U-Optimal Accelerated Life Test Scheme Considering Right Censored Data

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Abstract: The purpose of accelerated life test is to promote more failure units of test products in a shorter time, so as to predict the reliability of products under normal conditions based on the failure data collected under accelerated conditions. In view of the right censored data, some statistical optimality is considered. In the framework of generalized linear model, the semi-parametric Cox proportional hazard model is used to obtain the accelerated life test scheme under the U- and D-optimal criteria. From the perspective of prediction variance, the fitting effect is best when the shape parameter α is 1.5 in Weibull distribution. The uncertainty of model parameters is evaluated by Monte Carlo technique to verify the feasibility of the test scheme.

1 INTRODUCTION

A large number of products in the market have a long expected life. In order to ensure the stable performance of product life during the service period, it is unrealistic to completely observe the product life due to limiting factors. Therefore, accelerated life test (ALT) is introduced. ALT ensures that more products failure data can be obtained in a shorter time, and the reliability of products can be inferred by establishing statistical models. However, when faced with irregular experimental design areas, the regular design loses some statistical "optimality" and needs to be "tailored" design.

Censored data results from inaccurate observations of failure times. The definition of right censored data is that the starting time of the test is known, but the test fails to fail at the end of the test, thus, the survival time is longer than the observed time. There is a huge literature on ALTs. Song Wu et al (Wu, Lu, Li, 2021) briefly described the relevant theoretical knowledge of ALT. Under the assumption of lognormal distribution, Xiaopei Li et al (Li, Li, Liang, 2021) proposed the relationship between single stress variable and product life. Yi Dai et al (Dai, Liu, 2020) applied the maximum likelihood theory to design the optimal test under the condition that the product life obeys the minimum extreme value distribution. The literature described above are performed under single stress conditions. In fact, most product life is affected by multiple stress variables. Xu et al (Xu, Fei, 2007) discussed the dual-stress variables with no interaction between. Park and Yum (Park, Yum, 1996) assumed the interaction between stress factors and verified it. The process of obtaining the information matrix is particularly complex, thus, focusing on the hazard rate and under the assumption of proportional hazard model (PH), the ALT scheme is transformed into an optimization problem under the generalized linear model (GLM). For different statistical optimality, Guo and Pan (Guo and Pan, 2007) used GLM method to obtain the plan under D-criterion. Juan Wang (Wang, Ma, Wang, 2017) discussed the ALT scheme with 2 stress factors under the I-optimal criterion for interval censored data. In addition, due to the uncertainty of model parameters, literature (Dror, Steinberg, 2006; Ozol-Godfrey, Anderson-Cook, Robinson, 2008) has discussed the wrong designation of relevant parameters.

2 ACCELERATED LIFE TEST MODEL

The purpose of using D-optimal is to maximize the determinant of the expected information matrix. The goal of the U-optimal is to minimize the overall variance of the model parameter estimator. Specifically, the D-optimal criterion is expressed as:

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$$\xi^* := \underset{\xi}{argmax}\, |\boldsymbol{X}(\xi)'\boldsymbol{W}\boldsymbol{X}(\xi)|$$

U-optimal criterion:

$$\xi^* := \underset{\xi}{argmin} x'_{use} \cdot (\boldsymbol{X}(\xi)'\boldsymbol{W}\boldsymbol{X}(\xi))^{-1} \cdot x_{use}$$

x_{use} represents the stress under the use condition, $\boldsymbol{X}(\xi)$ represents the model matrix of $n\times p$, n and p represent testing numbers and model parameters, respectively, and $\boldsymbol{W}$ is the weight matrix related to the variance of the predicted life.

Under the assumption of PH, the failure function can be expressed as:

$$f(t) = h(t)R(t) = h_0(t)e^{x'\beta}(R_0(t))^{e^{x'\beta}}$$

$h_0(t)$ is underlying hazard function, $\boldsymbol{\beta}$ is the vector of regression coefficients, and $\eta = \boldsymbol{x}'\boldsymbol{\beta}$ is the linear prediction of the model. $R_0(t)$ is reliability function, the relationship with the cumulative hazard function is $R(t) = exp\,(-H(t))$. For the right censored failure time dataset, $(t_1,\ r_1)$,…$(t_i,\ r_i)$,…$(t_n,\ r_n)$, i=1, 2, 3, …n, t_i is the failure or survival time of the ith data, r_i is indicator variable of censored time. If the ith test unit fails, r_i takes the value 1; otherwise, it is 0. After simplification, the likelihood function can be expressed as:

$$L = \prod_{i=1}^{n}(f(t_i))^{r_i}(R(t_i))^{1-r_i} = \prod_{i=1}^{n}(h(t_i))^{r_i}R(t_i)$$

take the logarithm of both sides:

$$\begin{aligned} ln\,L &= \sum_{i=1}^{n}[r_i\,ln\,h(t_i) + ln\,R(t_i)] \\ &= \sum_{i=1}^{n}[r_i(ln\,h_0\,(t_i) + \boldsymbol{x}'_i\boldsymbol{\beta}) \\ &\quad + e^{x'\beta}\,ln\,R_0(t_i)] \end{aligned}$$

let $u_i = H(t_i, \boldsymbol{x}_i) = exp(\boldsymbol{x}'_i\boldsymbol{\beta})(-\ln R_0(t_i))$, this is:

$$ln\,L = \sum_{i=1}^{n}[r_i\,ln(\frac{1}{t_i}) + (r_i\,ln\,u_i - u_i)]$$

The form $r_i\,ln\,u_i - u_i$ can be regarded as the log-likelihood function form of Poisson distribution with mean u_i. So, in the GLM: indicator variable r_i can be regarded as poisson distribution with mean u_i, the connection function is the logarithmic function, $ln\,u_i = \eta_i +$ compensation term, the compensation term is $ln\,H_0(t_i)$.

In the GLM described above, using the elements u_i, $i = 1, \ldots n$ construct weight matrix, $\boldsymbol{W} = diag\{u_1, u_2, \ldots, u_n\}$, then the model estimation parameter $\widehat{\boldsymbol{\beta}}$ is:

$$Var(\widehat{\boldsymbol{\beta}}) = (\boldsymbol{X}(\xi)'\boldsymbol{W}\boldsymbol{X}(\xi))^{-1}$$

Among them, $(\boldsymbol{X}(\xi)'\boldsymbol{W}\boldsymbol{X}(\xi))^{-1}$ is the expected Fisher information matrix, and the number of elements in matrix $\boldsymbol{X}$ is $n\times(p+1)$:

$$\boldsymbol{X} = \begin{bmatrix} 1 & x_{1,1} & \cdots & x_{p,1} \\ 1 & x_{1,2} & \cdots & x_{p,2} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{1,n} & \cdots & x_{p,n} \end{bmatrix}$$

u_i in the weight matrix is a function of the failure time t_i. Therefore, it is more appropriate to express the weight matrix by the expected value of u_i: $\boldsymbol{W} = diag\{E(u)\}$:

$$\begin{aligned} E(u_i) = H(t_i, \boldsymbol{x}_i) &= exp(\beta_0 + \boldsymbol{x}'_i\boldsymbol{\beta}) \cdot E(t_i) \\ &= \lambda_0 e^{x'\beta} \cdot E(t_i) \end{aligned}$$

λ_0 is the failure rate, $\beta_0 = ln\lambda_0$, which is the intercept term of linear prediction. Explicitly censoring time, the expectation function is $E(t_i) = [1 - e^{-H(t_r, x_i)}]\frac{1}{\lambda_0 e^{x'_i\beta}}$.

Finally get the expected value:

$$E(u_i) = [1 - e^{-H(t_r, x_i)}]$$

3 THE EXAMPLE ANALYSIS

3.1 The Scheme Affected by Temperature and Humidity

Consider tests related to metal oxide semiconductors (Zhu, Elsayed, 2011). Assuming that the lifetime of the semiconductor is affected by two stress variables, temperature and humidity. 100 samples is planned to be tested, and the test is set to be censored at 50 hours. It is assumed that lifetime obeys the Weibull distribution, and the cumulative hazard function is $H(t, \boldsymbol{x}) = \lambda_0 t^{\alpha} e^{x'\beta}$, α is the shape parameter. Various factors are considered in this paper, and $\alpha = 1.5$ is taken as the value. In the test, under normal conditions, the temperature and humidity range is (25℃, 25%)-(45℃, 40%). In the ALT, set temperature range of (60℃-110℃), humidity range is (60%-90%), the temperature of natural stress level is expressed as $S_1 = 11605/T$, T is an unit with Kelvin, natural stress level of relative humidity is expressed as $S_2 = \ln h$, h is relative humidity. To normalize the processing, the following linear transformation of temperature and humidity is used: $x_1 = \frac{S_1 - S_1^H}{S_1^L - S_1^H}$, $x_2 = \frac{S_2 - S_2^H}{S_2^L - S_2^H}$, $S_H(0, 0)$ said the highest stress level, $S_L(1, 1)$

minimum levels of stress, x_1 and x_2 are the coding stress variables corresponding to S_1 and S_2. In this paper, the interaction effect of temperature and humidity is considered and the previous empirical formula is followed: $\eta = -4.086x_1 - 1.476x_2 + 0.01x_1x_2$.

Figure 1 and Figure 2 show the test protocol under U- and D-criterion and the contour plots of predicted variance in the use area. The process of calculating 100 test points is more complicated, using a clustering algorithm to aggregate the points into four different stress level combinations. Small square area of a graphic after the corresponding standardized test area, the origin (0, 0) corresponding to (110℃, 90%), point (1, 0)-(60 ℃, 90%), (0, 1)-(110 ℃, 60%), (1, 1)-(60 ℃, 60%). The circle diameter corresponds to the assigned sample size under the test conditions, and the contour lines outline the positions where the predicted variance are equal. Comparing the two figures, it can be seen that the predicted variance under the U-criterion scheme is smaller than that under the D-criterion; It can be seen from Figure 1 that under the U-criterion, the number of test samples assigned to point (1, 1) is the largest. The reason may be that the low stress level is closer to the normal operating conditions, and more test samples have been censored when they do not reach the high stress level. According to Figure 2, the number of samples distributed around each stress point under the D-criterion is roughly balanced.

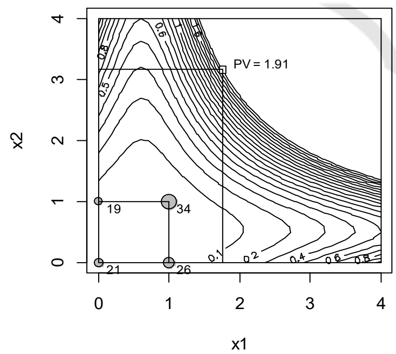

Figure 1: U-optimal criterion design plot.

The graphical evaluation tool further compares the test protocols under the U- and the D-criterion. In the FUS (Fraction of Design Space) plot, the predicted variance of the ordinate increases with the increase of the ratio of test areas, and the vertical red line represents the mean. The results show that the predicted variance of D-criterion is larger than that of U-criterion. In the VDUS (Variance Dispersion of Use Space) diagram, *ave*, *min* and *max* respectively represent the mean, minimum and maximum value. It can be clearly seen that the predicted variance value under the D-criterion test scheme is larger.

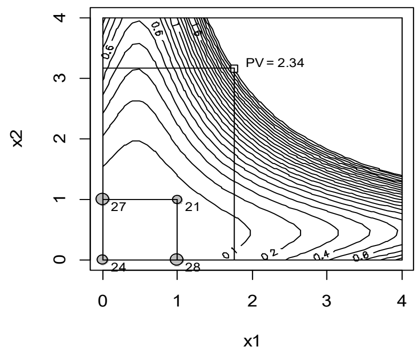

Figure 2: D-optimal criterion design plot.

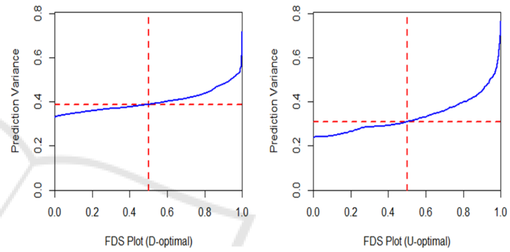

Figure 3: FDS design plot.

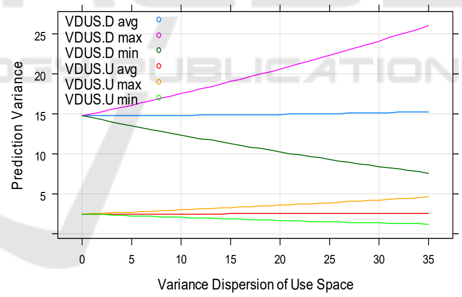

Figure 4: VDUS design plot.

The prediction results are expected to be accurate from the perspective of predicted variance, and the shape parameter α in Weibull distribution is an indeterminate variable. The above results are obtained under the assumption that $\alpha = 1.5$. In order to show the rationality of the method, the case of α taking other values is further discussed. When $\alpha = 1$, the predicted variance under D- and U-criteria are 4.33 and 3.58, respectively, and when $\alpha = 2$, the predicted variance are 2.14 and 1.92. The experimental results show that the predicted variance of the U-criterion is always smaller than that of the D-criterion for different values of α. In addition, in terms of sample num, when the value of α changes, the sample num under

D-criterion does not change significantly, on the contrary, the sample num under U-criterion changes significantly, especially the num of point (1, 1) decreases with the increase of the value of α. The results show that U-criterion has a more obvious influence on the value change.

3.2 Model to Evaluate

The coefficients specified are taken from previous experimental results; therefore, ALT protocol with assumed model coefficients needs to be evaluated. The previous scheme assumes that the real value of stress coefficient is not more than ±20% away from the set value. In this paper, Monte Carlo technology is used to analyze the uncertainty of the model coefficient, calculate the fluctuation range of the error, and verify the robustness in reverse.

The Monte Carlo technique uses repeated random sampling method to obtain numerical results, which is beneficial to the processing of complex tests. First, specify the right censored data type, input the sample size, expectation matrix, linear predictor coefficient and other relevant variables; Secondly, the GLM was fitted to obtain the values in the model matrix. Finally, Monte Carlo simulation is used to evaluate the intercept term, temperature coefficient x_1, humidity coefficient x_2, and interaction coefficient x_1x_2 in the linear predictor given the values of the running matrix and the statistical model fitted to the data. The expected test result is (0, 0, 0, 0). The actual test results are as follows: the intercept term change rate is 19.65%, x_1 is 19.27%, x_2 is 19.95%, x_1x_2 is 19.17%. The test results show that the change rate of each coefficient is less than 20%, thus, the error rate of the test scheme is acceptable. The coefficients in the linear predictor vary within the range, which will not affect the operation of the test scheme, and the scheme is still robust.

4 CONCLUSION

In this paper, we discuss the ALT scheme based on optimal criteria in the framework of GLM with right censored data. However, the method of parameter estimation is based on determining the failure data distribution, and the parameters are fixed. In fact, in many cases, the failure data are limited or non-existent, which makes it difficult to determine the data distribution. In this case, the Bayesian method is an option. In the following research, when the failure data are interval censored, Bayesian method is used to obtain the posterior distribution according to the prior estimation of parameters, so as to reduce the dependence of model parameters.

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