100（15）	S8R	0.050865	0.92%(6.69%)	6.69%

Note: The third column of parentheses is the average size of the unit, the sixth column of brackets is the error comparison between the numerical calculation value of S4 unit and the test result, P_c represents the result of numerical analysis

3.2 Type of Solid Unit

The plan of the solid unit mesh can be obtained by using the Create Bottom-Up Mesh in Mesh model in ABAQUS, to offset the shell unit mesh alone the thickness direction. And the material, mesh type and boundary conditions were redefined.

The unit types were set as 8-node linear solid unit (C3D8), 8-node reduced integral unit t (C3D8R), 20-node complete integral unit (C3D20), 20-node quadratic reduction integral unit (C3D20R), and 8-node linear non-coordinating mode solid unit (C3D8I), corresponding to plan 13, plan 14, plan 15, plan 16 and plan 17.

Mesh sizes with average sizes of 3mm, 5mm, 7mm, 9mm, 11mm, 13mm and 15mm were set to research mesh convergence, unit types were all C3D20R, the boundary condition were all four-point constraint, corresponding to plan 16, plan 18, plan 19, plan 20, plan21, plan22, plan23. The load and materials of all solid unit plans were consistent with the shell unit, constraint mode was four-point symmetric constraint, along the corresponding shell unit, and solid unit symmetrically offset 3 layers in the thickness direction. This above unit plan and numerical analysis results are shown in Table 3.

Table 3: Solid unit plan and numerical analysis results.

Plan	Number of units	Unit type	P_c（MPa）	$(P_c\text{-}P_e)/P_e$	$(P_c\text{-}P_{0a})/P_{0a}$
13	160272（3）	C3D8	0.067913	34.75%	42.45%
14	160272（3）	C3D8R	0.046867	-7.01%	-1.69%
15	160272（3）	C3D20	0.050621	0.44%	6.18%
16	160272（3）	C3D20R	0.050612	0.42%	6.16%
17	160272（3）	C3D8I	0.050728	0.65%	6.41%
18	58368（5）	C3D20R	0.050600	0.40%(1.02%)	6.14%
19	29160（7）	C3D20R	0.050580	0.36%(1.68%)	6.10%
20	17640（9）	C3D20R	0.050556	0.31%(2.72%)	6.05%
21	11832（11）	C3D20R	0.050892	0.98%(4.87%)	6.75%
22	8352（13）	C3D20R	0.051067	1.32%(8.03%)	7.12%
23	6300（15）	C3D20R	0.051160	1.51%(12.24%)	7.31%

Note: The second column of parentheses is the average size of the unit. The fifth column of brackets is the error comparison between the calculated value of the C3D8I unit and the test result. P_c indicates numerical analysis results.

3.3 Analysis of Numerical Analysis Results

3.3.1 Shell Unit Numerical Result Analysis

Five different types of shell elements were compared in plan 2 - plan 6, it would be found that the error between numerical calculation results and test results of universal shell unit (S4 and S4R), thin shell unit (S4R5 and S8R5) and thick shell unit (S8R) is less than 1% by comparing with Fishlowitz experimental values, considering the precision and efficiency of numerical calculations, S8R should be selected as the unit type for numerical analysis of circular toroidal shell stability under external pressure.

The numerical results of the average size of the seven units are compared in plan 6 - plan 12 for mesh convergence analysis, the comparison between the numerical calculation results of some unit types and the experimental results is shown in Fig. 3.

From Table 2 and Figure 3, it can be seen that for the S8R unit type, the mesh refinement operation has little effect on the convergence of the numerical analysis results, even if the division is very "rough" mesh, such as plan 12, the error between the numerical analysis results and the test results is still less than 1%, the superiority of the S8R in numerical analysis was proven again.

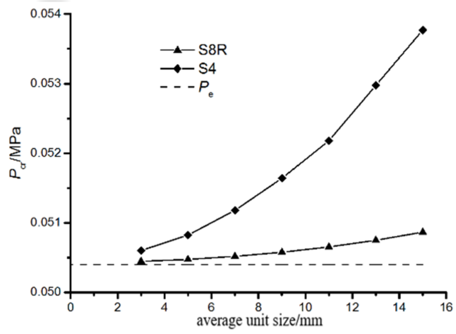


Figure 3: Comparison of numerical calculation results of some unit types with experimental results.

Combined with the comparison results of the universal shell unit S4, the unit average length is preferred to select the two unit numerical results and the test result error is less than 1% of the 5mm length dimension. In the circular toroidal shell, since $R>r$, it is more reasonable to evaluate the average size of the unit with taking r as a reference, so, 5% of the r size should be prioritized as average unit size for numerical analysis of circular toroidal shell stability under external pressure.

3.3.2 Solid Unit Numerical Result Analysis

By comparing plan 13 - plan 17, the results of numerical analysis using C3D8 and C3D8R were found to be significantly different from the experimental results of Fishlowite, especially the C3D8, the error was 34.75%. The numerical analysis results using the C3D20, the C3D20R, and the C3D8I are highly consistent with the results of using the shell unit, the error with the results of Fishlowite experimental was less than 1%.

Due to the huge computational workload of the C3D20, the relative computational efficiency is much lower than that of the C3D20R and the C3D8I, comparison of errors with the results of Fishlowite test, obviously, the C3D20R is the first choice for the numerical analysis of the stability of the circular toroidal shell under external pressure.

By comparing plan 16-plan 23, it can be found that for the C3D20R, with mesh refinement, numerical analysis and test results vary from large to small, then from small to large ,the error is the smallest when the average mesh size is 9mm, which is 0.31%.

However, considering the average size of 5mm, the calculation error of the C3D8I unit is close to 1% and the calculation efficiency of the unit is higher [18]. It should be mentioned that in the case of a small unit distortion (C3D8I unit is sensitive to distortion), a C3D8I (0.05r) unit with an average size of 5mm should be considered first; otherwise the C3D20R unit with an average size of 9mm (0.09r) should be chosen. The comparison between the numerical analysis results and the test results of C3D20R unit and C3D8I unit is shown in Fig. 4.

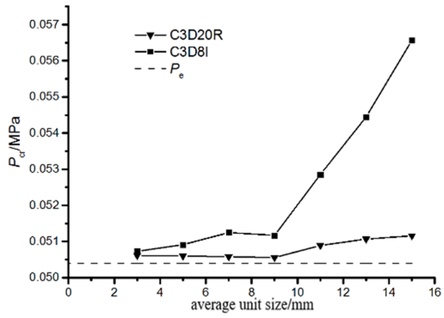


Figure 4: Comparison of numerical results and test results of different unit types.

It is worth noting that in the numerical calculation of all shell unit types, only the calculation result of the solid unit C3D8R is smaller than the test result.

It can be seen from Table 2, Table 3 and Figure 5, for the thin shell type to circular toroidal shell (such as this example), the numerical calculation accuracy and efficiency of the shell unit are higher than the solid unit.

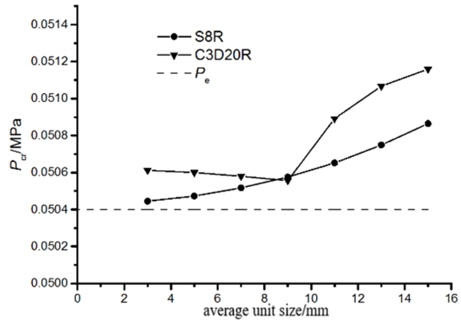


Figure 5: Comparison of numerical results and test results of shell elements and solid units.

It can find out that the error between numerical results and experimental results was within 1% (In addition to plan 13, 14) the results are highly consistent by synthesizing the last two columns of Tables 2 and 3(comparison of experimental results, Jordan formula results with numerical results) ,the error between the calculated value of the Jordan formula and the experimental result is 9% (Table 1), and the Jordan formula results are more conservative than the numerical results, it can be seen that the Jordan formula predicts the buckling load of the circular toroidal shell more easily, but numerical analysis methods are more accurate.

4 EFFECT OF PARAMETER *t/r* ON STABILITY OF CIRCULAR TOROIDAL SHELL UNDER EXTERNAL PRESSURE

The thickness of the circular toroidal shell has a great correlation with the buckling behavior of the shell, the theoretical formula and numerical solution of the buckling load of thin shell are discussed before in this paper, and the stability analysis of thick shell circular toroidal shell is carried out here. Its parameters are as follows:

R=60mm, r=24mm, t=2 mm, E=2500MPa, ν=0.4, among them， r/t=12 belongs to the thick shell category. The results of comparing Jordan formula are shown in Table 4.

Compared with the numerical calculation results, the numerical calculation result of S8R unit type has the smallest difference with Jordan formula results, for the circular toroidal shells of the given thick shell type, the buckling load P_{cr} should be 0.88973MPa. Comparing the numerical results of plan 1 with plan 2-5, the errors are 1.460%, 1.389%, 1.268% and 1.263% respectively. Although the error is small, it can still be proved that even for thick-shell type circular toroidal shells, the S8R is still suitable as a unit type for establishing a circular toroidal shell numerical solution under external pressure.

Table 4 solid unit plan and numerical analysis results

Plan	Average size of units	Unit type	P_c（MPa）	P_{0a}（MPa）	$(P_c-P_{0a})/P_{0a}$
1	1.2mm（0.05r）	S8R	0.88973	0.803688	10.71%
2	1.2mm（0.05r）	C3D8I（2layers）	0.90272	0.803688	12.32%
3	1.2mm（0.05r）	C3D8I（2layers）	0.90209	0.803688	12.24%
4	2.2mm（0.09r）	C3D20R（2layers）	0.90101	0.803688	12.11%
5	2.2mm（0.09r）	C3D20R（2layers）	0.90097	0.803688	12.10%

5 CONCLUSIONS

Comprehensive consideration of the calculation accuracy and efficiency of numerical simulation, it is recommended to use the S8R unit type with four-point constraint and unit average size of 0.05r to establish the shell unit numerical plan of the circular toroidal shell under external pressure.

Comparing shell unit and solid unit, it is recommended to choose a shell type in the circular toroidal shell in the thin shell category to establish a numerical model; for the thick shell category and the circular toroidal shell near the boundary line [20] between the thin shell and the thick shell (r=20t), the shell unit and the solid unit numerical model can be simultaneously established, and the unit type with smaller result is selected.

Compared with the Jordan formula, the numerical results differ from the experimental results by a small difference; Jordan formula can be used to predict buckling load when $t/r < 1/14$, this is more efficient; when $t/r \geqslant 1/14$, the numerical calculation method should be used to predict the buckling load.

ACKNOWLEDGEMENTS

This work was supported by the Natural Science Foundation of Jiangsu Province [grant number BK20211343].

REFERENCES

Flügge W., and Sobel, L. H., Locheed Missiles and Space Company Report 6-75-65-12 (Mar.1965).

Sobel L. H., and Flügge, W., AIAA, S (1967), 425-431.

Fishlowitz, E.G., NSRDC Rept.3338, April(1972), Dept.of the Navy, Naval Research and Development Center, Washington,D.C.

Jordan, P.F. Buckling of circular toroidal shells under hydrostatic pressure. AIAA Journal, 1973, 11(10): 1439-1441.

Du Q H, Cui W C, Zhang B. Buckling characteristics of a circular toroidal shell with stiffened ribs. Ocean Engineering, 2015, 108:325-335.

Du Q H, Cui W C, Wan Z Q. Nonlinear Finite Element Analysis of acircular toroidal shell with Ring-Stiffened Ribs Asme International Conference on Ocean. American Society of Mechanical Engineers, 2010.

Galletly G D, Blachut J. Stability of Complete Circular and Non-Circularcircular toroidal shells. Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, 1995, 209(4):245-255.

Jordan, P.F. Analytical and experimental investigation of pressurised circular toroidal shells. NASA CR-261, 1965.

Jordan, P.F. Stiffness of thin pressurised shells of revolution. Am. Inst. Aeronaut. Astronaut.1965,3,899-906.

Jordan, P. F. and Shelley, P. E. Stabilisation of unstable two-point boundary value problems. Am. Inst. Aeronaut. Astronaut.1966,4.923-924.

K.Hibbitt, S.Inc, ABAQUS-Theory and Standard User's Manual Version 6.3, USA,2006.

Zhang J, Gao J, Wang W. Investigation on mechanical properties of deep sea spherical pressure shell. China ship building, 2015,56(04):129-140.

Blachut,J., Jaiswal O R. On buckling of circular toroidal shells under external pressure. Computers & Structures, 2000, 77(3):233-251.

TOMAS A, TOVAR J P. The influence of initial geometric imperfections on the buckling load of single and double curvature concrete shells. Computers & Structures, 2012, 96(97):3445.

China Classification Society Diving System and Submersible Classification and Construction Specifications. BeiJing: People's communications publishing house, 1996.