A Vertex Degree-Based GRASP Approach for the Minimum Independent Dominating Set Problem

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Abstract: The Minimum Independent Dominating Set (MIDS) problem is a classical graph theory problem, with applications in sensors networks and database management systems. As a NP-hard problem, there is no exact solution that can be processed in polynomial time, unless $P = NP$. Some metaheuristic approaches have been proposed to tackle the problem in polynomial time. In this paper, we develop our own metaheuristic approach called GRASP+VD, a GRASP that uses vertex degree as its greedy function. We show that GRASP+VD outperforms the state-of-the-art approach drMIDS in all BHOSLIB dataset and in most of the DIMACS dataset.

1 INTRODUCTION

The Minimum Independent Dominating Set (MIDS) problem is a classical graph theory issue which the solution has shown to be a promising alternative to search for energy efficient wireless sensor network topologies (Santos et al., 2009); eliminate redundancy in wireless sensor and actor network (Akyildiz et al., 2004); among others (Alowa et al., 2022). Also, it is used on a new concept of sets used as a straightforward way to handle similarity in database management systems, from complex datasets. Some results towards this direction have been presented in Pola et al. (2015).

Given a Graph $G = (V, E)$, a dominating set D of G is a set of vertices such that every vertex in $V - D$ is adjacent to a vertex in D. Furthermore, a set I is independent if there is no pair of adjacent vertices in I. Thus, an independent dominating set (IDS) of $G$is a set that is both dominating and independent. The MIDS problem focus, therefore, to point out the smallest IDS in a graph.

Some variations of the MIDS problem have been studied in recent literature. For example, the independent perfect domination sets in Cayley graph (Wang et al., 2021), the open-independent, open-locating-dominating sets in complementary prism graphs (Capelle et al., 2022) and the IDS with obligation in graphs (Laforest and Martinod, 2022), to name a few.

Some exact algorithms have been exploited to resolve the MIDS problem (Johnson et al. 1988) (Liu and Song, 2006) (Gaspers and Liedloff, 2006) (Laforest and Phan, 2013) (Burgois et al., 2013). However, they are limited by the fact that its processing has proven to be NP-Hard (Irving, 1991), meaning that an exact solution cannot be processed in polynomial time lest $P = NP$ (Halldórsson, 1993). Even though these solutions guarantee optimal results, they are limited by the exponential processing time.

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As an usual alternative to process exponential-time algorithms, metaheuristics (Gendreau and Potvin, 2010) have also been conjectured in the literature to solve the MIDS problem. Some efforts exploit GRASP algorithms combined with a path cost function (GRASP+PC) (Wang et al., 2017), others use memetic algorithms (Wang et al., 2018) and local-search-based approaches (Haraguchi, 2018), including the state-of-the-art metaheuristic approach, drMIDS (Wang et al., 2020).

Although there are some metaheursitic approaches for the MIDS problem, we will show that their results have room for improvements in terms of solution quality. Here, we develop a new metaheursitic approach for the MIDS problem, called GRASP+VD, that uses vertex degree as a greedy function instead of the path cost used by GRASP+PC by Wang et al. (2017).

2 PROBLEM FORMULATION

In the subsequent, we shall ponder a graph $G = (V, E)$ as finite, undirected, with no multiple edges, and unlooped structure. We also consider $N(v)$ as the neighborhood of the vertex v, and the closed neighborhood as $N[v] = N(v) \cup \{v\}$. Then, we can define (Allan and Laskar, 1978):

Definition 2.1. A set $D \subseteq V$is a dominating set of G if $\forall v \in V - D, N(v) \cap D \neq \emptyset$.

Definition 2.2. A set $I \subseteq V$ is an independent set of G if $\forall u, v \in I, N(u) \cap \{v\} = \emptyset$.

Definition 2.3. A set $IDS \subseteq V$ is an independent dominating set of G if IDS is both an independent and a dominating set, that is, if it follows both Definitions 2.1 and 2.2.

We are now in position to introduce the MIDS problem, as follows.

Problem 2.1. Specified a graph $G = (V, E)$, the MIDS problem aims to identify the smallest independent dominating set in G , following Definition 2.3.

With the problem formally introduced, we can explain how we tackle it with our GRASP+VD approach.

3 THE GRASP+VD APPROACH

Algorithm 1 demonstrates the main blocks of a GRASP procedure. On lines 1 to 5, the algorithm runs n_iter times, where n_iter is the maximum number of iterations. On line 2, a solution is fabricated by the Greedy Randomized Construction (GRC) algorithm, receiving as input the graph G and α, a threshold parameter. When $\alpha = 0$ we have a totally greedy algorithm and when $\alpha = 1$ we have a totally random algorithm. Later, on line 3, the solution passes through a Local Search phase. Then, on line 4, the best-known solution S_best is updated if S is a reasonable solution (*i.e.*, attends to Definition 2.3) and better than S_best. The best-known solution S_best is returned on line 6.

Algorithm 1: GRASP

```
  Data: n_iter, G = (V, E), α, k
  Result: S_best
1 for k = 1, 2, ... n_iter do
2 |   S ← GreedyRandomizedConstruction(G, α);
3 |   S ← LocalSearch(G, α, S, k);
4 |   if |S| < |S_best| then
5 |   |   S_best ← S;
6 |   end
7 end
8 return S_best
```

The first step of the GRASP algorithm is the Greedy Randomized Construction, which is illustrated in Algorithm 2. Initially, on line 1, the solution is assumed to be empty. For each iteration of this stage, the collection of candidates is formed by all elements of the ground set that can be included to the partial solution that has being built, while not preventing the construction of a workable solution. In this case, the candidate set is the vertex set V, as illustrated by line 2. In this case, the greedy function is the vertex degree: the higher the vertex degree the lower the incremental cost. The assessment of the elements by the greedy evaluation function guides to the making of a restricted candidate list (RCL) formed by the best elements (this is the greedy aspect of the heuristic). On line 6, the best elements are defined by those with greedy function greater or equal then a parameter $\alpha \in [0,1]$ multiplied by the difference between the maximum and minimum value of the greedy function, summed with the minimum value. The element to be included into the incomplete solution is randomly selected from those in the RCL on line 7. Once the selected element is incorporated on line 8, the candidate list is updated. The above steps are repeated until there exists at least one candidate element on lines 3 to 10 (Resende and Ribeiro, 2010).

Algorithm 2: Greedy Randomized Construction (GRC).

Data: $G = (V, E), \alpha$
Result: S
$S \leftarrow \emptyset$;
$C \leftarrow V$;
while $C \neq \emptyset$ **do**
  Evaluate $d(v)\ \forall v \in C$;
  $d_{max} \leftarrow max\{d(v) | v \in C\}$;
  $d_{min} \leftarrow min\{d(v) | v \in C\}$;
  $RCL \leftarrow \{v \in C | d(v) \geq d_{min} + \alpha(d_{max} - d_{min})\}$;
  Select an element v^* from the RCL at random;
  $S \leftarrow S \cup \{v^*\}$;
  $C \leftarrow C - N[v^*]$;
end
return S;

The solutions created by a greedy randomized construction are not surely optimal. The local search phase can improve the constructed solution. A local search algorithm works in an interactive mode by successively replacing the current solution by a superior solution in its neighborhood. In this case, neighborhood refers to the solutions close to the current solution in the search space, not to be confused with vertex neighborhood. In this work, we chose the strategy of removing k elements at random from the current solution and reconstruct it by using Algorithm 2 with the remaining solution set. This simple procedure is illustrated in Algorithm 3.

Algorithm 3: Local Search.

Data: $G = (V, E), \alpha, k, S$
Result: S
Remove k elements from S at random;
$GRC(G, \alpha, S)$;

With the GRASP+VD algorithm fully explained, we can go on to the computational experiments and results.

4 COMPUTATIONAL EXPERIMENTS AND RESULTS

The following experiments were done using a personal computer with processor AMD Ryzen 5 2600 and 16 GB of RAM, running on Windows 10. The code was implemented in Python. We performed computational experiments to apply Algorithm 1 into the DIMACS (Johnson and Trick, 1996) and BHOSLIB (Xu et al., 2007) benchmark datasets, obtained in the Network Repository (Rossi and Ahmed, 2015).

The experiment consists in running the implemented GRASP+VD algorithm 100 times on each instance of the datasets, then calculating its average and minimum results, and comparing to the results of drMIDS. The outcomes are condensed in Tables 1, 2, and 3. The first column of the tables brings the instance name. The second and third columns bring the minimum and average length of the independent dominating set extracted by the GRASP+VD approach, respectively. The fourth column brings the standard deviation of the length of the IDS extracted. Finally, the last two columns bring the minimum and average length of the independent dominating set extracted by the drMIDS approach, respectively. The numbers in bold indicate that GRASP+VD showed equal or better performance in comparison with drMIDS for that instance. It can be noticed that GRASP+VD outperforms drMIDS in the entirety of the BHOSLIB dataset and in 82.7% of the DIMACS dataset.

These results show that GRASP+VD could be a better option than drMIDS in terms of finding IDS with minimum cardinality. By its applications, this would mean extracting better similarity sets, finding better energy efficient wireless sensor networks, and so on. It should be noted that GRASP+VD uses a simpler information about the vertices, the vertex degree, than the personalized path cost utilized by drMIDS as its greedy function.

Table 1: Experimental results of GRASP+VD and drMIDS on the BHOSLIB dataset.

	GRASP+VD			drMIDS	
Instance	min	mean	std	min	mean
frb100-40	**3**	**4.29**	0.537	43	44.35
frb30-15-1	**3**	**3**	0	11	11
frb30-15-2	**3**	**3**	0	11	11
frb30-15-3	**3**	**3.01**	0.100	11	11
frb30-15-4	**3**	**3**	0	11	11
frb30-15-5	**3**	**3.11**	0.314	11	11
frb35-17-1	**3**	**3.19**	0.394	13	13
frb35-17-2	**3**	**3.08**	0.273	13	13.03
frb35-17-3	**3**	**3.07**	0.256	13	13
frb35-17-4	**3**	**3.13**	0.338	13	13.29
frb35-17-5	**3**	**3.06**	0.239	13	13.65
frb40-19-1	**3**	**3.05**	0.219	15	15.39
frb40-19-2	**3**	**3.06**	0.239	15	15.03
frb40-19-3	**3**	**3.19**	0.394	15	15.03

frb40-19-4	**3**	**3**	0	15	15
frb40-19-5	**3**	**3.09**	0.288	15	15.19
frb45-21-1	**3**	**3.09**	0.288	17	17.77
frb45-21-2	**3**	**3.13**	0.338	17	17.87
frb45-21-3	**3**	**3.21**	0.409	17	17.39
frb45-21-4	**3**	**3.18**	0.386	17	17.55
frb45-21-5	**3**	**3.14**	0.349	17	17.45
frb50-23-1	**3**	**3.38**	0.488	19	19.94
frb50-23-2	**3**	**3.25**	0.435	19	19.9
frb50-23-4	**3**	**3.31**	0.465	19	19.9
frb50-23-5	**3**	**3.09**	0.288	20	20.03
frb53-24-4	**3**	**3.29**	0.456	20	20.9
frb53-24-5	**3**	**3.58**	0.496	20	21.06
frb59-26-1	**3**	**3.33**	0.473	23	23.61
frb59-26-2	**3**	**3.59**	0.494	23	23.9
frb59-26-3	**3**	**3.62**	0.488	23	23.84
frb59-26-4	**3**	**3.42**	0.554	23	23.94
frb59-26-5	**3**	**3.84**	0.368	24	24.19

Table 2: Experimental results of GRASP+VD and drMIDS on the DIMACS dataset I.

	GRASP+VD			drMIDS	
Instance	min	mean	std	min	mean
brock200-2	5	5.0	0.1	4	4
brock200-4	**3**	**3.3**	0.5	6	6
brock400-2	**3**	**3.5**	0.5	9	9
brock400-4	**3**	**3.0**	0	9	9
brock800-2	**4**	**5.0**	0.1	8	8
brock800-4	**4**	**4.9**	0.3	8	8
C1000-9	**2**	**2.4**	0.5	25	25.48
C125-9	**2**	**2**	0	14	14
C2000-5	**7**	7.7	0.4	7	7
C2000-9	**3**	**3**	0	32	32.03
C250-9	**2**	**2**	0	17	17
C4000-5	**8**	8.8	0.4	8	8
C500-9	**2**	**2**	0	21	21
DSJC1000-5	**6**	6.9	0.4	6	6
DSJC500-5	**5**	5.9	0.3	5	5
c-fat200-1	**13**	13.5	0.5	13	13
c-fat200-2	**6**	6.4	0.5	6	6
c-fat200-5	**3**	3	0.0	3	3
c-fat500-1	28	30	0.7	27	27
c-fat500-2	**14**	14.8	0.5	14	14
c-fat500-5	**6**	**6**	0	6	6
gen200-p0-9-44	**2**	**2**	0	16	16
gen200-p0-9-55	**2**	**2**	0	16	16
gen400-p0-9-55	**2**	**2**	0	20	20
gen400-p0-9-65	**2**	**2**	0	20	20
gen400-p0-9-75	**2**	**2**	0	20	20
hamming10-4	**8**	**8**	0	12	12
hamming6-2	**2**	**2**	0	12	12
hamming6-4	8	8	0	2	2
hamming8-2	**2**	**2**	0	32	32
hamming8-4	8	8	0	4	4

Table 3: Experimental results of GRASP+VD and drMIDS on the DIMACS dataset II.

	GRASP+VD			drMIDS	
Instance	min	mean	std	min	mean
johnson16-2-4	**3**	**3**	0	8	8
johnson32-2-4	**3**	**3**	0	16	16
johnson8-2-4	**3**	**3**	0	4	4
johnson8-4-4	**5**	**5**	0	7	7
keller4	**4**	**4**	0	5	5
keller5	**4**	**4**	0	9	9
keller6	**4**	**4**	0	15	17.16

MANN-a27	**2**	**2**	0	27	27
MANN-a45	**2**	**2**	0	45	45
MANN-a81	**2**	**2**	0	81	81
MANN-a9	**2**	**2**	0	9	9
p-hat1500-1	14	16.7	1.1	12	12.71
p-hat1500-2	**7**	10.0	0.9	7	7.68
p-hat1500-3	4	4.1	0.3	3	3
p-hat300-3	**3**	3	0.1	3	3
p-hat700-1	12	14.2	0.9	11	11
p-hat700-2	7	8.4	0.9	6	6
p-hat700-3	**3**	3.5	0.5	3	3
san1000	19	22.1	1.1	4	4
san200-0-7-1	**3**	**3.2**	0.4	6	6
san200-0-7-2	**4**	**4.5**	0.5	6	6
san200-0-9-2	**2**	**2**	0	16	16
san200-0-9-3	**2**	**2**	0	15	15
san400-0-5-1	13	15.2	0.9	4	4
san400-0-7-1	**4**	**4.8**	0.5	7	7
san400-0-7-2	**4**	**5.0**	0.7	7	7
san400-0-7-3	**5**	**5.1**	0.3	7	7

5 CONCLUSIONS

The Minimum Independent Dominating Set (MIDS) problem is a classical graph theory problem. The solution for this problem has applications in some areas, like sensors networks and similarity set extraction. There has been some work regarding approximated approaches for this problem, but there was room to improvement.

In this work, we suggested a novel GRASP+VD approach that uses vertex degree instead of path cost as greedy function, explained its functioning and made computational experiments to measure its performance against the competitor drMIDS approach. We demonstrated that GRASP+VD outperforms drMIDS in the entirety of the BHOSLIB dataset and in 84.2% of the DIMACS dataset.

For future research, we would like to experiment different types of metaheuristics to compare their performances with GRASP+VD. We suggest Ant Colony Optimization and Simulated Annealing as starting points.

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REFERENCES

Akyildiz, I., Kasimoglu, I., 2004. Wireless sensor and actor networks: research challenges. *Ad Hoc Networks,* 2 (4): 351 – 367.

Allan, R., Laskar, R., 1978. On domination and independent domination numbers of a graph. *Discrete mathematics*, 23 (2): 73-76.

Alowa, A., Fevens, T., Khamayseh, Y., 2022. Survival backup strategy for controller placement problem in software defined networking. *Computer Communications,* 185: 104-117.

Bourgeois, N., Della Croce, F., Escoffier, B., Paschos, V., 2013. Fast algorithms for min independent dominating set. *Discrete Applied Mathematics*, 161 (4): 558-572.

Capelle, M. R., Coelho, E. M. M., Foulds, L.R., Longo, H. J., 2022. Complexity results on open-independent, open-locating-dominating sets in complementary prism graphs. *Discrete Applied Mathemathics,* in press.

Gaspers, S., Liedloff, M., 2006. A branch-and-reduce algorithm for finding a minimum independent dominating set in graphs. In: *International Workshop on Graph-Theoretic Concepts in Computer Science*, Springer: 78-89.

Grandreau, M., Potvin, J., 2010. Handbook of metaheursitics, Vol. 2, Springer.

Halldórson, M., 1993. Approximating the minimum maximal independence number. *Information Processing Letters*, 46 (4): 169-172.

Haraguchi, K., 2018. An efficienct local search for the minimum independent dominating set problem. *arXiv preprint*. arXiv:1802.06478.

Irving, R., 1991. On approximating the minimum independent dominating set. *Information Processing Letters*, 37 (4): 197-200.

Johnson, D., Yannakakis, C., Papadimitriou, C., 1988. On generating all maximal independent sets. *Information Processing Letters*, 27 (3): 119-123.

Johnson, D., Trick, M., 1996. Cliques, coloring, and satisfiability: second DIMACS implementation challenge. *American Mathematical Society*, Vol. 26.
Laforest, C., Phan, R., 2013. Solving the minimum independent domination set problem in graphs by exact algorithm and greedy heuristic. *RAIRO – Operations Research*, 47: 199-221.
Laforest, C., Martinod, T., 2022. On the complexity of independent dominating set with obligation in graphs. *Theoretical Computer Science*, 904: 1-14.
Liu, C., Song, Y., 2006. Exact algorithms for finding the minimum independent dominating set in graphs. In: Asano, T. (eds) *Algorithms and Computation*. ISAAC 2006. Lecture Notes in Computer Science, vol 4288. Springer, Berlin, Heidelberg: 439-448.
Pola, I., Cordeiro, R., Traina, C., Traina, A., 2015. Similarity sets: a new concept of sets to seamlessly handle similarity in database management systems. *Information Systems*, 52: 130-148.
Resende, M., Ribeiro, C., 2010. Greedy randomized adaptative search procedures: Advances, hybridizations, and applications. In: *Handbook of metaheuristics*, Springer, 2010, pp. 283-319.
Rossi, R., Ahmed, N., 2015. The network data repository with interactive graph analytics and visualization. In: *AAAI* 2015.
Santos, A., Bendali, F., Mailfert, J., Duhamel, C., Hou, K., 2009. Heuristics for designing energy-efficient wireless sensor network topologies. *Journal of networks*.
Wang, X., Xu, S.-J., Li, X., 2021. Independent perfect dominating sets in semi-Cayley graphs. *Theoretical Computer Science*, 864: 50-57.
Wang, Y., Li, R., Zhou, Y., Yin, M., 2017. A path cost-based grasp for minimum independent dominating set problem. *Neural Computing and Applications*, 28: 143-151.
Wang, Y., Chen, J., Sun, H., Yin, M., 2018. A memetic algorithm for minimum independent dominating set problem. *Neural Computing and Applications*, 30: 2519-2529.
Wang, Y., Li, C., Yin, M., 2020. A two phase removing algorithm for minimum independent dominating set problem. *Applied Soft Computing*, 88.
Xu, K., Boussemart, F., Hemery, F., Lecoutre, C., 2007. Random constraint satisfaction: Easy generation of hard (satisfiable) instances. *Artificial Intelligence*, 171 (8): 514-534.