Cooperative Trajectory Optimization for Long-Range Interception with Terminal Handover Constraints

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Abstract: Motivated by the requirement of consistency engagement of long-range interceptors, a time cooperative guidance method based on feasible region is proposed. First, the multi-missile cooperative trajectory planning problem is established, considering the constraints in energy, heat protection and interception capacity. We transform the problem into subproblems of determination of coordinated time and trajectory optimization under time constraint. Based on the hp-adaptive pseudo-spectral method, the feasible region is analyzed and solved under different initial conditions. The RBF neural network was used to realize the online negotiation and prediction of cooperative cost. Numerical simulation shows the optimal cooperative trajectories can meet the constraint on cooperative rendezvous.

1 INTRODUCTION

The conflict between air threat and interceptors is becoming increasingly fierce in modern warfare, with the range of precision-guided weapons getting farther and farther. Interceptor missiles must also have the ability to intervene quickly in remote areas and accurately intercept various air targets (Wei, M, Cui, Z, & Li, Y, 2020; Wang, F. B, & Dong, C. H, 2013; Farooq, A, & Limebeer, D. J, 2002).

Unlike stationary or slow-moving targets, the target set of long-range air-defense missiles also includes high-mobility targets. Considering the long-range and target maneuvers, the handover area between midcourse and final interception is inevitably expanded. In order to realize information closure, it is used to add a trajectory planning phase at the end of the midcourse phase, so that the positions of multiple interceptors can meet the conditions of cooperative detection field splicing, and improve the capture probability of the seeker to the target.

The end stage of midcourse trajectory planning problem for long-range air-defense missiles is often described as a multi-constrained optimal control problem under finite feasible regions (GUO M,YANG F,LIU K,XIA G,YANG J, 2022); Because the collaborative detection constraints are involved, it is necessary to restrict the whole terminal state of the interceptor, including position, velocity, velocity angle and time. Meanwhile, the flight capability boundary of each interceptor should be considered comprehensively to find the cooperative trajectory that meets the state and terminal constraints. With the increase in the number of targets and interceptors, finding such a trajectory under multiple constraints via traditional method is time-consuming. Considering the target movement, it is difficult to meet the actual requirements of rapid response of interceptors online.

There are nonlinear coupling among time, speed, trajectory, constraint, and the horizontal and longitudinal plane in the energy descent phase. Nonlinear programming (NLP) tools can be used to solve these problems (Lv, S, Cai, M, & Zhou, D, 2019). Besides, the numerical trajectory optimization method can consider a variety of constraints and directly use the dynamic model of interceptors, which more truly reflects the mutual coupling between states and the restraint relationship of air defense missiles (Taub, I, & Shima, T, 2013).

In this paper, we study the coordination rendezvous problem of long-range air-defense missiles and trying to give a generalized structure to realize the online negotiation and prediction of cooperative cost. The key idea of this paper is to divide the negotiation and optimization of the trajectory into two subproblems: a) determination of coordinated time and b) trajectory optimization under

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time constraint. And solve by nonlinear programming tools. Based on the hp-adaptive pseudo-spectral numerical optimization method, the properties of feasible region are analyzed and fit the established database with RBF neural network. Numerical simulation shows the effeteness of the proposed method.

2 PROBLEM STATEMENT

2.1 Dynamic Model

Assuming the vehicle is a point of mass, the kinetic model of the i^{th} interceptor in the collaborative mission is:

$$\begin{aligned}
\dot{V}_i &= -\frac{D_i}{m_i} - g\sin\theta_i \\
\dot{\theta}_i &= \frac{Y_i}{m_i V_i} - \frac{g}{V_i}\cos\theta_i \\
\dot{\psi}_i &= -\frac{Z_i}{m_i V_i \cos\theta_i} \\
\dot{x}_i &= V_i\cos\theta_i\cos\psi_i \\
\dot{y}_i &= V_i\sin\theta_i \\
\dot{z}_i &= -V_i\cos\theta_i\sin\psi_i
\end{aligned} \tag{0.1}$$

Where: m , g , V , x , y , z are the mass, local gravitational acceleration, velocity, position components in the interceptor, respectively. The local trajectory inclination angle θ is the angle between the velocity vector and the local level, and the heading angle ψ, that is, the angle between the velocity vector and the local north direction. Y , Z , D are the lift, lateral force and resistance of the interceptor; the control parameters $U=[a_Y, a_Z]$ is the normal acceleration instruction in two directions, can be expressed as follows:

$$a_Y = Y/m \;,\quad a_Z = Z/m \tag{0.2}$$

2.2 Boundary Conditions and Constraints

Different from the re-entry trajectory planning problem for hypersonic gliding vehicles, the main feature of the long-range air-defense missile collaborative trajectories planning problem lies in the different constraints. Long-range air-defense missiles use light body and axisymmetric layout, with lower energy, smaller lift-drag ratio, thinner cylinder wall, poorer heat resistance than the glide vehicle, and the interception targets are high mobile aircraft targets. Their unique constraints can be summarized as follows:

I) Overload Constraints

Due to the axisymmetric layout of the air-defense missiles, its high-altitude overload capability is very limited, and the acceleration constraint of aerodynamic steering missile can not be simply considered as constant, but limited by the shell structure and the aerodynamic capacity. The maximum overload $n_{T\max}$ for the shell structure can be considered as the constant value, but the maximum aerodynamic overload capacity is subject to substantial aerodynamic pressure variation due to altitude and speed change. So, it is coupled together with speed, height and trajectory, it cannot be solved analytically (Cho, S. B, & Choi, H. L, 2022). The coupling relationship between overload constraints, control history, and terminal constraints can be expressed as the following formula:

$$[\rho, V_m] = f_1(U, t) \tag{0.3}$$

$$[U] = f_2(\mathbf{x}, \mathbf{x}_f, U_{\lim}, t) \tag{0.4}$$

$$[U_{\lim}] = f_3(V_m, \rho) \tag{0.5}$$

Where $\mathbf{x}$ refer to the state parameters, $U_{\lim}$ is the time varying control limitation.

II) Detection Constraints

The multi-missile cooperative detection of long-range air-defense missiles needs to create good detection conditions for the seeker. It has strict constraints on the position, difference in search time, difference in speed, speed angle and lower bounds of speed. They form the terminal constraints of long-range air-defense missiles. In addition, the interception object of air-defense missile interception is the aircraft-class high dynamic maneuver target. After the interceptor arrives in the handover area and the seeker is started, the target may maneuver at any time. The interceptor is required to have strong maneuverability and the final speed as much as possible, to reserve sufficient speed advantages and overload capacity for the final guidance.

2.3 Optimization Problem

Transforming the problem of collaborative guidance into optimal control is described as follows:

$$\min_{U(t)\in\Omega(U)} J(U) = -V_d$$

$$\text{s.t.} \left.\begin{aligned} \mathbf{X}_i &= \mathbf{X}_{fi} \\ V_i &\geq V_{i\min} \\ U_i &\leq U_{i\lim} \\ n_i &\leq n_{i\max} \\ \dot{U}_i &\leq \dot{U}_{i\lim} \\ \dot{Q}_i &\leq \dot{Q}_{i\max} \\ T_1 &= T_2 = \ldots = T_i \end{aligned}\right\} i = 1,2\ldots N \qquad (0.6)$$

$$\mathbf{X}_i = \left(X_i, Y_i, Z_i, V_i, \theta_i, \psi_i\right)^{\mathrm{T}}$$

Because of the nonlinearity aforementioned, the optimization problem has no analytic solution. It can only be solved via numerical methods.

3 FEASIBLE REGION FOR COLLABORATIVE DETECTION

As the number of interceptors in the bomb group increases, the dimension explosion phenomenon will appear, and it takes too long to solve the above collaborative planning problem directly by numerical methods. In order to give a general solution scheme suitable for the arbitrary number of interceptors, this paper divides the collaborative search problem into two subproblems: solving the collaborative search feasible region and constrained trajectory planning.

The difference between the collaborative search feasible domain and the aircraft accessible /recoverable region is that the former focuses on the state-space boundary when the aircraft arrives at the predicted handover point and focuses on its efficiency on the subsequent mission; the latter focuses on the space boundary of the aircraft, such as the hypersonic aircraft foot print problem and the interceptor interception area problem.

The Initial conditions are: $X(0) = 0$, $Y(0) = 35\,\text{km}$, $Z(0) = 0$, $V(0) = 2000 m/s$, $\theta(0) = 0°$, $\psi(0) = 0°$; The terminal conditions are: $X(t_f) = 200\text{km}$, $Y(t_f) = 20\,\text{km}$, $Z(t_f) = 0$, $V(t_f) = V_f \max$, $\theta(t_f) = 0$, $\psi(t_f) = 0°$. For the reason of retain the destruction ability to the target after detection, the minimum value of the final speed is limited as $V(t_f) \geq 1200 m/s$.

3.1 Unconstrained Feasible Region

The trajectory obtained without constraints is the ideal scenario of the flight profile. Reflects the best capability of the aerodynamic design of the aircraft. In this paper, the maximum final speed trajectory and the unconstrained feasible region are shown in Figure 1-4.

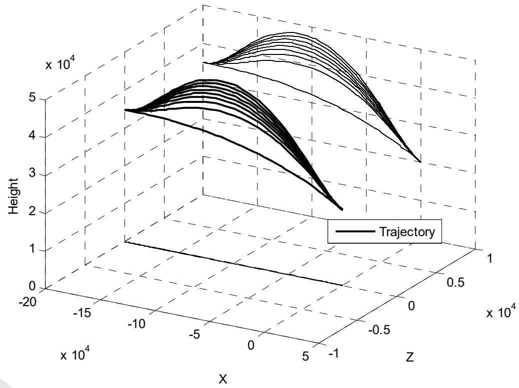

Figure 1: Unconstraint optimal trajectory for different arrival time.

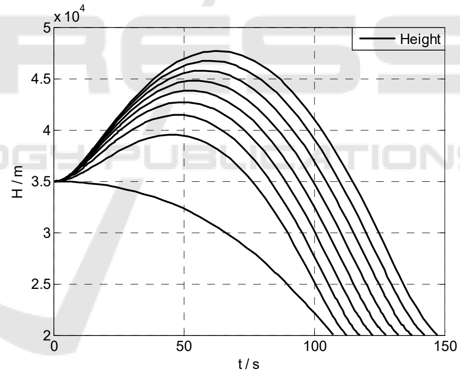

Figure 2: Height history.

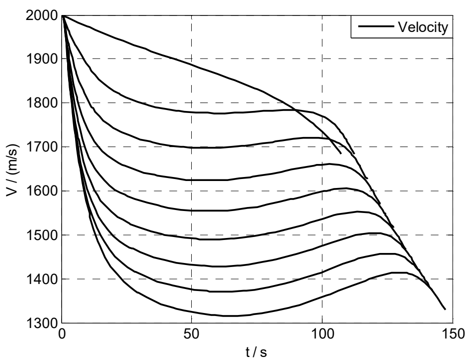

Figure 3: Velocity history.

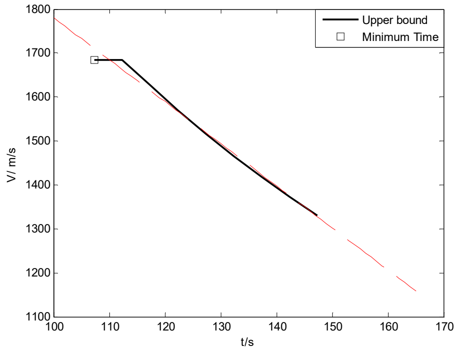

Figure 4: Maximum speed varies with time.

The simulation results show that after considering the passive decay characteristics, the unconstrained trajectory is only maneuver in the longitudinal plane. The trajectories seem to be in the form of parabolic trajectory. The former accomplishments related mostly concentrate on the maximum terminal speed, which is a critical factor for interception. Nevertheless, this paper further points out through simulation that the parabolic trajectory also can delay the arrival time with minimum velocity cost. It has certain reference significance for the subsequent design of long-range air-defense missile coordinated trajectories.

In addition, it can be seen that, after the maximum speed can afford from parabolic trajectory, the speed decreases while arrive time increases, and the linear characteristics appear within a certain range. The linear slope has obvious physical meaning: the velocity cost of delay per second. This linear correspondence will be further analyzed later.

3.2 Constrained the Feasible Region

After considering all the constraints of the long-range air-defense missiles, the adjustable range of the shift state is affected by the constraints, and it is reduced accordingly. As shown in Figure 5~8:

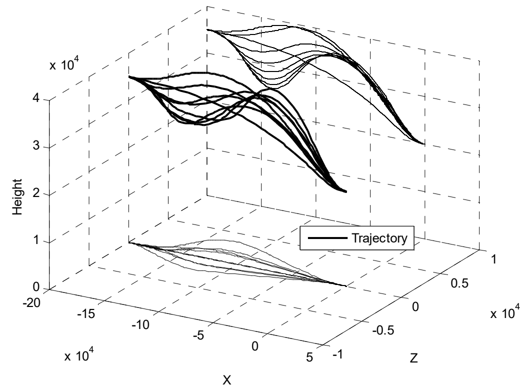

Figure 5: Constrained optimal trajectory for different arrival time.

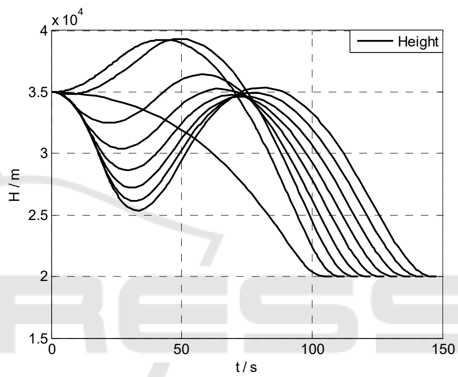

Figure 6: Height history.

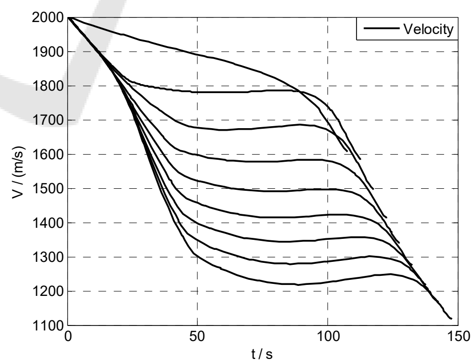

Figure 7: Velocity history.

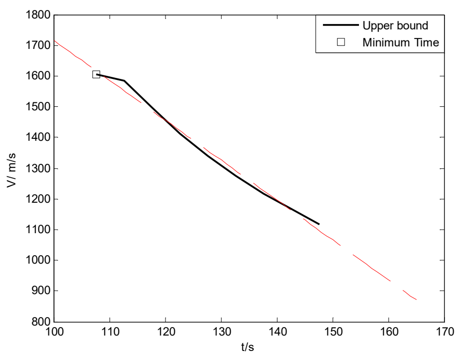

Figure 8: Maximum speed varies with time.

Compared with the unconstrained trajectory above. It can be found that a) the trajectory changes not only in the longitudinal plane, but also in the horizontal plane. b) Because of the overload limitation, the trajectory is in the form of double-parabolic in the longitudinal plane, where only have once in unconstrained scenario. c) Maximum speed varies with time still shows linear feature. Furthermore, it is worth noticing that numerous simulations show that it is unsensitive to the disturbance on lift and drag coefficients, so it is reasonable to fit the relationship linearly.

4 RBF NEURAL NETWORK

The Radical Basis Function (RBF) is one of the multidimensional spatial interpolation techniques proposed by Powell in 1985. In 1989, Jackson demonstrated that RBF neural networks constructed by RBF function as hidden layer neurons have the ability to consistently approximate any nonlinear continuous function. The RBF neural network has the advantages of simple structure, explicit training algorithm, and fast learning convergence. It is widely used in the field of pattern recognition (Lampariello, F, & Sciandrone, M, 2001) and nonlinear control (Yang, H., & Liu, J, 2018).

The function commonly used in RBF neural networks is Gaussian function, so the activation function in a radial basis function neural network can be expressed as:

$$R\left(x_p - c_i\right) = \exp\left(-\frac{1}{2\delta_i^2}\left|x_p - c_i\right|^2\right) \qquad (4.1)$$

The neural network structured as Figure 9 can get the output as:

$$y_i = \sum_{i=1}^{k} w_i \exp\left(-\frac{1}{2\delta_i^2}\left|x_p - c_i\right|^2\right) \quad i = 1,2\ldots n \quad (4.2)$$

Where, $x_p \in \left(x_1^{\,p}, x_2^{\,p} \ldots x_n^{\,p}\right)$ is the pth input sample, c_i is the node center of hidden layer, w_{ij} is the connection weight between the hidden layer to the output layer, y_i is the actual output of the ith node.

The solution of the feasible domain of the long-range air-defense missile can be transformed into a nonlinear function regression problem with six inputs and three outputs. The input variables are: speed, height, range, lateral deviation and two initial deviation Angle, and the output variables are the maximum speed, corresponding time and the slope K.

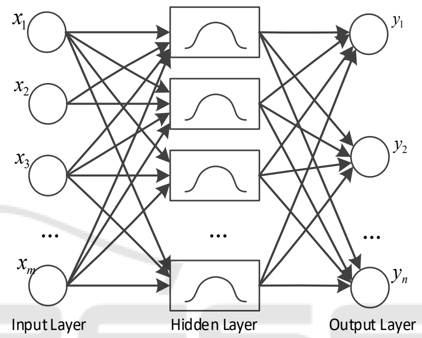

Figure 9: Schematic diagram of the RBF neural network.

The learning algorithm of the RBF neural network needs to solve the problem having three aspects, the center of the basis function, the variance, and the weights between the hidden layer to the output layer. using self-organized selection center method as the training method for RBF neural network. It can be divided into two stages, one is the self-organized learning stage, this stage using K-average clustering method determine the center c_i and variance σ_i , it is a nonlinear optimization process; In second stage, in order to find the appropriate weight for the output neurons, using gradient descent method for training neurons in output layer.

4.1 Training and Validation of RBF Neural Network

Optimize 2700 sets of data points offline over the possible range of initial conditions. The convergence criteria of the loss function is set to be 4e-7, the training and the testing result is shown in Figure 10~12:

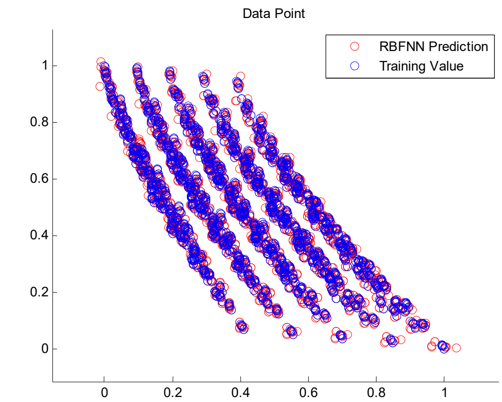

Figure 10: RBF neural network fitting result.

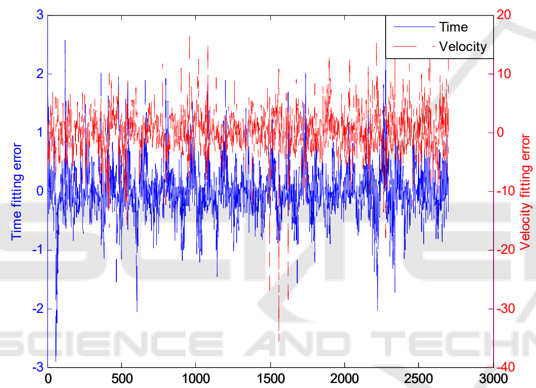

Figure 11: Fitting error in RBF neural network.

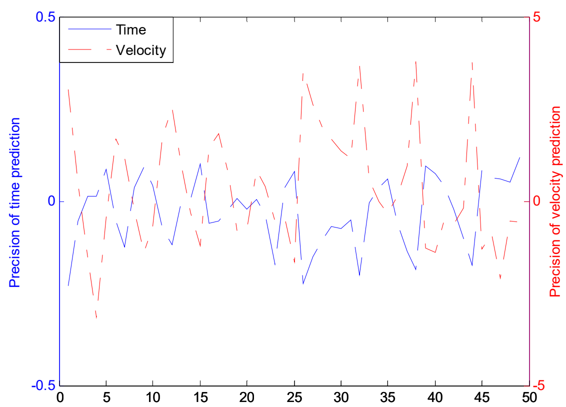

Figure 12: Generalization capability of RBF neural network.

4.2 Cooperative Guidance Strategy Based on RBF Neural Network

The negotiation and optimization process of consistency engagement as shown in Figure 13.

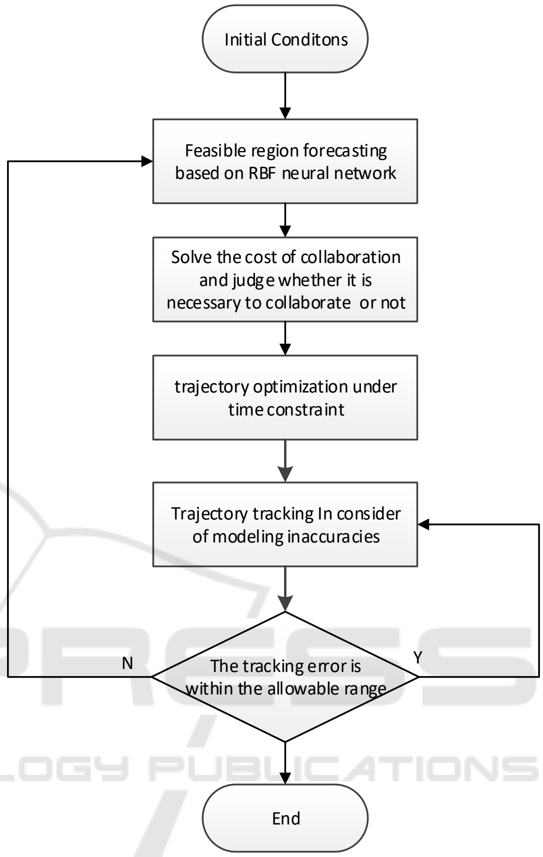

Figure 13: Negotiation and optimization based on RBF neural network.

5 SIMULATION RESULTS

The typical trajectory simulation of SM-6 missile is given as an example, which satisfies all the constraints and requirements on consistency engagement. The initial parameters of the three interceptors are shown in Table 1. And the forecast results obtained through the neural network are shown in Table 2. Because the time can only be extended but not shorten, the coordination time is set as (4.3), and the multiple interceptors plan their own feasible trajectory. The planning results are shown in Figure 14~16:

$$t_f = \max\left(t_{f1}, \ldots, t_{fi}\right) \quad (4.3)$$

Table 1: Initial conditions of the interceptors.

Missile Number \ Trajectory Parameter	R_{L0}	θ_0	h_0	V_0	Z_0	ψ_0
M_1	240km	1°	35km	2000m/s	300m	2°
M_2	200km	–1°	37km	1930m/s	500m	–2°
M_3	220km	–2°	33km	2070m/s	0m	0°

Table 2: Forecast results of the RBF neural network

Missile Number \ Trajectory Parameter	t_f	V_f	κ
M_1	131.1 s	1558.9 m/s	-9.25
M_2	111.2 s	1541.2 m/s	-12.6
M_3	117.1 s	1586.9 m/s	-10.4

According to the formula (4.3), we can get $t_f = 131.1s$

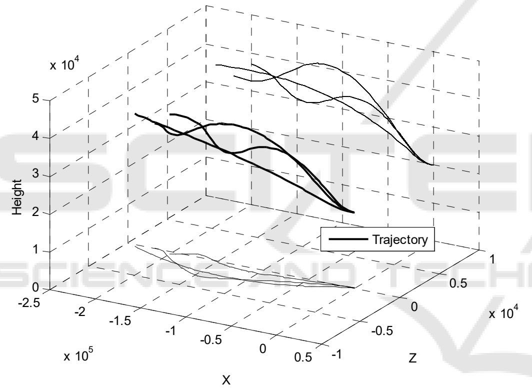

Figure 14: Cooperative trajectory for different interceptors.

Judging by the trajectory, all the interceptors can reach the target with terminal constraints. And they maneuver in both the horizontal and longitudinal planes.

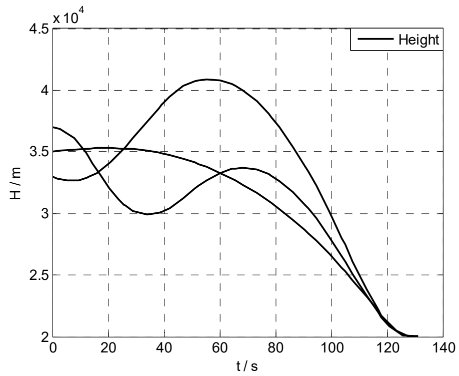

Figure 15: Height varies with time.

In longitudinal planes, M_3's trajectory is in the form of single high-parabolic ballistics, and M_2's is in the form of double-parabolic. They both extend the encounter time while reducing terminal speed losses.

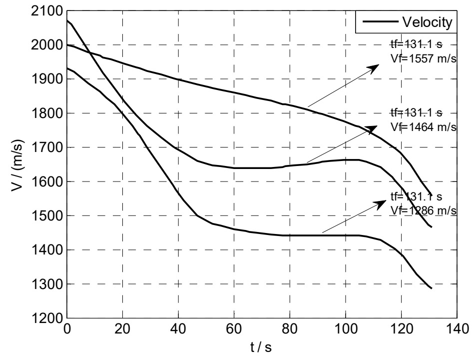

Figure 16: Velocity varies with time.

The optimal cooperative trajectories and can satisfied the constraints and can adjust the terminal time to realize the collaborative rendezvous of all interceptors.

6 CONCLUSIONS

This paper proposes a method of time cooperative guidance method based on feasible region to meet the collaborative rendezvous problem for long-range air-defense missile. First, the trajectory optimization problem is decomposed into two subproblems of determination of the coordinated time and trajectory optimization under time constraint. Based on hp-adaptive pseudo-spectral based solution, RBF neural networks were trained to realize the online negotiation and prediction. Trajectory simulation shows that the proposed method can quickly negotiate the terminal conditions and realize the multi-missile cooperative planning for long-range air-defense missiles.

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