

# Simulation of Korea Composite Stock Price Index (KOSPI) for the First 20 Years in the 21<sup>st</sup> Century Using Random Walk

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**Abstract:** The Korea Composite Stock Price Index (KOSPI) can be regarded as a representative of the economic development not only in South Korea but also in the emerging markets. Thus, the KOSPI and its derivatives are the objective of many studies with models from neural network model to stochastic model, whose computation can be considered as a part of the computational finance, in particular with the Monte Carlo simulation. In this study, we apply the random walk model to simulate KOSPI for the first 20 years in the 21<sup>st</sup> century with division into five sub-periods because whether or not KOSPI follows a random walk is closely related to the efficient market hypothesis (EMH). At first, we use the random walk in the 1/-1 format to simulate KOSPI in a simplified format for 2020, and then we use the random walk in the decimal format to simulate the real-life KOSPI for five different periods with increment of five-year data each. The simulation is done using the Monte Carlo algorithm to generate random numbers with 100,000 seeds for each sub-period of KOSPI simulation. The results show that the simulation can fit for short periods of time and can follow KOSPI for a longer period of time.

## 1 INTRODUCTION

The Korea Composite Stock Price Index (KOSPI) is an index, which represents the economic development not only in South Korea but also in the emerging markets. Consequently, it is the objective of many studies (Na, Sohn, 2011; Lee, Lim, 2011; Kim, Kim, 2004; Kim, Bang, 2014). Moreover, the KOSPI derivatives are also the subject of studies (Ko, 2012; Han, Guo, Ryu, Webb, 2012).

Theoretically, the studies on KOSPI can help us understand the human behaviour in stock market, the underlined mechanisms for the KOSPI movement, etc. Practically, the studies on KOSPI can help investors and fund managers for better following KOSPI and allocate their assets. Technically, various mathematical and statistical models have been applied to KOSPI studies, for example, neural network (Lee, Lim, 2011; Lee, Yoo, Jin, 2007) and stochastic method (Kim, Kim, 2004). In this context, we are particularly interested in the random walk model, which was too used in KOSPI studies (Aggarwal, 2018; Yoon, Kim, 2018).

To a broader sense, the issue of whether a stock index follows a random walk is a disputing subject

for years with approval and disapproval in different stock markets around the world (Charles, Darné, 2009; Abraham, Seyyed, Alsakran, 2002; Gilmore, McManus 2003). As a matter of facts, most studies using the statistical tests to determine the random walk hypothesis, such as unit root tests (Aggarwal, 2018), but it is rare to use a random walk to simulate a particular stock index. Nevertheless, both statistical tests and real-life case simulation are equally important because they reflect the same mechanism from two different angles.

In this study, we use the random walk to directly simulate the KOSPI for the first 20 years in the 21<sup>st</sup> century to add more pieces of evidence to this hot debating issue from real-life simulation aspect.

## 2 MATERIALS AND METHODS

### 2.1 KOSPI Data

The daily KOSPI for the first 20 years in the 21<sup>st</sup> century is obtained from a branch of Yahoo Finance. The KOSPI from 2001 to 2020 includes 4935 trading days with open, high, low, close, adjusted close, and

volume. We choose the daily close as our target for random walk simulation. In order to get more practical knowledge on the simulations, the 20-year KOSPI are furthermore divided into five sub-groups: there are 248 daily closes for 2020, 1220 for 2016-2020, 2456 for 2011-2020, 3700 for 2006-2020, and 4935 for 2001-2020.

## 2.2 Random Walk Model

The classical random walk is a path obtained by tossing a fair coin continuously (Feller, 1968): define a side of coin and the other side of coin as 1 and -1, record 1 or -1 for each tossing, and finally add the recorded 1 and -1 together. In graphic presentation, the  $x$ -axis is time (number of tossing of coin), and the  $y$ -axis is the addition of recorded 1 and -1. As the tossing of coin is a random event, its addition is a series of random events, which construct a random walk.

## 2.3 KOSPI in a Simplified Form

Similar to the consideration in random walk, the KOSPI can also be presented in a similar way, i.e. if KOSPI close in a day is higher or lower than that in its previous day, we record 1 or -1 for the day, and finally we add these 1 or -1 step-by-step along the time course. Graphically, this operation will have the  $x$ -axis as time and  $y$ -axis as the addition of recorded 1 or -1. Basically, this graph is a simplified KOSPI when we consider only its up or down movement daily. In this type of simulation studies, we use the random walk to fit this simplified KOSPI profile.

## 2.4 Random Walk in Decimal Format

Although the classical random walk is just related to 1 or -1, we should expand its concept to decimals to accommodate KOSPI because KOSPI is in decimal form. Conceptually, this means that we toss an item with many sides, each of which represents a number in decimals. This is possible because we do not toss a physical coin to generate a random walk but use a computer to generate a series of random numbers, which in fact are decimal numbers. Actually, we have to round the decimal numbers to integer in order to construct a classical random walk. Hence, we can simply use a computer to generate a series of random numbers, and then add them along the time course, which is a random walk in decimal format useful to compare with the real-life KOSPI.

## 2.5 Simulation

The simulation is done by means of computation with random numbers generated by Monte-Carlo algorithm. The generated random numbers are rounded to integers for simplified KOSPI and without rounding for real-life KOSPI. Thereafter, the random walk in both 1/-1 and decimal formats are compared with the simplified and real-life KOSPI. This process is continued until the random walks are very approximate to the simplified and real-life KOSPI. Because the Monte-Carlo algorithm requires a seed to generate a series of random numbers, 100 thousand seeds ranging from 0 to 10 are used to find the best simulation in this range.

## 3 RESULTS AND DISCUSSION

Table 1 explains how to perform the random walk simulation in both 1/-1 and decimal formats. Columns 1 and 2 are the first 10 trading days in 2020 and their corresponding KOSPI close. Column 3 is a list comparing whether a KOSPI close is larger or smaller than that in its previous day in terms of the 1/-1 format. For example, 2176.46, the KOSPI close on January 3, 2020 is larger than 2175.17, the KOSPI close on January 2, 2020, so 1 is assigned to the second cell in column 3. Column 4 is the addition of each cell in column 3, resulting in a KOSPI in the 1/-1 format. Column 5 is the random numbers generated by Monte Carlo simulation using SigmaPlot (SPSS Inc., 1986-2001) with a seed of 0.78654, which is the best one of 100,000 seeds. Column 6 is the comparison of whether the generated random number is larger or smaller than its preceding random number in the 1/-1 format. Column 7 is the classical random walk by adding each number in column 6, and is compared with column 4 for goodness-of-fit. The last two columns describe how to perform a random walk simulation in the decimal format. Column 8 is the random numbers generated by Monte Carlo simulation using SigmaPlot with one of the following ten seeds: 7.30548, 7.30549, . . . 7.30557. The command for generation of random numbers generally includes the generated number of random numbers, seed, upper and lower ranges, and we use the standard deviation of the KOSPI close in 2020 upper and lower ranges. Column 9 is the random walk in the decimal format by adding each random number in column 8, and then we can compare the KOSPI close value in column 2 with the random walk simulation in column 9.

Table 1: Simplified KOSPI and random walks in 1/-1 and decimal formats.

Date	KOSPI close	Compare preceding close	Random walk in 1 or -1 format	Generated random number	Compare preceding random number	Random walk in 1 or -1 format	Generated random number	Random walk in decimal format
Jan 2, 2020	2175.17			-0.56759			18.76995	
Jan 3, 2020	2176.46	1	1	0.10146	1	1	16.42251	2191.59
Jan 6, 2020	2155.07	-1	0	0.12706	1	2	0.70776	2192.3
Jan 7, 2020	2175.54	1	1	0.81422	1	3	-13.88377	2178.42
Jan 8, 2020	2151.31	-1	0	-0.32088	-1	2	-17.16913	2161.25
Jan 9, 2020	2186.45	1	1	0.03117	1	3	-16.45686	2144.79
Jan 10, 2020	2206.39	1	2	0.66874	1	4	-22.7432	2122.05
Jan 13, 2020	2229.26	1	3	-0.90877	-1	3	34.00345	2156.05
Jan 14, 2020	2238.88	1	4	-0.33833	1	4	7.08048	2163.13
Jan 15, 2020	2230.98	-1	3	-0.50799	-1	3	-20.66977	2142.46

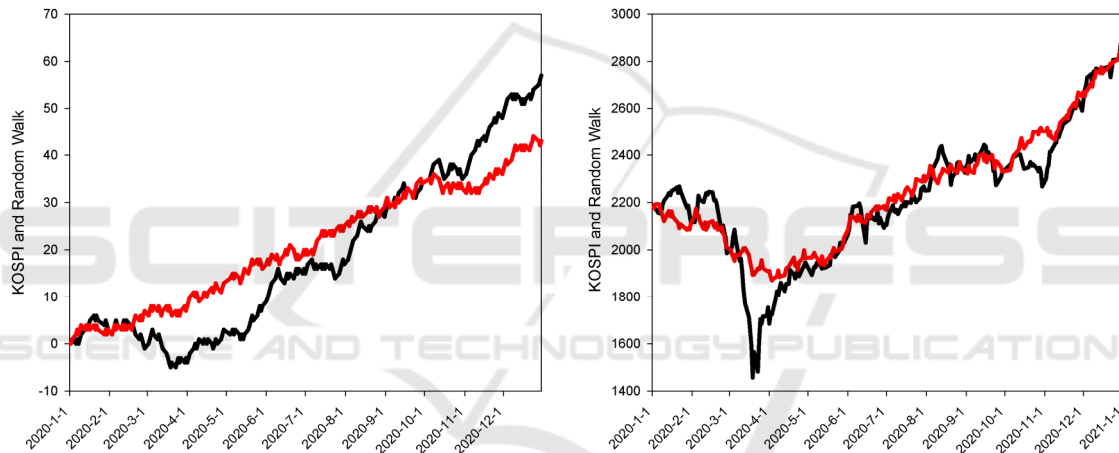


Figure 1: KOSPI and random walk simulation. On the left panel, KOPIS in simplified format (black line) and random walk in the 1/-1 format (red line) with a seed of 0.78654. On the right panel, KOSPI (black line) and random walk in the decimal format with one of seeds: 7.30548, 7.30549, . . . 7.30557

Exactly following the steps demonstrated in Table 1, we can construct the random walk simulation of KOSPI graphically for different periods of KOSPI and its simulation..

The left panel of Figure 1 shows the simplified KOSPI and its random walk simulation in the 1/-1 format. The initial several steps are exactly the same as we show in from column 1 to column 6 in Table 1. Because of simplification of KOSPI, the range in y-axis is not as large as real-life KOSPI range. As seen in the left panel, the random walk simulation increases monotonically, but the simplified KOSPI fluctuates largely for the first half year of 2020, which marks the initial lockdown due to Covid-19 pandemic. Because the random walk goes either one step up or one step down and there are 248 trading

days in 2020, the random walk theoretically has a  $(\frac{1}{2})^{248}$  chance to fit the KOSPI without any difference although this chance is extremely low.

The right panel of Figure 1 shows the real-life KOSPI and its random walk simulation in decimal format. Evidently, the random walk misses the big fall of KOSPI in March 2020, which is totally due to the unexpected Covid-19, on the one hand. On the other hand, it does follow the uptrend from May 2020 until December 2020 although there is a small dip in October. Interestingly, there are ten seeds, which can produce the same results. This may suggest the possibility that random number generator using Monte Carlo algorithm is not sensitive to the increment at 0.00001. However, this possibility may not be valid because the increment of 0.00001 does

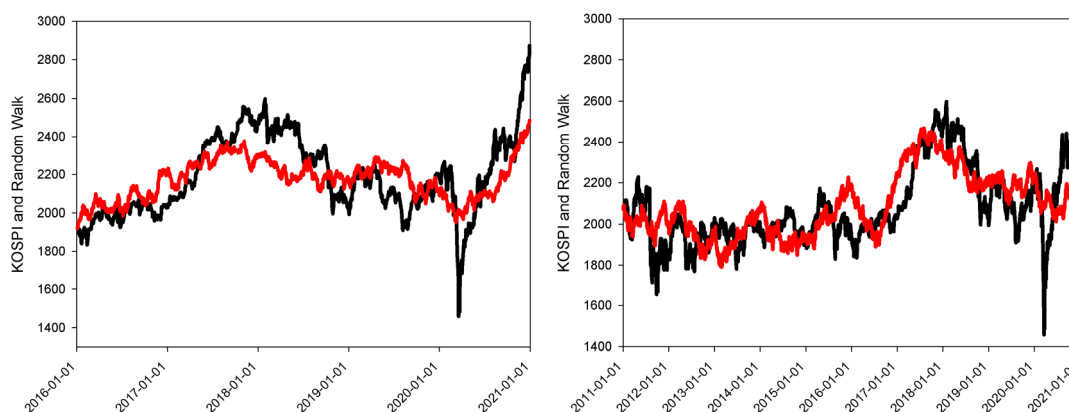


Figure 2: KOSPI (black line) and random walk simulation (red line) for the period from 2016 to 2020 with seed of 1.32353 (left panel), and for the period from 2011 to 2020 with one of seeds: 7.73709, 7.73710, . . . 7.73718 (right panel).

have effects in other simulations. In fact, we used the increment of 0.000001 in the past, but we frequently find the same result from a series of seeds with increment of 0.000001. Therefore, we choose the increment of 0.000001 in our recent studies to reduce the computational time.

Comparing two panels in Figure 1, we sometimes feel that we can phase out the random walk simulation in the 1/-1 format in the future because it does have a certain chance to occur, i.e.  $(\frac{1}{2})^{248}$ , but the random walk in decimal format also has a certain possibility for a perfect fit. Technologically, we have no need to pay great attention to the upper and lower ranges in the random walk simulation in the 1/-1 format because the generated random numbers should be rounded after comparison. In contrast, the choice of upper and lower ranges becomes difficult when there is an unexpected rise or fall in KOSPI because these rises and falls do not rend a big change in standard deviation, which are used in our studies.

The left panel in Figure 2 illustrates the random walk simulation on KOSPI for five years. There are two periods of uptrend, a slow one and a rapid one. For the slow uptrend from 2016 to 2018, the random walk simulation still can follow its paces step-by-step. For the rapid uptrend from April 2020 to December 2020, the random walk does not move fast enough to catch up with the final phase of uptrend in December 2020.

The right panel in Figure 2 depicts the random walk simulation on KOSPI for ten years. As seen in this panel, the KOSPI holds on the relatively small fluctuations from 2011 to the middle of 2017. For this period of time, the random walk simulation can follow the trend moving horizontally, and even follow the uptrend in 2018, but fails to follow the sharp uptrend at the end of 2020. When looking the three real-life random walk simulations from the right

panel in Figure 1 to left and right panels in Figure 2, we can see a general tendency, that is, it becomes harder and harder to catch up with the final uptrend in the end of 2020.

Similar to what happens in the right panel in Figure 1, there are also ten seeds in the right panel in Figure 2, which generate the same result. More importantly, there is no big difference between the seeds for the right panel in Figure 1 and the seeds for the right panel in Figure 2. These are very suggestive because it implicates that there is a possibility to use a seed to simulate two periods of KOSPI opening up the possibility to predict the future KOSPI movement using the same seed.

The left panel in Figure 3 pictures the random walk simulation on KOSPI for fifteen years from 2006 to 2020. For this period of time, we can see that KOSPI actually goes up although there are two big falls in 2008, which is the financial crisis, and in 2020, which is the Covid-19 pandemic. Clearly, the random walk cannot reach as deep as the real-life KOSPI in these two big falls.

The right panel in Figure 3 describes the random walk simulation on KOSPI for twenty years from 2001 to 2020. For this longest period of simulation in this study, the KOSPI goes up from 500 approaching to 3000. The small fluctuations appear insignificant in such a long scale, however, their contribution to the statistical description is large because they significantly decrease the standard deviation. Therefore, it is impossible for Month Carlo algorithm to generate very large random numbers to cope with the two great falls in 2008 and 2020, and subsequently the biggest jump at the end of 2020. Indeed, the simulation in the right panel of Figure 3 is encouraging because it does follow the KOSPI trend for 20 years.

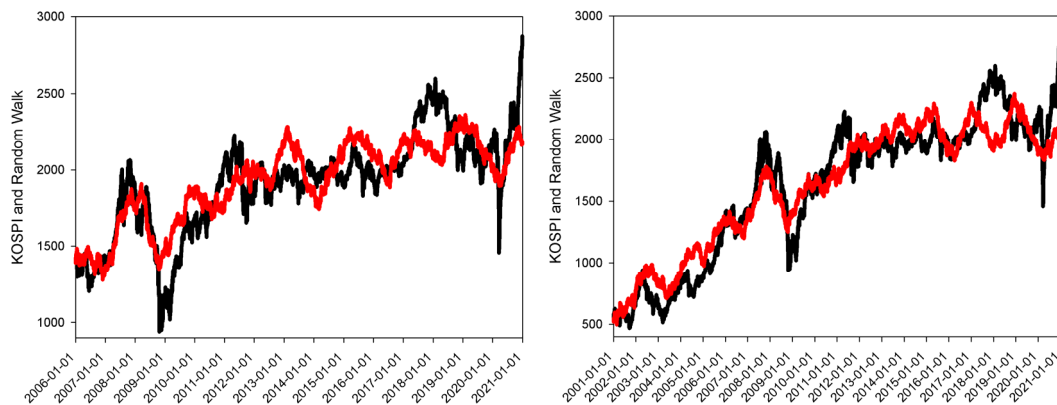


Figure 3: KOSPI (black line) and random walk simulation (red line) for the period from 2006 to 2020 with a seed of 2.62207 (left panel), and for the period from 2001 to 2020 with a seed of 0.03275 (right panel).

The issue of whether or not a stock market index or an individual stock follows a random walk is the objective in many studies, but the difficulty is that random walk cannot reveal the underlined mechanism in stock market. Nevertheless, randomness can be the underlined mechanism for stock movement, not only because various known and unknown factors affect the movement of stock market, but also randomness is considered to stay at the heart of nature (Everitt, 1999). However, the simulation actually does a phenomenological job. In fact, regression is also a phenomenological tool to build a possible and potential cause-effect relationship, but random walk does link any cause to the effect. Yet, the random walk has few parameters, i.e. seed, upper and lower ranges, so it may encompass uncountable known and unknown factors, which can be considered random because we cannot define their cause-effect relationship.

Oftentimes, the issue of whether or not a stock index can be described by a random walk model is related closely to the efficient market hypothesis (EMH), especially, the weak form market efficiency. The knowledge gap is progressively filled up with various statistical tests as well as the real-life random walk simulations. The issue of whether or not the simulation is dependent on time length is yet to be proved because simulations for different period of time result in different results.

In foreseeable future, we hope to solve several problems with random walk simulation, for example, how to choose suitable upper/lower ranges, whether to use a percentage will be more helpful than the real-life data, etc. At any rate, much work needs to be done in the future.

## 4 CONCLUSION

In this study, we use the random walk model to simulate KOSPI for the first 20 years in the 21<sup>st</sup> century because the hypothesis of whether a stock index can be described using a random walk model is still unsolved. Therefore, it is necessary to conduct this type of studies to different stock indices and different individual stock. In fact, statistical tests on random walk go along this way. Our results demonstrate that the random walk model can simulate at least the general trend of KOSPI, but far from accurately and precisely follow the real-life KOSPI although there is an extremely small possibility to do so. We wish to continue our studies along this line in near future.

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