






Robust Optimization for Climatological Emergency Evacuation

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Abstract: Natural disasters are very common nowadays. Therefore, human lives are lost, and economical resources are destroyed, so, it is important to plan actions to mitigate these unwanted effects. The uncertainty associated to these phenomena is large. The solution shall somehow be robust, for instance the value of the losses shall be relatively small for a sufficient large set of possible cases. This contribution will provide an overview on the scenarios based robust mathematical model for the treatment of climatological emergencies models to assist in the task of decision making for natural disasters with emphasis on evacuation work.

1 INTRODUCTION

Mathematical modelling of complex logistics systems in the context of climatological emergencies management is currently an difficult problem because the uncertainty inherent to data received from an emergency. (Behl and Dutta 2019; Beresford and Pettit 2021; Rodríguez-Espindola, Albores, and Brewster 2018; Yáñez-Sandivari, Cortés, and Rey 2021; Zhang and Liu 2021).

Climatological phenomena (Clarke, E. L. Otto, and Jones 2021) cause great physical damage and material losses due to natural events or phenomena such as earthquakes, hurricanes, floods, landslides, tsunamis, and others.

A classic humanitarian logistics (HL) model envisages pre-emergency and post-emergency stages (Yáñez-Sandivari et al. 2021) (See Figure 1).

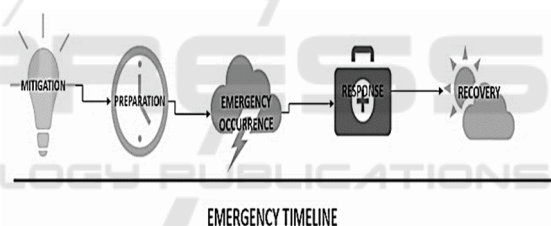




Figure 1: Emergency stages.


Prior to an emergency, mitigation; consist in the idea of help reduce the risks of large-scale events.


Preparedness requires having a clear idea of what actions need to be taken once an emergency occurs. Response and recovery are post-emergency stages. This paper will focus pre-emergency stages.


The problem of evacuation has recently been addressed mainly in hurricane and flood emergencies, see for example (Dalal and Uster 2021). A robust approach to problem P entails a robust optimization model which may even be non-linear, which would lead to greater complexity at the time of being solved.

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Considering any optimization problem P such that (See eq. 1).

$$\begin{aligned}
 (P): \min\{f(x)\} \\
 \text{s. t. } F(x) \leq 0 \\
 x \in X
 \end{aligned}
 \tag{1}$$

Where $F: \mathbb{R}^n \rightarrow \mathbb{R}^m$ represents the problem constraints, $f: \mathbb{R}^n \rightarrow \mathbb{R}$ is the objective function and the variable space is such that $X \subseteq \mathbb{R}^n$.

Starting from the problem P , the uncertainty can be formalized by considering a family of uncertain scenarios $P(u)$ such that (See eq. 2):

$$\begin{aligned}
 (P(u)): \min\{f(x, u)\} \\
 \text{s. t. } F(x, u) \leq 0 \\
 x \in X
 \end{aligned}
 \tag{2}$$

Where $F(\cdot, u): \mathbb{R}^n \rightarrow \mathbb{R}^m$, $f(\cdot, u): \mathbb{R}^n \rightarrow \mathbb{R} \forall u \in \mathbb{R}^M$, which describes that a scenario u is permitted to occur.

The formalization discussed in (Goerigk 2012), suggests that it is at this point that the values that u can take in the optimization problem $P(u)$, are not known but; it is assumed that u is known to be in a given uncertainty set $\mathcal{U} \subseteq \mathbb{R}^M$ representing the probable scenarios of the analysis and the uncertainty optimization problem.

Some authors (Akbari, Valizadeh, and Hafezalkotob 2021; Ben-Tal, Ghaoui, and Nemirovski 2009; Cao et al. 2021; Dönmez et al. 2021; Fakhrzad and Hasanzadeh 2020; Goerigk 2012; Mahtab et al. 2021; Seraji et al. 2021) revived the conceptual approach of robust modelling (Goerigk and Schöbel 2011) given around the 1960's (Gupta and Rosenhead 1968; Rosenhead, Elton, and Gupta 1972).

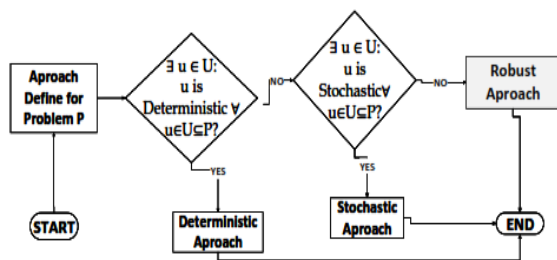


Figure 2. Basic procedure for detecting the robust approach in a problem

Roughly speaking, this research aims to propose a generic linear robust optimization model for climatological emergencies with emphasis on the evacuation task with the vulnerable population as an unknown or uncertain parameter with the particularity using scenarios in climatological

management considering the specific case of evacuation.

2 APPROACHES FOR REPRESENTING UNCERTAINTY

Mathematical models describing emergency situations have been presented in the literature, for example;(Cao et al. 2021), proposes a post-disaster relief model considering sustainability, multi-period, hierarchical relationships, equity, diffuse and insufficient supplies, split and unsplit demand, multi-repository and multi-destination. (Seraji et al. 2021) presents a two-stage multi-objective mathematical programming model for resource location and distribution.

In (Mahtab et al. 2021) is proposed a robust stochastic humanitarian logistics model to assist decision-makers in pre- and post-disaster management.

(Zhang and Liu 2021) proposes a mathematical simulation model based on the vehicle routing problem with uncertain transport time for a post-emergency period.

(Akbari et al. 2021) proposes a mathematical simulation model based on the vehicle routing problem with uncertain transport time for a post-emergency period.

(Dönmez et al. 2021) proposes a comprehensive review of the research conducted on the problems of locating facilities under uncertainty in a humanitarian context.

(Yáñez-Sandivari et al. 2021) conducts a comprehensive review of recent literature on humanitarian logistics and disaster response operations.

In (Fakhrzad and Hasanzadeh 2020), the author analyzes the importance of logistics networks in strategic decisions for emergency relief distribution using a mathematical model for stock shortages and pre-disaster decision support.

Other approaches use fuzzy optimization, neutrosophic solutions and even the modeling of these events with possibilistic optimization (Mohammadi et al. 2020; Özceylan and Paksoy 2014; Paydar and Saidi-Mehrabad 2014; Saati et al. 2015). However, with the searches performed, there is no model that integrates the various robust uncertainty management approaches.

One of the fundamental problems detected in the previous contributions continues to be the uncertainty

and quality of the model as we seek an integral mathematical model that can absorb a humanitarian logistics problem under various approaches.

2.1 Scenarios for Robust Optimization

The concept established by Ben-Tal (Ben-Tal et al. 2009) show feasibility for all scenarios as conservative in nature. This conceptualization is not always possible to apply given the complex data structure of a system (Goerigk and Schöbel 2011).

In (Kouvelis and Yu 1997) a framework for working with scenarios is formalized.

In (Kouvelis and Yu 1997) a clear definition of the case of discrete scenarios with the different types of robustness for mathematical optimization models is proposed.

It is important to note the importance of the concept of robustness referred to by (Kouvelis and Yu 1997) from (Mulvey, Vanderbei, and Zenios 1995) perspective:

- A mathematical program solution is robust with respect to optimality (it is called a robust solution) if it remains close to the optimum for any input data scenario to the model.
- A solution is robust with respect to feasibility if it remains close to feasible for any realization scenario (it is called model robust).

For a better theoretical understanding of this approach, see (Goerigk and Schöbel 2011; Kouvelis and Yu 1997).

2.2 Robust Stochastic Optimization

In (Mulvey et al. 1995) an attempt is made to give a robust answer to the issue of stochasticity through (RSO) so, let P be any (LP) with an uncertainty coefficients constraint (eq. 3.ii) such that (see eq. 3).

$$(P): \min\{C^T x + d^T y\} \forall x \in \mathbb{R}^{n_1}, y \in \mathbb{R}^{n_2}$$

$$\begin{aligned} (i) \quad & s. t. \quad Ax = b \\ (ii) \quad & Bx + Cy = e \\ (iii) \quad & x \geq 0, y \geq 0 \end{aligned} \quad (3)$$

For a set of $\Omega = \{1, 2, \dots, s\}$ scenarios are associated another set $\{d_s, B_s, C_s, e_s\}$ of coefficients of the control constraints where the probability of the occurrence of the scenario p_s is such that $\sum_{s=1}^S p_s = 1$.

Thus, the optimal solution of (eq. 3) may be robust with respect to "optimality" or robust with respect to "feasibility", in the first case; if it remains close to the optimum for any realization of the scenario $s \in \Omega$ and it's called "Robust Solution". In

the second case, if the solution remains "almost feasible" for any realization of the scenario $s \in \Omega$ and it's called "Robust Model" solution.

The model proposed in (Mulvey et al. 1995) then seeks to use an alternative through the stochastic solution of linear programming, introducing a set of control variables $\{y_1, y_2, \dots, y_s\} \forall s \in \Omega$ and another set of error vectors $\{z_1, z_2, \dots, z_s\} \forall s \in \Omega$ to measure the infeasibility contained in the control constraints considering the following formulation $P(\xi)$ of the model of (RO) (eq. 4).

$$P(\xi): \min \begin{cases} \sigma(x, y_1, y_2, \dots, y_s) \\ + \omega \rho(z_1, z_2, \dots, z_n) \end{cases}$$

$$\begin{aligned} (i) \quad & s. t. \quad Ax = b \\ (ii) \quad & B_s x + C_s y_s + Z_s = e_s \quad \forall s \in \Omega \\ (iii) \quad & x \geq 0, y_s \geq 0 \quad \forall s \in \Omega \end{aligned} \quad (4)$$

where the first term $\sigma(x, y_1, y_2, \dots, y_s)$ of the objective function measures the optimality of robustness, the penalty term being a measure of model robustness, the second term for $\rho(z_1, z_2, \dots, z_n)$ is a function for penalizing violations of control constraints in some scenarios, and ω represents the goal programming weight used to derive a range of compensatory responses for model robustness. $P(\xi)$ prevents there being a single option for an aggregate objective function with multiple ξ scenarios such that $\xi = C^T x + d^T y$ becomes a random variable taking the value $\xi_s = C^T x + d_s^T y_s$ with probability p_s .

$$\sigma(\cdot) = \sum_{s \in \Omega} p_s \xi \quad (5)$$

In summary, this is the point of the author's contribution and where it is suggested to use the mean value function of stochastic linear programming $\sigma(\cdot)$ as the aggregation function of the model (see eq. 5).

3 ROBUST MODEL FOR CLIMATOLOGICAL EMERGENCY

To model the process, we will consider the stochastic problem for two stages according to the two operational moments described above and the following five scenarios:

1. Precipitation and intensity increases.
2. Precipitation and decreasing intensity.
3. Winds and increasing intensity.
4. Winds and decreasing intensity.

5. Sea penetrations.

The estimated probabilities of each of these scenarios are input data. N possible evacuation centers, the location and capacity of each are known.

Model and Notation:

Let $J=\{1,\dots,N\}$ be the set of possible evacuation candidate center (CC).

K_j : Cost of conditioning the (CC) considering

C_j : Capacity of candidate center j (CC), $j \in J$).

I : Set of localities with vulnerable affected population.

pel_a : Population of locality a vulnerable to rainfall, $a \in I$.

$pell_a$: Population of locality a vulnerable to intense rainfall, $a \in I$.

pev_a : Population of locality a vulnerable to wind, $a \in I$.

$pevf_a$: Population of locality a vulnerable to strong wind, $a \in I$.

pvp_a : Population of location a vulnerable to penetrations, $a \in I$.

$S=\{1,\dots,5\}$: Set of described scenarios.

Decision variables:

x_{ajt}^s : Number of people from location a to be in CC_j if scenario s in stage t .

Model for each scenario:

Restrictions: Evacuate all vulnerable. At least 20% in case of heavy rain and at least 10% in case of non-heavy rain. Similarly in case of winds. All vulnerable in danger of penetration.

Objective: For each scenario s decide how many people from each location are evacuated in stage 1 and 2 (mitigation and preparation):

Rainfall:

$$\sum_{j \in J} x_{aj1}^u \geq prob_u \cdot pell_a \quad a \in I \quad (6)$$

$$\sum_{j \in J} x_{aj1}^l \geq prob_l(pel_a - pell_a)$$

By wind:

$$\sum_{j \in J} x_{aj1}^{vf} \geq prob_{vf} \cdot pev_a \quad a \in I \quad (7)$$

$$\sum_{j \in J} x_{aj1}^v \geq prob_v \cdot (pev_a - pev_{fa}) \quad a \in I$$

Sea penetration:

$$\sum_{j \in J} x_{aj1}^p = pvp_a \quad a \in I \quad (8)$$

Capacity constraints at the centers:

$$\sum_s \sum_a x_{aj2}^s + \sum_s \sum_a x_{aj1}^s \leq C_j \quad (9)$$

Objective Function:

$$\min \{ \sum_{a=1}^2 \sum_{j \in J} \sum_{a \in I} K_{jt} X_{jt} \} \quad (10)$$

3.1 Case Simulation

It is considered a cyclonic type of emergency in which there are 4 localities affected by this entity (Table 1) with their respective probabilities for each scenario at each stage and a possible candidate evacuation center for each location.

Table 1: Data locality vulnerable people.

Locality	Stage	Rain		Wind		Sea Penetration
		Normal	Intense	Normal	Intense	Normal
L1	Mitigation	25	5	20	-	-
	Preparation	-	-	-	6	-
L2	Mitigation	20	-	20	-	50
	Preparation	20	-	-	-	-
L3	Mitigation	30	-	10	-	85
	Preparation	-	6	-	30	-
L4	Mitigation	25	-	-	-	-
	Preparation	-	-	-	30	20

The affected localities have people vulnerable to rains, winds, and sea penetration (probable scenarios of the emergency) and some occurrence probability for some scenarios (Table 2).

Table 2: Probability assigned when scenario happen.

Locality	Stage	Rain			Wind			Sea Penetration			
		Normal	Intense		Normal	Intense		Normal	C1	C2	C3
L1	Mitigation	.3	.4								
	Preparation					.6					
L2	Mitigation	.7						.1	.6	.2	
	Preparation					.3					
L3	Mitigation										
	Preparation		.7								
L4	Mitigation							.4	.3	.1	
	Preparation				.2						

The response seeks to minimize the costs of evacuating vulnerable people in each locality by considering the likely scenarios.

AIMMS version 4.89.2.5 under community license was used to emulate the following results. (Table 3) shows the set of decisions that the decision-maker must make to mitigate the effects of the example problem, while reducing evacuation costs for the planned centers a relationship can be visualized between the scenario that occurred and the people to be evacuated considerer Center Evacuation Capacity as $\{ C1 : 175, C2 : 90, C3 : 312 \}$ with the unitary person evacuation costs $\{ C1 : \$ 50, C2 : \$ 48, C3 : \$ 72 \}$.

A robust solution is sought for all scenarios, to exemplify the random case selected, in the case of sea penetration, an affected population of 85 people is visualized in L3 (Table 1); however, there is no

probability of sea penetration for this location, which is contemplated in the decision not to evacuate people in L3 due to sea penetration.

Table 3: Decision Variable and objective results.

Z:	\$ 3516
X(L1,C1,Mitigation, Rainfall):	8
X(L1,C1,Mitigation, Intense Rainfall):	2
X(L1,C2,Preparation, Intense Wind):	4
X(L2,C1,Mitigation, Sea Penetration):	3
X(L2,C2,Mitigation, Rainfall):	14
X(L2,C2,Mitigation, Sea Penetration):	15
X(L2,C2,Preparation, Intense Wind):	2
X(L2,C3,Mitigation, Sea Penetration):	5
X(L3,C2,Preparation, Intense Rainfall):	5
X(L4,C1,Mitigation, Sea Penetration):	5
X(L4,C2,Mitigation, Sea Penetration):	4
X(L4,C3,Mitigation, Sea Penetration):	2

The contributions of this research are moderate and are in full development with the aim of using applied robust optimization models to mitigate the effects of a climate catastrophe.

4 CONCLUSIONS AND FUTURE WORK

This contribution shows partial theoretical results on robust optimization models applied to the management of climatological emergencies related to doctoral research in progress at the University of Havana, Cuba.

It is expected soon to obtain specialized simulations for the construction of a decision tool for climatic catastrophes with uncertainty management with different approaches.

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