Neural Network Interpretation of Bayesian Logical-Probabilistic Fuzzy Inference Model

Gulnara I. Kozhomberdieva[1][a], Dmitry P. Burakov[2][b] and Georgii A. Khamchichev[1][c]

[1]*Department of Information and Computing Systems, Emperor Alexander I St. Petersburg State Transport University, Moskovsky pr., 9, Saint Petersburg, 190031, Russia*

[2]*Department of Information Technology and IT Security, Emperor Alexander I St. Petersburg State Transport University, Moskovsky pr., 9, Saint Petersburg, 190031, Russia*

Keywords: Artificial Neural Network, Multilayer Neural Network, Fuzzy Neural Network, Neuro-Fuzzy Network, Fuzzy Inference, Fuzzy Logic, Bayesian Logical-Probabilistic Model of Fuzzy Inference, Bayesian Approach, Probabilistic Logic, Bayes' Theorem.

Abstract: The paper discusses the possibilities of using the Bayesian logical-probabilistic model of fuzzy inference, previously proposed, researched and software implemented by the authors, in a neural network context. A multilayer structure of a neuro-fuzzy network based on a Bayesian logic-probabilistic model is presented. According to the authors, the proposed network structure is comparable to the well-known Takagi–Sugeno–Kang and Wang–Mendel neuro-fuzzy networks. An example shows which network parameters can be used to train it.

1 INTRODUCTION

Currently, the world is experiencing another wave of neural networks popularity as the most dynamically developing area in the field of artificial intelligence. Impressive achievements in this area are primarily associated with the rapid increase in computing power and the emergence of super-large data sets used to train artificial neural networks.

The previous wave of interest to neural network technologies in artificial intelligence, during the 1990s and 2000s, was marked by successful attempts to hybridize intelligent information processing systems (especially in automatic control and regulation systems), combine the advantages of fuzzy inference systems and neural networks in the so-called *fuzzy neural (hybrid) networks* (Yarushkina, 2004; Rutkovskaya et al., 2013; Osovsky, 2018).

The effectiveness of the neural network apparatus is determined by their approximating ability, due to which neural networks are universal functional approximators capable of implementing any continuous functional dependence based on training. At the same time, the disadvantages of neural networks include the inability to explain the output result, because the knowledge accumulated by the network are distributed among neurons in the form of weight coefficient values.

Systems with fuzzy logic are deprived of this drawback; however, already at the stage of their design, there are required expert knowledge about the method of solving the problem of control or regulation, the formulation of rules, and membership functions. Therefore, there is no possibility to train such systems.

The combination of neural network and fuzzy approaches in hybrid systems allows, on the one hand, to bring the training ability and the parallelism of calculations that are inherent to neural networks to fuzzy inference systems. On the other hand, it allows to strengthen the intellectual capabilities of neural networks by linguistically interpretable fuzzy decision-making rules (Yarushkina, 2004; Rutkovskaya et al., 2013; Souza, 2020).

At that, there are distinguished two types of hybrids: neuro-fuzzy networks (NFN) and fuzzy

[a] https://orcid.org/0000-0002-5499-8473
[b] https://orcid.org/0000-0001-7488-1689
[c] https://orcid.org/0000-0002-6747-8514

Kozhomberdieva, G., Burakov, D. and Khamchichev, G.
Neural Network Interpretation of Bayesian Logical-Probabilistic Fuzzy Inference Model.
DOI: 10.5220/0011901700003612
In *Proceedings of the 3rd International Symposium on Automation, Information and Computing (ISAIC 2022)*, pages 50-56
ISBN: 978-989-758-622-4; ISSN: 2975-9463

neural networks (FNN). Hybrid fuzzy neural networks (FNN) are networks (similar to the structures of classical neural networks) based on fuzzy neurons with fuzzy inputs and outputs and/or fuzzy weights. Neuro-fuzzy networks (NFN) can be defined as multilayer neuro-network fuzzy systems that use a fuzzy rule base to calculate the output signal and provide the ability to adaptively adjust the parameter values fed to the parametric layers.

In the overview article (Souza, 2020) as well as in the works (Sinha and Fieguth, 2006; Wu et al., 2020; Kordestani et al., 2019; Siddikov et al., 2020; Zheng et al., 2021; Fei et al., 2021; Manikandan and Bharathi, 2017; Caliskan et al., 2020; Chertilin and Ivchenko, 2020; Vassilyev et al., 2020), the numerous examples of both types of hybrid networks usage are presented, that indicates the relevance and intensity of modern research and development in this field.

The paper discusses the possibility of using a Bayesian logic-probabilistic model (BLP model) of fuzzy inference in the structure of a multilayer neuro-fuzzy network (NFN). The model was proposed (Kozhomberdieva, 2019) at the International Conference on Soft Computing and Measurement (SCM'2019, St. Petersburg, Russia), researched and software implemented by the authors of this report (Kozhomberdieva and Burakov, 2019; Kozhomberdieva and Burakov, 2020; Kozhomberdieva et al., 2021). A demonstration example of solving the problem of fuzzy inference is given. The example shows which network parameters can be used to train it. According to the authors, the proposed network structure is comparable to the well-known Takagi–Sugeno–Kang and Wang–Mendel neuro-fuzzy networks (Osovsky, 2018).

2 NEURO-FUZZY NETWORK BASED ON THE BAYESIAN LOGICAL-PROBABILISTIC MODEL

Let us give a brief description of the BLP fuzzy inference model proposed and described in details by the authors in (Kozhomberdieva, 2019; Kozhomberdieva and Burakov, 2019; Kozhomberdieva and Burakov, 2020; Kozhomberdieva et al., 2021). The BLP fuzzy inference model is based on the use of probabilistic logic and the Bayes' Theorem when performing fuzzy inference according to a scheme similar to the well-known Mamdani model.

The original principle of the BLP model is the transformation of the base of fuzzy rules represented by the Boolean functions (BF) into a set of probabilistic logic functions (PLF). The PLF arguments are the membership functions values of the input linguistic variables (LV) terms and the calculated values are used as conditional probabilities $P(e|H_k)$, $k = 1, \dots, K$, which determine the correspondence degrees of the values set of the input variables $x_1, \dots, x_N$ ("crisp" evidence) to assumptions about the truth of the Bayesian hypotheses $H_1, \dots, H_K$, corresponding to the values set of the output LV. The conditional probabilities are used to determine the posterior Bayesian probability distribution $P(H_k|e)$, $k = 1, \dots, K$ on a set of hypotheses. The resulting posterior probability distribution is used at the final stage of fuzzy inference – when defuzzifying the value of the output LV.

We note an important feature of the BLP model – the requirement that the number of fuzzy rules and the number of terms of the output LV, which determines the set size of the Bayesian hypotheses, coincide. If necessary, the set of rules is reduced by combining all rules with the same conclusion into one fuzzy rule. The combined rule is a disjunction of antecedents of the combined rules, and the combined rule weight is defined as the arithmetic mean of the combined rules weights.

To go from representations of fuzzy rules in the BF form to their representations in the PLF form, Boolean functions are transformed to orthogonal (ODNF) or perfect (PDNF) disjunctive normal form. The rules for the formal transition from the BF that specified in the PDNF or ODNF to the corresponding PLF are following (Ryabinin, 2015):

1) logical variables $z_1, z_2, \dots, z_m$ are replaced with the corresponding probabilities $p_1, p_2, \dots, p_m$;

2) instead of negations $\overline{z_i}$, $1 - p_i$ are used;

3) conjunctions and disjunctions are replaced with arithmetic multiplication and addition, respectively.

The posterior probability distribution on the set of hypotheses is calculated by an equation based on the Bayes' Theorem equation:

$$P(H_k|e) = \frac{w_k \cdot P(e|H_k)}{\sum_{l=1}^{K} w_l \cdot P(e|H_l)}, \qquad (1)$$

where K is the number of Bayesian hypotheses (output LV terms) equals to the number of PLF used to evaluate the truth degree of evidence in favor of each hypothesis, w_k is the weight of the k-th rule, $w_k \in [0, 1]$. In the equation (1), there are no prior probabilities used in the classical Bayes' Theorem equation, since in the context of fuzzy inference, the prior probability distribution on the set of hypotheses is assumed to be uniform (the hypotheses are equally probable).

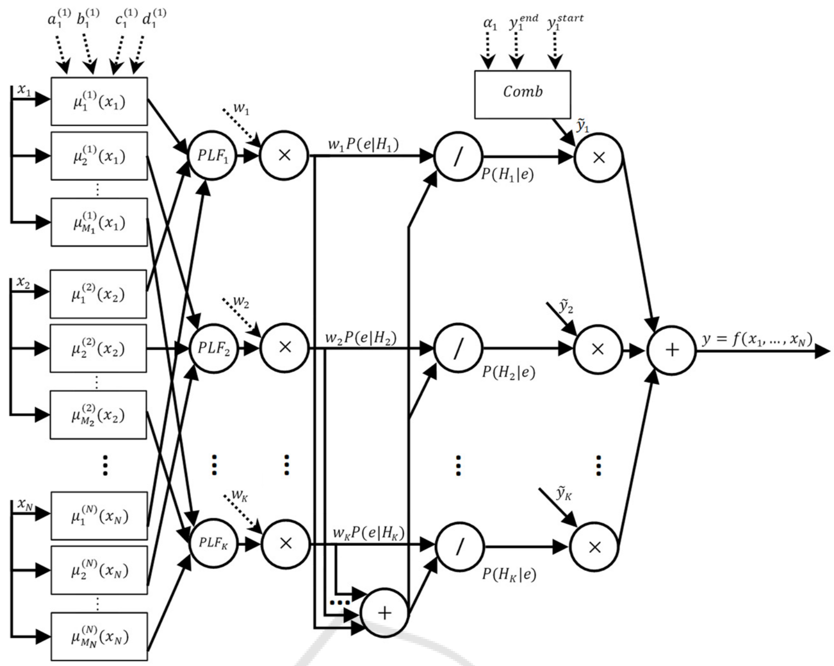


Figure 1: Structure of a neuro-fuzzy network based on a BLP model.

During defuzzification, the final value of the output variable y is determined as the mathematical expectation (average value) of a discrete random variable $\hat{y}$:

$$y = M(\hat{y}) = \sum_{k=1}^{K} \tilde{y}_k \cdot P(H_k|e), \quad (2)$$

where $P(H_k|e)$ is the k-th element of the posterior probability distribution calculated by the equation (1), and $\tilde{y}_k$ is the characteristic value of the corresponding k-th term of the output LV, which is by default taken as the central point of the interval on which this term is defined.

Figure 1 shows the structure of a neuro-fuzzy network based on the BLP model, the neurons of which perform the operations necessary to calculate the value of some output function $y = f(x_1, \ldots, x_N)$ using a set of input variable values $x_1, \ldots, x_N$. The network has seven layers:

- *the first (parametric) layer* performs separate fuzzification of each input variable, determining the membership function values for each fuzzy rule. In order to simplify the figure, the parameters to be adapted within the network training process are schematically indicated in Figure 1 only for the trapezoidal membership function $\mu_1^{(1)}(x_1)$, further used in the *example* in paragraph 3;
- *the second (non-parametric) layer* calculates, basing on a set of rules transformed into a PLF set, the conditional probabilities values $P(e|H_k)$, $k = 1, \ldots, K$. Depending on the task solved by the neuro-fuzzy network, the fuzzy rule base can be either formed by an expert, or (in the absence of linguistic information) is generated using a well-known universal algorithm for constructing a fuzzy rule base based on numerical data (Wang and Mendel, 1992; Rutkovskaya et al., 2013);
- *the third (parametric) layer* multiplies the results obtained from the second layer on the weight coefficients of the fuzzy rules $w_k \in [0, 1]$, which can be used as parameters within the network training process;
- *the fourth (non-parametric) layer* consists of a single adder neuron that calculates the sum of the weighted conditional probabilities $P(e|H_k)$, $k = 1, \ldots, K$, given from the third layer;
- *the fifth (non-parametric) layer* consists of neurons that perform the division operation in accordance with the equation (1) to obtain posterior Bayesian probability distribution $P(H_k|e)$, $k = 1, \ldots, K$, on the set of hypotheses that the output LV has some assigned value from its term-set;
- *the sixth (parametric) layer* consists of neurons each of that multiplies the probability $P(H_k|e)$, $k = 1, \ldots, K$ on the corresponding characteristic value $\tilde{y}_k$ of the output LV term. To calculate the characteristic value, a convex combination of two boundary points y_k^{start} and y_k^{end} of the corresponding interval of the output variable scale is applied. The combination coefficient $\alpha_k \in [0, 1]$

defines the shift of the characteristic value of the term within the interval. Parameters y_k^{start}, y_k^{end} and α_k are the network settings, the use of which is shown in the example in paragraph 3;

- *the seventh (non-parametric) layer* consists of a neuron-adder that generates the final value of the output variable $y = f(x_1, \ldots, x_N)$ in accordance with the equation (2).

3 EXAMPLE OF SOLVING THE PROBLEM AND SETTING UP THE NEURO-FUZZY NETWORK

As an explanatory example, we use the well-known demonstration problem "Dinner for Two", which, despite the simplicity of the solution, completely allows the authors to show the possibilities of using a neuro-fuzzy network based on the BLP model as a universal approximator of continuous functional dependence based on training.

Let it be necessary to develop an expert system to determine the tips amount to be left to the waiter of the establishment, depending on the level of service and the ordered dishes cooking quality. The visitor estimates the service and food quality on a 10-point scale, and the amount of tips paid – as a percentage (from 0 to 25% of the cost of dinner). This fuzzy model is included in the MATLAB demo examples (https://www.mathworks.com/help/fuzzy/fuzzy-inference-process.html), but in this paper it is presented in the edition used by the authors earlier in (Kozhomberdieva, 2019; Kozhomberdieva and Burakov, 2019; Kozhomberdieva and Burakov, 2020).

In the fuzzy inference system, the corresponding LVs for the estimated indicators *Service* and *Food* are formulated, the membership functions of their terms are defined on the indicator scales, and a system of fuzzy rules is formed that uses statements about the LV values in antecedents and conclusions. Graphs of the membership functions of the input LVs are shown in Figure 2.

The scale of the output variable *Tip*, in accordance with the conditions of the problem, is divided into three non-overlapping intervals [0, 5], [5, 20], [20, 25], corresponding to the linguistic values "small", "average" and "big", respectively. Note that the definition of membership functions for the terms of the output LV in a neuro-fuzzy network based on the BLP model is not required.

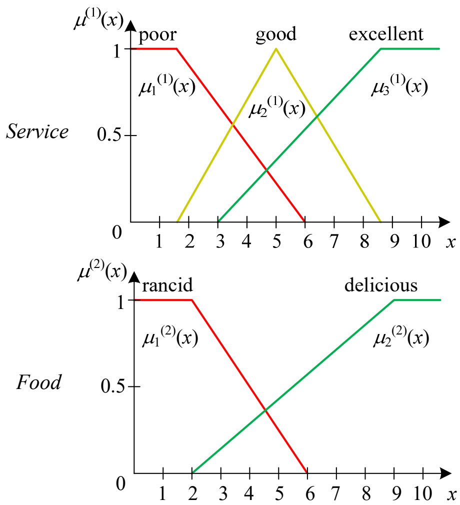


Figure 2: Membership functions $\mu_j^{(i)}(x)$ of input LV terms.

The following fuzzy rules are used:

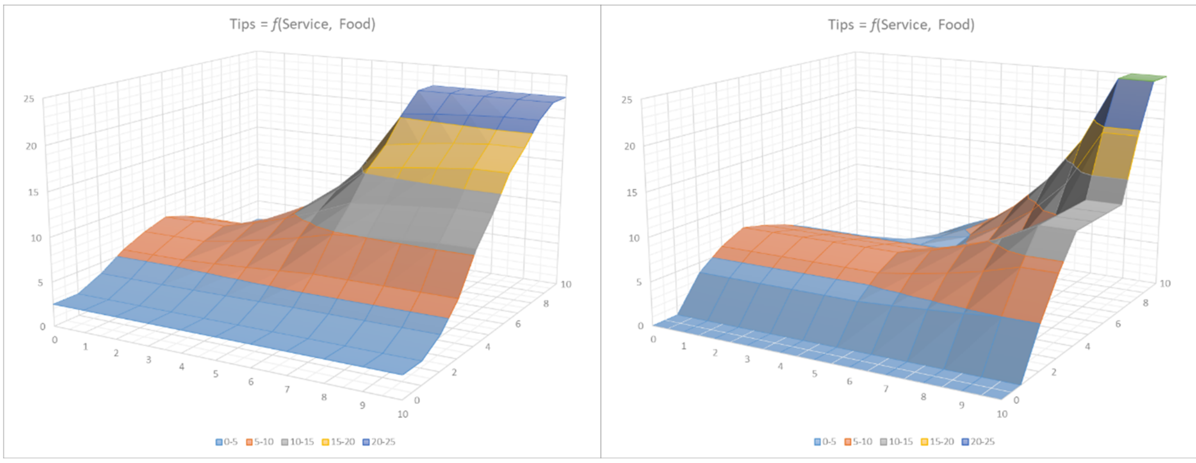
1) IF *Service* is "poor" OR *Food* is "rancid" THEN *Tip* is "small";
2) IF *Service* is "good" THEN *Tip* is "average";
3) IF *Service* is "excellent" AND *Food* is "delicious" THEN *Tip* is "big".

These rules are firstly presented as BFs, specified in the PDNF, and then transformed into a set of PLF. Obtained probabilistic functions are used to calculate conditional probabilities that estimate the degree to which a set of values of input variables x_1, and x_2 ("crisp" evidence) fits the assumptions about the truth of Bayesian hypotheses about the value ("small", "average", or "big") of the output LV:

$$P(e|H_1) = \mu_1^{(1)}(x_1) + \mu_1^{(2)}(x_2) - \mu_1^{(1)}(x_1) \cdot \mu_1^{(2)}(x_2),$$
$$P(e|H_2) = \mu_2^{(1)}(x_1),$$
$$P(e|H_3) = \mu_3^{(1)}(x_1) \cdot \mu_2^{(2)}(x_2).$$

For example, in the calculations we use the input values of the quality of service and food estimates $x_1 = x_2 = 5$. Let us set all the fuzzy rules weights w_k equal to 1, and as the characteristic values of the output LV terms, we will take by default the average value of the boundary points corresponding to the terms of the intervals y_k^{start} and y_k^{end} on the output variable scale. Then, for given membership functions (see Fig. 2), the posterior Bayesian probability distribution $\{P(H_k|e)\}, k = 1, 2, 3$, calculated by the equation (1) will be represented by the set of values $\{0.26, 0.64, 0.10\}$, and the desired tip size according to the equation (2) will be $y = 11\%$.

Let us transform the fuzzy inference system built to solve the "Dinner for Two" problem using the BLP model into a neuro-fuzzy network, the structure of which corresponds to the network structure in



(a) (b)

Figure 3: Surface plot of the output function $y = f(x_1, x_2)$. a) – the values of the parameters are presented in Table 1, b) – the values of the parameters are presented in Table 2.

Figure 1. Recall that the network is trained by changing the parameters on the parametric layers, and specify which parameters are used:

1. The membership functions $\mu_j^{(i)}(x_i)$ of the input LV terms are trapezoids described by four parameters $a_j^{(i)}, b_j^{(i)}, c_j^{(i)}, d_j^{(i)}$, which are the *x*-coordinates of the vertices of the trapezoids on the membership functions graphs (the triangular membership function is considered as a special case of the trapezoidal, when $b_j^{(i)} = c_j^{(i)}$);
2. Weight coefficients of fuzzy rules $w_k \in [0, 1]$ (by default they are taken equal to 1);
3. Boundary points of the intervals y_k^{start} and y_k^{end} corresponding to the terms of the output LV on the output variable scale, as well as the bias coefficients of the characteristic value for each term $\alpha_k \in [0, 1]$ (by default they are taken equal to 0.5). The characteristic value of the term $\tilde{y}_k$ used in the equation (2) is calculated as a convex combination of boundary points:

$$\tilde{y}_k = (1 - \alpha_k) \cdot y_k^{\text{start}} + \alpha_k \cdot y_k^{\text{end}}. \quad (3)$$

Table 1: Example 1 of configuring network settings.

Input LV	Service			Food	
Terms	Poor	Good	Excell.	Rancid	Delicious
Parameters of the membership functions and their values					
$a_j^{(i)}$	0	1.5	3	0	2
$b_j^{(i)}$	0	5	8.5	0	9
$c_j^{(i)}$	1.5	5	10	2	10
$d_j^{(i)}$	6	8.5	10	6	10
Weights of fuzzy rules					
w_k	1		1		1
Output LV	Tip				
Terms	Small		Average		Big
Parameters of terms output LV and their values					
y_k^{start}	0		5		20
α_k	0.5		0.5		0.5
y_k^{end}	5		20		25
$y = f(5, 5) = 11\%$					

To demonstrate the possibilities of parameters setting up a neuro-fuzzy network based on the BLP model, Table 1 shows the values of the parameters and the result of calculating the value of the output function $y = f(x_1, x_2)$ for the example considered above, and Table 2 – for the example with changed network parameters.

Table 2: Example 2 of configuring network settings.

Input LV	Service			Food	
Terms	Poor	Good	Excell.	Rancid	Delicious
Parameters of the membership functions and their values					
$a_j^{(i)}$	0	1	7	0	2
$b_j^{(i)}$	0	4	9	0	9
$c_j^{(i)}$	1	4	10	5	10
$d_j^{(i)}$	4	9	10	9	10
Weights of fuzzy rules					
w_k	0.75		1		0.5
Output LV	Tip				
Terms	Small		Average		Big
Parameters of terms output LV and their values					
y_k^{start}	0		5		20
α_k	0		0.5		1
y_k^{end}	5		20		25
$y = f(5, 5) = 6.5\%$					

Figures 3.a and 3.b show plots of the resulting surfaces, which differ markedly from each other.

4 CONCLUSIONS

Numerous examples of the use of neuro-fuzzy networks in automatic control and regulation systems published in open sources testify to the relevance and intensity of modern research and development in this field.

The paper presents the structure of a neuro-fuzzy network based on the BLP model of fuzzy inference, previously proposed, researched and software implemented by the authors. An example shows which network parameters can be used to train it.

According to the authors, the proposed seven-layer network structure with three parametric layers is comparable to the well-known Takagi–Sugeno–Kang and Wang–Mendel neuro-fuzzy networks.

When choosing an appropriate fuzzy rule base at the stage of network building and then training it, a network based on a BLP model can be used as a universal approximator of a continuous functional dependence. The authors plan to continue research in this direction.

REFERENCES

Caliskan, A., Cil, Z. A., Badem, H., Karaboga, D. (2020). Regression based neuro-fuzzy network trained by ABC algorithm for high-density impulse noise elimination. *IEEE Transactions on Fuzzy Systems*. 28(6):1084–1095.

Chertilin, K. E., Ivchenko, V. D. (2020). Configuring adaptive PID-controllers of the automatic speed control system of the GTE. *Russian Technological Journal*, 8(6):143–156 (in Russian).

Fei, J., Wang, Z., Liang, X., Feng, Z., Xue Y. (2021). Fractional sliding mode control for micro gyroscope based on multilayer recurrent fuzzy neural network. *IEEE Transactions on Fuzzy Systems*, 30(6):1712–1721.

Kordestani, M., Rezamand, M., Carriveau, R., Ting, D. S., Saif, M. (2019). Failure diagnosis of wind turbine bearing using feature extraction and a neuro-fuzzy inference system (ANFIS). In Proc. *Int. Work-Conf. Artif. Neural Netw*., pp. 545–556.

Kozhomberdieva, G. I. (2019). Bayesian logical-probabilistic model of fuzzy inference. *Mezhdunarodnaya konferentsiya po myagkim vychisleniyam i izmereniyam [International Conference on Soft Computing and Measurements]*. 1:35–38 (in Russian).

Kozhomberdieva, G. I., Burakov, D. P. (2019). Bayesian logical-probabilistic model of fuzzy inference: stages of conclusions obtaining and defuzzification. *Fuzzy Systems and Soft Computing*. 14(2):92–110 (in Russian).

Kozhomberdieva, G. I., Burakov, D. P. (2020). Combining Bayesian and logical-probabilistic approaches for fuzzy inference systems implementation. *Journal of Physics: Conference Series*. Volume 1703, 012042.

Kozhomberdieva, G. I., Burakov, D. P., Khamchichev, G. A. (2021). Decision-Making Support Software Tools Based on Original Authoring Bayesian Probabilistic Models. *Journal of Physics: Conference Series*. Volume 2224, 012116.

Manikandan, T., Bharathi, N. (2017). *Hybrid neuro-fuzzy system for prediction of stages of lung cancer based on the observed symptom values. Biomedical Research*, 28:588–593.

Osovsky, S. (2018) *Neural networks for information processing, trans. from Polish. by I. D. Rudinsky [Neironnye seti dlya obrabotki informacii, per. s pol'sk. I. D. Rudinskogo]*, Goryachaya Liniya – Telekom. Moscow, 2nd edition, 448 p. (in Russian).

Rutkovskaya, D., Pilinsky, M., Rutkovsky, L. (2013). *Neural networks, genetic algorithms and fuzzy systems: trans. from Polish. by I. D. Rudinsky [Neironnye seti, geneticheskie algoritmy i nechetkie sistemy, per. s pol'sk. I. D. Rudinskogo]*, Goryachaya Liniya – Telekom. Moscow, 2nd edition, 384 p. (in Russian).

Ryabinin, I. A. (2015). Logical probabilistic analysis and its history. *International Journal of Risk Assessment and Management*, 18(3-4):256–265.

Siddikov, I. X., Umurzakova, D. M., Bakhrieva, H. A. (2020). Adaptive system of fuzzy-logical regulation by temperature mode of a drum boiler. *IIUM Engineering Journal*, 21(1):185–192.

Sinha, S. K., Fieguth, P. W. (2006). Neuro-Fuzzy Network for the Classification of Buried Pipe Defects. *Automation in Construction*, 15:73–83.

Souza, P. V. C. (2020). Fuzzy neural networks and neuro-fuzzy networks: A review the main techniques and applications used in the literature. *Appl. Soft Comput*. 92, 106275.

Vassilyev S. N., Pashchenko F. F., Durgaryan I. S., Pashchenko A. F., Kudinov Y. I., Kelina A. Y., Kudinov I. Y. (2020). Intelligent Control Systems and Fuzzy Controllers. I. Fuzzy Models, Logical-Linguistic and Analytical Regulators. *Automation and Remote Control,* 81(1): 171–191.

Vassilyev S. N., Pashchenko F. F., Durgaryan I. S., Pashchenko A. F., Kudinov Y. I., Kelina A. Y., Kudinov I. Y. (2020). Intelligent Control Systems and Fuzzy Controllers. II. Trained Fuzzy Controllers, Fuzzy PID Controllers. *Automation and Remote Control,* 81(1):922–934.

Wang, L. X., Mendel, J. M. (1992). Generating Fuzzy Rules by Learning from Examples. *IEEE Transactions on Systems, Man, and Cybernetics*, November/December 1992, 22(6):1414–1427.

Wu, X., Han, H., Liu, Z., Qiao, J. (2020). Data-knowledge-based fuzzy neural network for nonlinear system identification. *IEEE Transactions on Fuzzy Systems*, 28(9):2209–2221.

Yarushkina, N. G. (2004). *Fundamentals of the theory of fuzzy and hybrid systems [Osnovy teorii nechetkikh i* *gibridnykh system],* Finansy i statistika. Moscow, 320 p. (in Russian).

Zheng, K., Zhang, Q., Hu, Y., Wu, B. (2021). Design of fuzzy system-fuzzy neural network-backstepping control for complex robot system, *Information Sciences*, 546:1230–1255.