



Crime-related Threats on Transportation Safety: A Mathematical Model of Risk Assessment

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Abstract: The article is devoted to the crime risks assessment as a possible option to counter threats to security in transport. The model used is demonstrated on the example of a specific criminal case. The problem in question developed mathematical model which describes the danger of a person. It is an attempt to solve an problem by numerical methods – to made the model of individual criminal behavior. This makes it possible to prevent recidivism of persons who have been sentenced to probation.

1 INTRODUCTION

The transport security system predetermines and interconnects several security systems of a lower rank, including criminological security.

Transport sector, including rail transport, provides the basic conditions for the life of society, but crime as a negative social and legal phenomenon is also an integral part of public life. Socio-economic transformations associated with the development of technology entail the transformation of crime, and this pattern is clearly manifested in the relationship between crime and transport.


Accordingly, in modern realities, the task is to increase the effectiveness of the impact on the sources of threats to criminological security - crime in all its manifestations, including the personality of the offender and criminogenic factors. In this paper, the authors focused on the study of the identity of the offender as a carrier, the subject of this threat in order to predict the risks of criminal behavior.


In the natural sciences, the task of researching any phenomenon or process is usually reduced to solving algebraic, differential, or other equations, which contain a large amount of quantitative information about the process under study. An

accurate description of the process under study usually allows one to model the process for other possible conditions as well.

Recently, mathematical models have become widely used in various not only technical sciences, but also in economics, medicine, biology, and in the humanities. Much less publications using mathematical models are found in legal sciences (Srivastav, 2020; Syed, 2013). It seems that the use of mathematical models and methods will help solve a significant class of problems that arise in the social system, in particular, in the field of criminological prevention. In criminology, mathematical methods for processing statistical information (Farrington, 2016) and correlation analysis began to be used to assess the possibility of relapse. There are also probabilistic predictive approaches to assessing the risk of relapse (Skeem, 2016). It is known that criminological studies reveal that some personality attributes are associated with criminal behavior (Miller, 2001).

Although the connection between past and future criminal behavior has been found to some extent (Farrington, 2003), in practical terms, predicting the individual criminal behavior of a person is an exceptionally difficult task. This is since human

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behavior is determined by a plurality of factors that have different influences on it. It should be emphasized that a criminological forecast is always probabilistic in nature, since with respect to any person it is impossible to say with an absolute guarantee that he will commit or not commit a crime. But the main factor influencing the possibility of committing a crime, including recidivism, is the individual danger to the personality of the offender, which is formed during life, which consists not only in the fact that the person turned out to be able to commit an already actually committed crime, but also in the fact that he is able to under certain conditions and continue to do so.

In this work, the proposed mathematical model proceeds from the fact that some aspects of the complex behavior of an individual can be formalized in some system from a multitude of simultaneously interacting factors that affect the personality. Individual behavior can be formalized on the basis of very simple rules for the interaction of independent components that negatively affect the personality and determined using mathematical equations. For this, a hypothesis is introduced that the probability of individual criminal behavior of a person is proportional to the total recorded number of crimes in the period of interest. This hypothesis allows us to create an equation in which the unknown is the personality dangerousness depending on time. Time discretization of the process of observing a person allows the resulting integral equation to be reduced to linear algebraic equations that are easily solved.

2 FORMULATION OF THE PROBLEM AND ACCEPTED CONCEPTS

To model individual criminal behavior, we introduce several functions that have a quantitative value. Let us introduce the personality function C_c (Characteristic of Criminal) as a negative characteristic of the subject's personal qualities that increase the likelihood of relapse. Let us assume that the function C_c will be determined by several known values obtained from information during observation in a certain period about the person under study. Let us assume that the value of this function may vary within $0 \leq C_c \leq 1$.

We also introduce the function S_d (Social determinants). This function will also have a known quantitative value and will change within $0 \leq S_d \leq 1$.

The S_d function will be determined by the influence of negative social processes taking place in society, affecting the subject and increasing the likelihood of relapse. We assume that the function S_d will be determined by several known values obtained from statistical sources.

The introduction of the function C_c and S_d , which vary from zero to one, is like the damage function in the mechanics of materials, which is used in the study of a damaged material. It is known that the influence of various types of damage on the characteristics of the deformation process is carried out with the help of a scalar function ω – a measure of damage. The value of ω , which varies from the value $\omega=0$ for an undamaged material, $\omega=1$ – for a destroyed material.

For mathematical modeling of individual criminal behavior, we introduce the function D (Personality dangerousness). This function will determine the danger of a person, i.e. propensity to commit a crime. We accept that the function will be determined from the solution of the problem of modeling the process of the criminal behavior of the subject, before and at the time of the commission of the crime.

Since the personality of a criminal can differ in different suggestibility – susceptibility to various social processes, we introduce the coefficient s (Suggestibility). We accept that this coefficient is determined by the psychophysiological characteristics of the subject's personality and will have a quantitative value within $0 \leq s \leq 1$. At the same time, we assume that at $s=0$ the subject is not receptive, but at $s=1$ it is strongly receptive to various social processes taking place in society.

Thus, any person at every moment of time will be under the influence of negative formed personal qualities and several social negative processes taking place in society. Given the different susceptibility of the individual, the negative "load" on a person can be represented as a sum of influences $(C_c + sS_d)$ that affects a person at any given time.

3 ACCEPTED HYPOTHESES AND RESOLUTION EQUATIONS

Let us introduce the concept of risk assessment of the commission of a crime R (Risk assessment) as a product of functions $D \times (C_c + sS_d) = R$. The meaning of this concept lies in the fact that under the same external conditions, a crime will be committed by a person who has a greater function of the danger of

personality D . Since the risk of a crime is a probabilistic value that depends on time t , then we define it as the probability of committing a crime P_R under the prevailing conditions, and, therefore,

$$D(t) \times [C_c(t) + sS_d(t)] = P_R(t) \quad (1)$$

Consider the ultimate crime scenario. If a crime has occurred, then, therefore, the probability of its commission is equal to one

$$P_R(t) = 1 \quad (2)$$

Suppose that a set of formed personal qualities and a spectrum of social negative processes has the highest possible importance, therefore

$$[C_c(t) + sS_d(t)] = 2. \quad (3)$$

We normalize equation (3), dividing it to 2

$$[C_c(t) + sS_d(t)] \times 2^{-1} = 1. \quad (4)$$

Considering equations (1) and (4) we obtain that the function of personality dangerousness D at the time of the crime under the most unfavorable external conditions is equal to one

$$D(t) = 1. \quad (5)$$

Since all parameters of equation (1) depend on the time t , integrating it

$$\int_0^{\Delta T} D(t) \times 2^{-1} [C_c(t) + sS_d(t)] dt = \int_0^{\Delta T} P_R(t) dt. \quad (6)$$

Here ΔT is the period of observation of the subject.

Equation (6) is an integral equation that contains unknown functions personality dangerousness D and the probability of committing a crime P_R .

It is known that in mechanics and physics in solving problems that are described by differential and integral equations, research objects are divided into a few elements in which constant mathematical values are taken or some known functions. This technique allows you to solve many scientific and applied problems (Grigorenko, 1999; Emelyanov I. G. 2009; Emelyanov I. G. 2018).

It is known that the integral equation can be represented as a system of linear algebraic equations. We divide the period of monitoring the subject of ΔT on N steps over time (day, month, year, ...) in which the functions entered above will have a constant value. Thus, the time step will be equal

$$t_i = \Delta T \times N^{-1} \quad (i = 1, \dots, N) \quad (7)$$

For each step in time, we write equations (6)

$$D_i \times 2^{-1} [C_{ci} + sS_{di}]_{t_i} = P_{Ri} t_i \quad (8)$$

Accepting a permanent and dimensionless time step, equation (8) will take the form

$$D_i \times 2^{-1} [C_{ci} + sS_{di}] = P_{Ri} \quad (9)$$

In the system (9) in the right part, the likelihood of committing a crime within each time period is presented. Since we will explore cases with an intentional crime at the end of the observation period, the probability of its offense will be equal to one. Therefore, we will have an additional equation

$$\sum_{i=1}^N P_{Ri} = 1. \quad (10)$$

In system (9), on the right side, the probabilities of committing a crime by some subject from a certain group of offenses that occurred in the period of interest are presented. We accept the hypothesis that the probability of a single crime from a certain observation group is proportional to the probability of the totality of crimes committed in a certain industry over a certain period of time. Therefore, one can write

$$P_{Ri} = k p_i \quad (11)$$

Here k – unknown coefficient of proportionality, p_i – the probability of committing crimes committed in a certain industry for a certain period. The probability of committing crimes in a certain territory for a certain period can be defined as

$$p_i = \frac{Cr_i}{Pop_u} \quad (12)$$

Here Cr_i – the number of intentional crimes in a certain industry in a certain time, P_{OP} – the total population.

Considering the expression (11) and (12), system (9) will have the form

$$D_i \times 2^{-1} [C_{ci} + sS_{di}] = \frac{k}{P_{OPU}} Cr_i \quad (13)$$

From equation (10) one can determine the coefficient k for the right side of equation (13)

$$\frac{k}{P_{OPU}} \sum_{i=1}^N Cr_i = 1 \quad (14)$$

Considering relations (14), system (13) takes the form

$$D_1 \times 2^{-1} [(C_{c1} + sS_{d1})] = \frac{Cr_1}{\sum_1^N Cr_i} \tag{15}$$

$$D_N \times 2^{-1} [(C_{cN} + sS_{dN})] = \frac{Cr_N}{\sum_1^N Cr_i}$$

4 EXAMPLE OF USING THE PROPOSED METHOD

Let us define the function personality dangerousness D of the offender for the criminal case committed in 2014. Criminal case № 1-649/2014 <https://sudact.ru/regular/doc/O6J6wFXEKtrt/>.

The period of observation of the subject 2010 - 2014, $\Delta T = 5$ years. $t_i = 1$ year is a dimensionless time for collecting statistical information.

Table 1 shows the state of crime in the Russian Federation in thousands of registered crimes.

Table 1: The state of crime in the Russian Federation.

Year	2010	2011	2012	2013	2014
Cr_i	2628,8	2404,8	2302,2	2206,2	2166,4

To determine the personality function C_c , we will assume that the personal qualities of the subject under study will be characterized by a certain set of diskette values obtained from statistical sources. For this case, we assume that a person can be characterized by 20 ($k = 20$) values that contribute to a crime and are presented in Table 2. Since the maximum value of $C_c = 1$, then, assuming the hypothesis of the same influence of each factor on the commission of a crime, each component will "weigh" $C_{Ci} = C_c \times k_i^{-1} = 0.05$. The accepted hypothesis means that in the presence of all 20 negative factors, a person has the maximum probability of an offense.

Therefore, we will assume that the personal qualities of the subject during five years of observation are constant and equal to $C_c = 0.35$. It should be added that the quantitative values in this table 2 may change during the observation period, which can be considered in equation (15).

We accept that the function of the influence of negative social processes on the personality of the subject under study S_d will be characterized by a set

of diskette values obtained from statistical sources.

For this case, we will assume that four ($k = 4$) negative social factors contributing to crime can influence a person. These are international migration, lack of cash income, the number of unemployed aged 15-72 and urbanization. Information on these processes for the observation period from 2010 to 2014 is available on the website <https://www.gks.ru/statistic>.

Table 2: Characteristics of the criminal

1.	The presence of psychopathy and psychopathic conditions – 0
2.	Organic lesions of the central nervous system, brain contusions, mental anomalies – 0
3.	Aldolization and narcotization – 0.5
4.	Emotional instability – 0
5.	Social degradation, marginalization – 0
6.	Ethnic, religious intolerance – 0
7.	Belonging to a criminal subculture – 0
8.	Belonging to other marginal groups, informal groups of an illegal orientation – 0
9.	Low level of material security – 0.5
10.	Lack of work – 0.05
11.	Employment in unskilled labor – 0.5
12.	Not married – 0.5
13.	Lack of secondary education – 0.5
14.	The presence of a criminal record – 0
15.	Bringing to criminal liability – 0
16.	Lack of housing – 0
17.	Belonging to the urban population – 0.5
18.	Cynicism, legal nihilism – 0
19.	Unresolved life conflicts – 0
20.	Facts of domestic violence – 0

Statistical information on international migration in the number of people for 2010-2014 is calculated using the formula

$$S_{di} = \frac{M_i - M_{min}}{M_{max} - M_{min}} \tag{16}$$

Here M_i, M_{max}, M_{min} – the current, maximum, and minimum value of international migration

Table 3 shows the normalized distribution of international migration calculated by formula (16) (a function of negative social processes occurring in society S_d).

Table 3: International migration.

Year	2010	2011	2012	2013	2014
S_d	0.	0.413	0.566	0.727	1.

Similarly to international migration, statistical information is processed on the deficit of money income (in billion rubles), the number of unemployed aged 15-72 years (in thousands of people) and urbanization (in thousands of people). Table 4 - Table 6 provides information on the normalized distribution of the deficit of money income, the number of unemployed and urbanization (total urban population growth) calculated by formula (16), respectively.

Table 4: Deficit of income

Year	2010	2011	2012	2013	2014
S_d	0.04	0.477	0.	0.415	1.

Table 5: Number of unemployed

Year	2010	2011	2012	2013	2014
S_d	1.	0.624	0.145	0.149	0.

Table 6: Urbanization

Year	2010	2011	2012	2013	2014
S_d	0.355	0.	0.505	1.	1.

For the maximum factor S_d to be equal to one, as defined above, the values in tables 3-6 must be reduced by a factor, therefore $S_d \times k_2^{-1}$.

Considering the above calculations and setting a low coefficient of the subject's susceptibility to social factors $s=0.25$, system (15) takes the form

$$\begin{aligned}
 D_1 \times 2^{-1}[(0.35 + 0.25 \times 0.348)] &= \frac{2628.8}{11708.4} \\
 D_2 \times 2^{-1}[(0.35 + 0.25 \times 0.378)] &= \frac{2404.8}{11708.4} \\
 D_3 \times 2^{-1}[(0.35 + 0.25 \times 0.304)] &= \frac{2302.2}{11708.4} \quad (17) \\
 D_4 \times 2^{-1}[(0.35 + 0.25 \times 0.572)] &= \frac{2206.2}{11708.4} \\
 D_5 \times 2^{-1}[(0.35 + 0.25 \times 0.75)] &= \frac{2166.4}{11708.4}
 \end{aligned}$$

Figure 1 shows the law of change function personality dangerousness D over the observation period from 2010 to 2014. Curves 1,2,3 correspond to the calculation for the susceptibility coefficient $s=0.25$ $s=0.5$ $s=0.75$.

Thus, it can be stated that for the subject of the considered criminal case, the function personality

dangerousness D for not repeating the crime, under equal external conditions, should be less than $D < 0.405$.

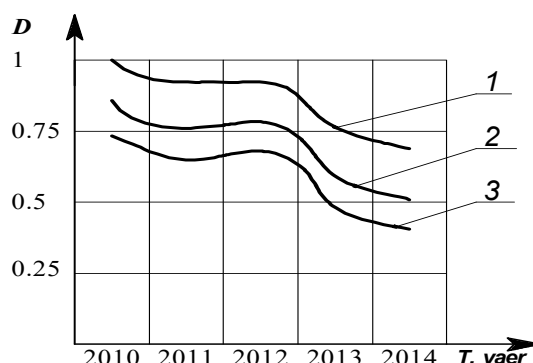


Figure 1: The law of change function personality dangerousness D over the observation period

5 DISCUSSION OF RESULTS SOLUTIONS AND CONCLUSIONS

In this article, the function personality dangerousness was found based on a specific crime. It should be emphasized that we propose to extend our model only to cases of probation, that is, in this model there are no changes in environmental factors, the offender is not imprisoned.

The paper proposes a mathematical model that describes the function personality dangerousness. Quantifying this function makes it possible to assess the risks of relapse in probationers based on extrapolation of social determinants influencing crime and predictions of the state of crime, as well as data on changes in personality traits.

Based on the extrapolation of social determinants influencing the crime, and forecasts of the state of crime, as well as data on the individual in the time - the term of probation, knowledge of the function personality dangerousness will make it possible to assess the risks of relapse. If the function exceeds $D > 0.405$, this dramatically increases the risk of recurrence and requires the subjects to respond proactively and take preventive measures.

Thus, numerical modeling of individual criminal behavior through a comprehensive account of external and internal factors affecting the individual's behavior can be an effective way to predict such processes. The method can be used to counter security threats in transport.

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