An Auto-Paired Two-Dimensional DOA Estimation Method for Two Uniform Linear Arrays

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Abstract: In this paper, an auto-paired two-dimensional DOA estimation method is proposed. This method uses two nonparallel uniform linear arrays to construct three cross-correlation matrices, and between these cross-correlation matrices there exists some linear transformation matrices which can be used for obtaining a special matrix with the paired azimuth and elevation angles information. Then, the propagator method principle will be used to obtain the special matrix, and by calculating the eigenvalues of the matrix, the auto-paired DOA can be estimated. The simulations show that the proposed method has better estimation performance with a good paring manner than two other methods for two parallel uniform linear arrays, and greater advantage in computational complexity.

1 INTRODUCTION

The direction of arrival (DOA) estimation is an import problem in array signal processing technology and widely used in wireless communication, sonar, and other fields (Fayad et al., 2017 and Wen et al., 2020). In decades, many liner space processing methods have been used in DOA estimation for the parallel array structure, such as multiple signal classification method (MUSIC) (Liu et al., 2018), estimation of signal parameters via rotational in variance techniques method (ESPRIT) (Herzog and Habets, 2019), propagator method (PM) (Xia et al., 2015), DOA matrix method (Dai et al., 2019), etc. To solve the non−positive-definite problem of direction matrix caused by the parallel array structure, such as coprime array (Li et al., 2018 and Chen et al., 2021), nested array (Liu et al., 2021 and Zheng and Mu, 2020), many methods have been proposed in recent years by combing the compressed sensing methods with liner space processing methods. Besides, for the radio signal is propagated in three-dimensional space, the DOA estimates generally contain two angles, such as azimuth and elevation angles, which should be correct paired in the DOA estimation progress.

For two uniform linear arrays, an auto-paired two-dimensional (2-D) DOA algorithm based on PM method is proposed in this paper. Unlike the array structure considering in many methods, the structure of the two uniform linear arrays is not parallel structure, and it will be used for constructing three cross-correlation matrices between which there exists some linear transformation matrices with auto-paired angles information. The proposed method makes use of the PM method principle to obtain the linear transformation matrices, and then completes the paired DOA estimation by performing eigenvalue decomposition of the linear transformation matrix. For the method proposed in this paper only uses the cross-correlation matrix of the array output to perform the DOA estimation, it will have a lower computational complexity.

2 SYSTEM AND SIGNAL MODEL

The two uniform linear arrays structure is shown in Figure 1, and the spacing of horizontal array located in the X axis is d_x, and the spacing of the uniform array located in the X-Y plane is d of which the horizontal spacing is $d_x/2$ and the vertical spacing is d_y. Then the position coordinates of the two uniform linear arrays are $(i\times d_x, 0)$ and $(j\times d_x/2, j\times d_y)$, $0\leq i\leq M$, $1\leq j\leq M+1$, respectively. The K far-field narrowband plane radio waves impinge on the arrays shown in Figure 1 with angles (θ_1, ϕ_1), (θ_2, ϕ_2), …, (θ_K, ϕ_K) where θ_i and ϕ_i are the elevation and azimuth angles

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of ith incident signal and $0\le\theta_i<\pi/2$, $-\pi/2\le\phi_i<\pi/2$. In this paper, the number K of radio source signal and the wavelength λ is assumed as known, and $K<M$, $d_x=d_y=\lambda/2$.

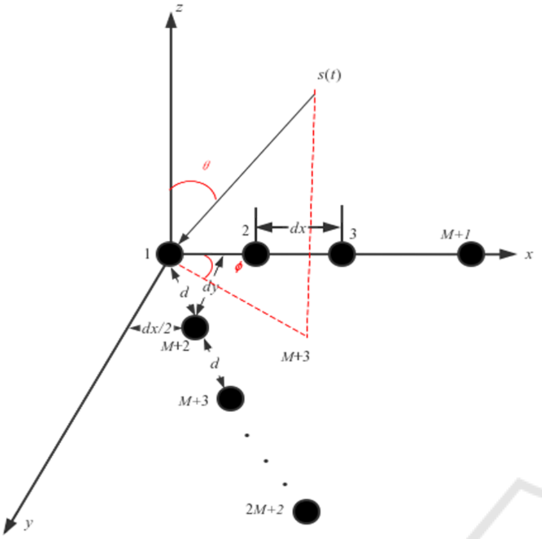


Figure 1: The two uniform linear arrays structure and direction of arrival of signal.

Then, let $\mathbf{X}_{11}(t)$, $\mathbf{X}_{12}(t)$, $\mathbf{X}_{21}(t)$ and $\mathbf{X}_{22}(t)$ are the output of the first M components and last M components of the two uniform linear arrays, respectively, which can be written as:

$$\begin{aligned}\mathbf{X}_{11}(t) &= [x_1(t), x_2(t), \ldots\ldots, x_M(t)]^T \\ &= \mathbf{A}_1\mathbf{S}(t) + \mathbf{n}_1(t)\end{aligned} \tag{1}$$

$$\begin{aligned}\mathbf{X}_{12}(t) &= [x_2(t), x_3(t), \ldots\ldots, x_{M+1}(t)]^T \\ &= \mathbf{A}_1\mathbf{\Phi}_1\mathbf{S}(t) + \mathbf{n}_2(t)\end{aligned} \tag{2}$$

$$\begin{aligned}\mathbf{X}_{21}(t) &= [x_{M+2}(t), x_{M+3}(t), \ldots\ldots, x_{2M+1}(t)]^T \\ &= \mathbf{A}_2\mathbf{S}(t) + \mathbf{n}_3(t)\end{aligned} \tag{3}$$

$$\begin{aligned}\mathbf{X}_{22}(t) &= [x_{M+3}(t), x_{M+4}(t), \ldots\ldots, x_{2M+2}(t)]^T \\ &= \mathbf{A}_2\mathbf{\Phi}_2\mathbf{S}(t) + \mathbf{n}_4(t)\end{aligned} \tag{4}$$

$\mathbf{n}_1(t)$, $\mathbf{n}_2(t)$, $\mathbf{n}_3(t)$ and $\mathbf{n}_4(t)$ are the $M\times 1$dimensional white Gaussian noise vectors with zero means and variance σ^2, $\mathbf{S}(t)$ is the $M\times 1$dimensional source signal vector with zero means, $\mathbf{A}_1$ and $\mathbf{A}_2$ are the direction matrices, $\mathbf{\Phi}_1$ and $\mathbf{\Phi}_2$ are the diagonal matrices, which are:

$$\mathbf{A}_1 = [\mathbf{a}_1(\theta_1,\phi_1), \mathbf{a}_1(\theta_2,\phi_2), \ldots\ldots, \mathbf{a}_1(\theta_K,\phi_K)] \tag{5}$$

$$\mathbf{A}_2 = [\mathbf{a}_2(\theta_1,\phi_1), \mathbf{a}_2(\theta_2,\phi_2), \ldots\ldots, \mathbf{a}_2(\theta_K,\phi_K)] \tag{6}$$

$$\mathbf{\Phi}_1 = diag[\exp(-j2\pi\frac{d_x}{\lambda}\alpha_1), \ldots, \exp(-j2\pi\frac{d_x}{\lambda}\alpha_K)] \tag{7}$$

$$\begin{aligned}\mathbf{\Phi}_2 = diag[&\exp(-j2\pi(\frac{d_x}{2\lambda}\alpha_1 + \frac{d_y}{\lambda}\beta_1)), \ldots, \\ &\exp(-j2\pi(\frac{d_x}{2\lambda}\alpha_K + \frac{d_y}{\lambda}\beta_K))]\end{aligned} \tag{8}$$

$$\alpha_i = \sin\theta_i\cos\phi_i, \quad \beta_i = \sin\theta_i\sin\phi_i \tag{9}$$

$\mathbf{a}_1(\theta_i, \phi_i)$ and $\mathbf{a}_2(\theta_i, \phi_i)$ are the direction vectors of the ith source signal respectively, which are:

$$\begin{aligned}\mathbf{a}_1(\theta_k,\phi_k) = [&1, \exp(-j2\pi\frac{d_x}{\lambda}\alpha_k), \ldots \\ &, \exp(-j2\pi\frac{(M-1)d_x}{\lambda}\alpha_k)]\end{aligned} \tag{10}$$

$$\begin{aligned}\mathbf{a}_2(\theta_k,\phi_k) = [&\exp(-j2\pi\frac{\frac{d_x}{2}\alpha_k + d_y\beta_k}{\lambda}), \ldots \\ &, \exp(-j2\pi\frac{\frac{M+1}{2}d_x\alpha_k + (M+1)d_y\beta_k}{\lambda})]\end{aligned} \tag{11}$$

3 THE AUTO-PAIRED DOA ESTIMATION METHOD

3.1 The DOA Estimation Method Based on PM

It is clear from Equations (5), (6), (10) and (11) that both the direction matrices $\mathbf{A}_1$ and $\mathbf{A}_2$ are the Vandermonde matrices. Thus, $\mathbf{A}_1$ and $\mathbf{A}_2$ can be assumed as the non-singular matrices, and let $\mathbf{A}_c$ be the first K rows of $\mathbf{A}_1$, and $\mathbf{A}_d$ be the other M-K rows. According to the propagator method, $\mathbf{A}_c$ is a full rank square matrix, and for the matrices $\mathbf{A}_c$ and $\mathbf{A}_d$, there exists a propagator matrix $\mathbf{P}$ which have:

$$\mathbf{P}^H\mathbf{A}_c = \mathbf{A}_d \tag{12}$$

Then the cross-correlation matrices of $X_{11}(t)$, $\mathbf{X}_{12}(t)$, $\mathbf{X}_{21}(t)$ and $\mathbf{X}_{22}(t)$ can be written as:

$$\begin{aligned}\mathbf{R}_1 &= E[\mathbf{X}_{21}(t)\mathbf{X}_{11}^*(t)] = \mathbf{A}_2\mathbf{R}_\mathbf{S}\mathbf{A}_1^H \\ &= \mathbf{A}_2\mathbf{R}_\mathbf{S}\left[\mathbf{A}_c^H \quad \mathbf{A}_d^H\right] = \mathbf{A}_2\mathbf{R}_\mathbf{S}\mathbf{A}_c^H[\mathbf{I}_{K\times K} \quad \mathbf{P}] \\ &= \mathbf{A}_2\mathbf{R}_\mathbf{S}\mathbf{A}_c^H\mathbf{P}_1 = \mathbf{R}_c\mathbf{P}_1 \approx \frac{1}{N}\sum_{i=0}^{N-1}\mathbf{X}_{21}(i)\mathbf{X}_{11}{}^*(i)\end{aligned} \tag{13}$$

$$\begin{aligned}\mathbf{R}_2 &= E[\mathbf{X}_{22}(t)\mathbf{X}_{11}^*(t)] = \mathbf{A}_2\mathbf{\Phi}_2\mathbf{R_S}\mathbf{A}_1^H \\ &= \mathbf{A}_2\mathbf{R_S}\mathbf{\Phi}_2\mathbf{A}_c^H\mathbf{P}_1 = \mathbf{A}_2\mathbf{R_S}\mathbf{A}_c^H\left(\mathbf{A}_c^H\right)^{-1}\mathbf{\Phi}_2\mathbf{A}_c^H\mathbf{P}_1 \\ &= \mathbf{R}_c\mathbf{P}_2 \approx \frac{1}{N}\sum_{i=0}^{N-1}\mathbf{X}_{22}(i)\mathbf{X}_{11}^*(i)\end{aligned} \tag{14}$$

$$\begin{aligned}\mathbf{R}_3 &= E[\mathbf{X}_{22}(t)\mathbf{X}_{12}^*(t)] = \mathbf{A}_2\mathbf{\Phi}_2\mathbf{R_S}\mathbf{\Phi}_1^H\mathbf{A}_1^H \\ &= \mathbf{A}_2\mathbf{R_S}\mathbf{A}_c^H\left(\mathbf{A}_c^H\right)^{-1}\mathbf{\Phi}_2\mathbf{\Phi}_1^H\mathbf{A}_c^H\mathbf{P}_1 = \mathbf{R}_c\mathbf{P}_3 \\ &\approx \frac{1}{N}\sum_{i=0}^{N-1}\mathbf{X}_{22}(i)\mathbf{X}_{12}^*(i)\end{aligned} \tag{15}$$

where $\mathbf{R}_c$ is the sub-matrix of $\mathbf{R}_1$ with the first K columns, $\mathbf{R}_s$ is the covariance matrix of source signal vector $\mathbf{S}(t)$ and $\mathbf{I}_{K\times K}$ is the $K\times K$ dimensional identity matrix. For the snapshot number N of output is finite, only the approximate matrices of cross-correlation matrices $\mathbf{R}_1$, $\mathbf{R}_2$ and $\mathbf{R}_3$ can be obtained which can be still used for the DOA estimation.

The matrices $\mathbf{P}_1$, $\mathbf{P}_2$ and $\mathbf{P}_3$ in Equations (13), (14) and (15) can be obtained by the the minimization problem as:

$$\mathbf{P}_1 = \mathbf{R}_c^+\mathbf{R}_1 = (\mathbf{R}_c^H\mathbf{R}_c)^{-1}\mathbf{R}_c^H\mathbf{R}_1 \tag{16}$$

$$\mathbf{P}_2 = \mathbf{R}_c^+\mathbf{R}_2 = (\mathbf{R}_c^H\mathbf{R}_c)^{-1}\mathbf{R}_c^H\mathbf{R}_2 \tag{17}$$

$$\mathbf{P}_3 = \mathbf{R}_c^+\mathbf{R}_3 = (\mathbf{R}_c^H\mathbf{R}_c)^{-1}\mathbf{R}_c^H\mathbf{R}_3 \tag{18}$$

where $\mathbf{R}_c^+$ is the pseudo-inverse matrix of $\mathbf{R}_c$.

Then using Equations (13), (14) and (15), an estimation matrix $\mathbf{\Psi}$ can be obtained as:

$$\begin{aligned}\mathbf{\Psi} &= (\mathbf{P}_2+\mathbf{P}_3)\mathbf{P}_1^+/2 = (\mathbf{P}_2+\mathbf{P}_3)\mathbf{P}_1^H(\mathbf{P}_1\mathbf{P}_1^H)^{-1}/2 \\ &= \left(\mathbf{A}_c^H\right)^{-1}(\mathbf{\Phi}_2+\mathbf{\Phi}_2\mathbf{\Phi}_1^H)\mathbf{A}_c^H/2\end{aligned} \tag{19}$$

where the diagonal matrix $\mathbf{\Phi}_2+\mathbf{\Phi}_2\mathbf{\Phi}_1^H$ in Equation (19) can be written as:

$$\begin{aligned}\mathbf{\Phi}_2+\mathbf{\Phi}_2\mathbf{\Phi}_1^H = diag[&2\cos(\pi\frac{d_x}{\lambda}\alpha_1)\exp(-j2\pi\frac{d_y}{\lambda}\beta_1) \\ &,...,2\cos(\pi\frac{d_x}{\lambda}\alpha_K)\exp(-j2\pi\frac{d_y}{\lambda}\beta_K)]\end{aligned} \tag{20}$$

It is clear from Equations (19) and (20) that the eigenvalue matrix of matrix $\mathbf{\Psi}$ is a diagonal matrix which contains the paired angles information. Therefore, by performing the eigenvalue decomposition and calculating the magnitude and phase terms of eigenvalues, the azimuth and elevation of source signal can be auto-paired and estimated without extra pairing progress which means the complexity can be reduced. Let ω_k be the kth eigenvalues of matrix $\mathbf{\Psi}$, then the azimuth and elevation can be estimated by:

$$\hat{\theta}_k = \arcsin\left(\left[\frac{2*\arccos(|\omega_k|)}{\pi}\right]^2+\left[\frac{\arg(\omega_k)}{\pi}\right]^2\right) \tag{21}$$

$$\hat{\phi}_k = \arctan\left(\frac{\arg(\omega_k)}{2*\arccos(|\omega_k|)}\right) \tag{22}$$

The major steps of the proposed method can be summarized as follows:

Algorithm 1: Estimate the 2-D DOA of source signal

Require: the N output snapshot of all array components

Obtain the cross-correlation matrices via Equations (13), (14) and (15), and let $\mathbf{R}_c$ be the sub-matrix of $\mathbf{R}_1$ with the first K columns

Calculate the matrices $\mathbf{P}_1$, $\mathbf{P}_2$ and $\mathbf{P}_3$ via Equations (16), (17) and (18).

Obtain the estimation matrix $\mathbf{\Psi}$ via Equation (19).

Perform eigenvalue decomposition of matrix $\mathbf{\Psi}$, and estimate the azimuth and elevation via Equations (21) and (22).

3.2 Complexity Analysis

Since the matrix multiplication is much more complicated than matrix addition, the complexity of the DOA estimation method almost depends on the matrix multiplication operation. For the proposed method, the complexity of cross-variance matrices calculation is $O(3M^2N)$, the complexity of the matrices $\mathbf{P}_1$, $\mathbf{P}_2$ and $\mathbf{P}_3$ calculation is $O(K^3+2MK^2+3M^2K)$, the complexity of matrix $\mathbf{\Psi}$ is $O(2MK^2+2K^3)$, and the complexity of eigenvalue decomposition is $O(K^3)$, then the total complexity is $O\{4K^3+4MK^2+3M^2K+2M^2N\}$. For comparison, Li's method (Li et al., 2012) and Luo's method (Luo et al., 2017) are considered, both of which are based on PM method and suit for two parallel uniform linear arrays without extra pairing progress. The complexity of Luo's method is $O\{4K^3+4MK^2+(2M)^2K+MK+(2M)^2N\}$, and the complexity of Li's method is $O\{3K^3+(8M-1)K^2+(4M-1)K+(2M+1)^2(N+K)\}$. Table 1 shows the complexity comparison with different snapshot number N where M is set as 8 and K is set as 3. It is clear that the complexity of the proposed method is nearly half of two other methods, and it means the proposed method has much more advantage in complexity.

Table 1: The total complexity of three estimation method with different snapshot number N.

	Proposed method	Luo's method	Li's method
N=100	O(13772)	O(26788)	O(30508)
N=200	O(26572)	O(52388)	O(59408)
N=300	O(39372)	O(77988)	O(88308)
N=500	O(64972)	O(129188)	O(146108)
N=1000	O(128972)	O(257188)	O(290608)

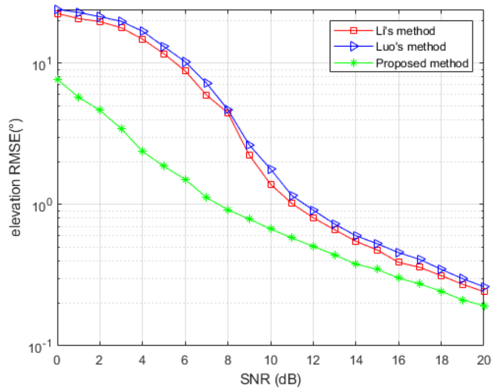
4 SIMULATION

In this section, the simulations of proposed method are given. The carrier frequency f_c is set as 1GHz, the source signals are the independent Gaussian random signals, d_x and d_y are set as half of wave length, and the total number of arrays is $2M+2=18$. For each simulation, there are 500 Monte Carlo simulation trails. The root mean square error (RMSE) of azimuth and elevation estimation which can be used for the estimation performance comparison is defined as:

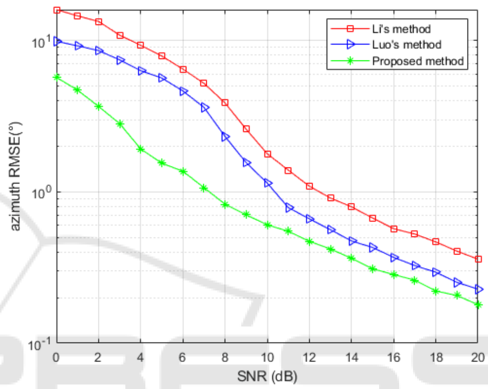
$$RMSE_\theta = \sqrt{\frac{1}{500K}\sum_{l=1}^{500}\sum_{k=1}^{K}(\hat{\theta}_{k,l}-\theta_k)^2} \quad (23)$$

$$RMSE_\phi = \sqrt{\frac{1}{500K}\sum_{l=1}^{500}\sum_{k=1}^{K}(\hat{\phi}_{k,l}-\phi_k)^2} \quad (24)$$

Figure 2 shows the RMSE of angles estimation and the comparison of three estimation methods versus different signal-noise ratio (SNR), where the snapshot number is 200, the azimuth and elevation angles of three source signal are (35°,25°), (40°,10°), (20°, -5°), respectively.



(a) Elevation estimation performance



(b) Azimuth estimation performance

Figure 2: The estimation RMSE of three methods versus different SNR.

It is clear in Figure 2 that the estimation RMSE of the proposed method decrease when SNR increase, which means the estimation performance of proposed method improves. With the comparison of Luo's method and Li's method which are suite for two parallel array structure generally used for 2-D DOA estimation, when SNR is 8dB, the RMSE of proposed method is below 1°, and that of the other two methods is above 1°, and when SNR is below 8dB, the RMSE of proposed method is still below the other two methods, and it is clear that the proposed method has much better estimation performance while the array structure used in this proposed method is two uniform linear arrays unlike Luo's method and Li's method.

Figure 3 shows the DOA estimation results when there are four source signals with angles (35°, -40°), (55°,10°), (15°,10°), and (45°,30°), respectively, where the SNR is set as 10dB and the snapshot number N is 200. It is clear in Figure 3 that the DOA estimates are very close to the true DOA which are marked in red in Figure3, and that indicates the proposed method can estimate the azimuth and elevation angles with a good pairing manner while the complexity is reduced.

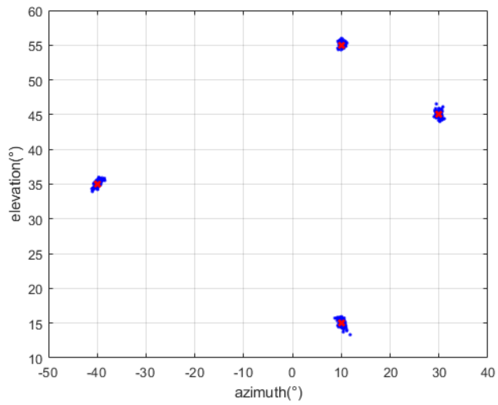


Figure 3: Distribution of DOA estimates with four source signals.

5 CONCLUSIONS

In this paper, an auto-paired 2-D DOA estimation method based on PM method has been proposed for two uniform linear arrays. The two uniform linear arrays is not parallel which can be used for constructing a linear transformation matrix containing the auto-paired angles information by using PM method principle and the cross-correlation matrices of array output. Then, by calculating the eigenvalue of the linear transformation matrix, the proposed method can obtain the paired DOA estimates. The simulations show that the proposed method have a better DOA estimation performance and lower complexity.

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