

# Design of Foundations for Buildings and Bridges using Nonlinear Soil Model

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**Abstract:** The article substantiates the need to design foundations of bridge supports on non-rocky soil base taking into account physically and geometrically nonlinear features of soil deformation. Dusty-clayey soils (sandy loams, loams, clays, and their older and stronger formations - saprolites) of semi-hard, hard, and often tough-plastic consistency, serve as a reliable and economical soil basis for various structures, including bridges. According to Article 16 of Technical Regulations on the Safety of Buildings and Structures (Federal Law № 384-FZ), as well as paragraphs 5.1.11 of SP 22.13330.2016 ("Soil bases of Buildings and Structures") in this case design must be carried out taking into account the nonlinear deformation of soils. The use in design of a theory (design model) that corresponds to physical essence of soils instead of nominal for them, physically linear (Hooke) model, makes it possible to increase radically reliability of structures and obtain at least a twofold saving in the cost of foundations.

## 1 INTRODUCTION

Soils: hard rocks and non-hard soils (sands, sandy loams, loams, clays) are the only solid natural environment, one of four (except liquid, gaseous and electromagnetic) natural environments at Earth and, obviously, at other planets. From an engineering point of view, soil base is bearing element of all structures, as a rule, in natural, and sometimes in an artificially improved state. If soil base consists of non-rocky soils, then it turns out to be the most deformable load-bearing element of the structure. But often due to technical, economic, hydrological reasons, it is impractical (for example, when dense soils or rocks are very deeply buried) to pass through non-rock soil by drilling or driven piles. In the case of using pile foundations on friction piles (not resting on rocky base), as well as non-piled foundations, it is necessary to predict (calculate) their settlements and other deformations of foundations (for example, their incline and difference in settlement with neighboring foundations, which is especially sensitive when providing reliability and working capacity of bridge spans. This situation imposes increased requirements on resolving capabilities of soil base design model, but in fact, on design soil model, in other words,

mechanical and mathematical deformation model of non-rock soil (for rock soil at main positions it is identical to deformation model of concrete, metal and rubber, which means to Hooke-Young's linear deformation theory) must corresponds as much as possible to real mechanics of non-rocky soil, determined in turn by its physical nature, which is so complex and multifactorial that even generally accepted in mechanics principle of replacement in deformation theory of real soil structure by an ideal continuous medium, which means introduction instead of real forces between particles of stresses, as forces acting on an infinitely small (idealization) area and relative deformations in an infinitely small (idealization) volume, creates an error in predicting soil deformations by about 25...30% (for metals, for example, as much more homogeneous formations, the error of such replacement is up to 5%).

More serious errors arise in prediction of internal forces (stresses) and, because of this, in external force interactions of neighboring objects (for example, in pressure of foundation on its soil base). But when designing, not only quantitative values are important, but even more important is character of their distribution, for example, unevenness, and this is determined the more reliably, the more adequate is

mechanical and mathematical model to mechanics of soil, by the way, in this case, quantitative values are also more reliably determined due to decrease in absolute terms of error from replacing real environment in design with continuous one. Approximately same assessment of the complexity of geotechnical design belongs to famous Anglo-Polish engineer and geotechnical scientist Zenkevich, a specialist in computer-aided design of soil bases of offshore oil platforms, who actually excluded an accurate prediction of soil deformation during loading even in the case of using the most advanced soil model, but at the same time who pointed out the need for ability of design specialists to model well the mechanical properties of soil which are necessary to solve the problem (Zienciewich, 1978). Unfortunately, in the 1920s, when important problem of reliable design of soil bases for large and complex industrial and civil objects arose, no other deformation theories except Hooke's theory (the theory of linear, more precisely, linear elastic deformation), especially for soil, did not exist (Terzaghi, 1961). However, back in 1798, the Swiss-Russian mathematician Fuss proposed an engineering method for predicting depth of wheel rut of carriages, carts and cannon carriages, which was important for Petersburg soils (Fuss, 1798), implemented later, in 1872 (Fuss, 1798) by Saxon scientist Winkler in a linear formula for predicting settlements of railway sleepers and, accordingly, deflections of railway tracks (Winkler, 1872) at the place of pressure application  $P$ :  $S = P / C_z$  (here  $C_z = \text{const}$  is a coefficient of proportionality, which Winkler called "bed coefficient" on not quite correct analogy with stiffness of sofa springs). But in the 1920s, after many checks (Kurdyumov, 1894; Minyaev, 1916; Germer's experiments with a round pressure area, 1932; Bernatsky, 1935), this formula was rejected as a possible mechanical and mathematical deformation soil model for design of structures (Terzaghi, 1961): firstly, due to the absence of a relationship between relative deformations  $\epsilon_{ij}$  and stresses  $\sigma_{ij}$  for an elementary but representative volume of soil medium (sample), which reduces Fuss-Winkler formula to some isolated boundary condition which is not in agreement with mechanism of internal deformation of soil massif, that, in turn, does not allow to carry out a full-fledged analysis of its deformation, for example, analysis of the effect of load influence on adjacent sections of soil massif and on neighboring structures; secondly, the hypothesis of constancy of the stiffness coefficient  $C_z$  was not confirmed, which is a consequence of previous defect of Fuss-Winkler's formula. In this regard, the use of Fuss-Winkler's

formula for calculating deformations of soil base in some widely advertised and currently used programs (for example, in programs "LIRA", "SCAD" and others) for any method of determining the value of stiffness coefficient  $C_z$  contradicts to basic principles of mechanics and is explained by failure of developers of these programs in their attempts to apply correct soil base model. The presence of this formula in SP 22.13330.2016 ("Soil bases of buildings and structures") (Gostroyizdat of Russia, 2017) is some kind of temporary compromise, which, however, for example, in Yekaterinburg has already led to emergencies associated with incorrect calculation of soil bases at several objects. In fact, Fuss-Winkler formula, due to its purpose and method of derivation, can be correctly used only for an approximate, and therefore actually estimated forecast of lateral displacement of a driven pile (actually more is not required) and for approximately same type of analysis of elastic work of soil under action of a not very intensive dynamic load from industrial equipment (but not from a much more intensive train load). In general, and it should be well known to engineers, that veracity and reliability of geotechnical calculations as for any other mechanical calculations, are ensured by using four groups of resolving relations: 1) equilibrium-motion equations, Newton, 1650; 2) geometric relations of compatibility of deformations and displacements in the framework of the theory of continuous medium, Cauchy, 1820s; 3) generalized physical relationships between relative deformations and stresses, Genki, 1920s; and, most importantly (Bell, 1984), 4) obtained from results of special experiments (tests), relations between relative deformations and stresses in a conventionally elementary (small), but representative volume (sample) of a solid formation, including for soil medium. These relations in turn determine type and value of rigidity of this solid formation, in this case of soil. But Fuss-Winkler's formula  $S = P / C_z$  does not belong to any of these four groups of resolving relations, even as defining stiffness relation, despite its outward likeness to Hooke-Young stiffness relation  $\epsilon = \sigma / E$ , including the constancy in both formulas of coefficients of proportionality  $C_z$  and  $E$ . As indicated above, there is no in Fuss-Winkler formula to contrast to Hooke-Young formula direct connection between values included in Fuss-Winkler formula and values included in other resolving relations. It is for this reason that Austrian-American geotechnician Terzaghi, founder of International Geotechnical Society, in absence of other deformation theories, as well as in full absence at that time devices for obtaining defining relationships

between relative deformations and stresses for soils and effective computing means, has proposed in 1920s (Terzaghi, 1961) to apply for predicting deformations of soils had being applied at that time for all materials, Hooke's theory (now known as theory of linear deformation), which mathematically closed above mentioned system of resolving relations. During that period of large-scale development of industrialization and housing construction all over the world, adoption of mathematically correct design model for soil and soil base has much intensified building design in all directions. But contradiction that arose due to insurmountable circumstances at the beginning of XX-th century (primarily due to the lack of a mechanical and mathematical design model adequate to mechanical properties of soils) and turned into a serious problem at the beginning of XXI-th century in condition of construction of high-rise and other uniquely complex objects, and especially bridges, is that for calculating deformations of soils was adopted mechanical and mathematical deformation model of incompatible with them on physics and mechanics metals, moreover, in many aspects this unreliable and dangerous approach is still preserved, including, and that is especially seriously threatens to technical safety, in regulatory documents, including SP 35.13330.2011 "Bridges and Pipes", although since 2009 this unacceptable situation was actually blocked by article 16 of Federal Law No. 384-FZ (Technical Regulations on Safety of Buildings and Structures), which requires the use in geotechnical design of adequate for non-rocky soils physically nonlinear deformation model (Federal Law № 384-FZ, 2010).

## 2 MAIN FEATURES OF PHYSICALLY NONLINEAR SOIL DEFORMATION

Obviously contradictory proposal of Terzaghi about adoption for calculation of deformations of soils with loose internal connections deformation theory of, absolutely opposite to them in physical and mechanical properties, dense materials (metals and even rubber) could not but raise numerous questions of geotechnical specialists, that even demanded later, in 1948 year, when Terzaghi's proposal, again due to insurmountable circumstances (absence, on one hand, a proven, soil deformation model and accessible, with the necessary power of computing means, and on other hand, urgent need on prompt restoration of destroyed by war and construction of new objects),

nevertheless, was adopted in USSR (Gersevanov, 1948), to apply this illogism, namely, having identifying value of Young's modulus  $E$ , which is consistent for materials, with inconsistent, as it was later found out for soils (Lushnikov, 1969; Ruppeneit, 1973), value of modulus of deformation with same designation  $E$ . although the physico-mechanical essence of these two values is absolutely different what is more, they are not even close analogs, as it was actually assumed in 1948 (Gersevanov, 1948).

Young's modulus  $E$  by definition (according to the technology of converting results of experiments of Hooke and his followers with metals and other similar materials into mechanical and mathematical formula of Young) reflects exclusively only direct proportionality between applied unidirectional force and resulting unidirectional deformation (and even adopted later, more general in relation to Young's formula, Genki's ratios with coefficient Poisson  $\nu$  did not change physical and mechanical essence of Young's modulus  $E$ ); modulus of soil deformation  $E$  as a constant for any method of its determination, due to the nonlinear features of deformation of soils, inevitably includes an element of disproportionality between applied forces and resulting deformation, moreover, multidirectional, which just creates effects of physical and geometric nonlinearity, and also effects of contraction and dilatancy, one of a manifestations of which is, for example, a multiple difference in values of modulus of deformation obtained from different methods of soil compression testing (settlement plate, pressuremeter, odometer) (Gersevanov, 1948; Lushnikov, 1969; Ruppeneit, 1973), which, in principle, cannot be when determining Young's modulus, for example, for metal. True, unreliable are formulas themselves (Lame and Schleicher) for calculating the modulus of soil deformation according to GOST 20276-2012 ("Soils. Methods for in-situ determination of strength and deformability characteristics") (GOST 20276–2012, 2013), since they are derived from the deformation theory not of soil, but of metal, that is, from Hooke's theory of linear deformation, There are also many other serious inconsistencies from application of physically linear theory of deformation to physically nonlinear soils, including, recently noted even in textbooks, significant differences for all types of buildings and for all types of soils between calculated and actual values of deformations of soil bases (Ukhov, 2002). At the same time, back in 1939 ... 1940 Leningrad scientist Botkin, obviously disagreeing with Terzaghi's proposal to accept the theory of deformation of metals and rubber for calculating deformation of soils, performed tests on

samples of sand and clay soils in a special triaxial compression device recently created in Germany - stabilometer, received deformation graphs of sandy and clay soils (Botkin, 1939; Botkin, 1940), depicted in the form of diagrams in Figure 1b (Botkin, 1939; Botkin, 1940). Thus, firstly physically nonlinear type of deformation of soils was investigated, which was different from the physically linear type of deformation proposed by Terzaghi, also shown diagrammatically in Figure 1a.

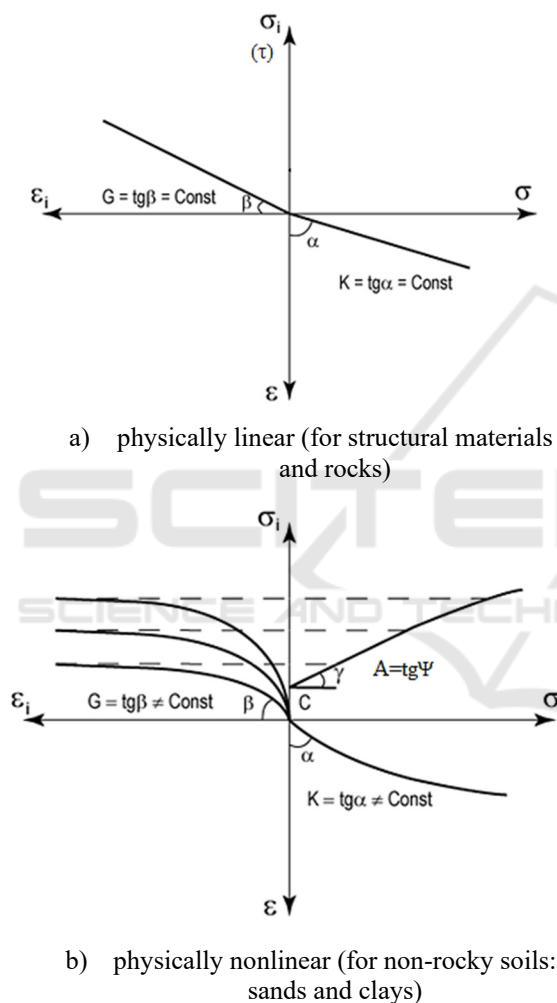


Figure 1: Types of deformation.

Names of types of deformation (that time physically nonlinear type of deformation was known and was studied only for soils; for rocks, by analogy with already studied concrete, metals and rubber, the physically linear type of deformation was adopted) were given in 1950s according to the form of given diagrams, but physically, for example, physically nonlinear deformation, consists in dependence of

rigidity (resistance to deformation) of a solid medium or solid material on their internal stress-strain state (herein after SSS) and decrease of this rigidity as stress-strain state approaches to the limit state for a given solid formation, namely to its strength (resistance to destruction). Physical essence of physically linear (Hooke's) deformation, on the contrary to nonlinear deformation, consists in independence from internal stress-strain state, i.e., in the constancy of the rigidity of a solid medium or solid materials throughout the entire deformation process due to density and strength of their internal bonds. In fact, in deformations of all solid formations (natural and artificially improved soils, as well as solid materials, there is a factor of physical nonlinearity to one degree or another, sometimes in certain areas of deformation (for example, area of yielding for steel). But only in soils, both in natural and in artificially transformed states, due to the fragility and looseness of internal bonds physically nonlinear type of deformation, in its essential understanding, is present at all stages of deformation without exception. Thus, it is obviously that design model for a soil should be at a minimum reflect physical and geometric nonlinearities, which is quite definitely declared by paragraphs 5.1.11, 5.1.12, 5.3.3 and paragraphs of Appendix C of SP 22.13330.2016 "Soil bases of buildings and structures" (Gosstroyizdat of Russia, 2017), and especially by Article 16 of Technical Regulations on Safety of Buildings and Structures (Federal Law № 384-FZ, 2010), which excludes the use of formulas of the theory of a linearly deformable soil with Young's modulus (modulus of deformation) E for designing soil bases, which from the standpoint of real soil deformation as physically nonlinear medium and hence requirements of above paragraphs of SP 22.13330.2016 and Federal Law № 384-FZ, cannot be used at designing of soil bases, especially since this value at different points of soil base, as in plan and in depth due to the factor of physical nonlinearity (dependence of stiffness on stress-strain state) is significantly different: naturally, the question arises about the localization of soil base section, for which value of modulus of deformation E is given in reports of geological engineers. There is no answer within the framework of a nominal, and therefore unreliable for soil, physically linear model (the theory of a linearly deformable medium). But this answer can be easily found within the framework of a physically nonlinear model adequate to mechanical properties of soil. Phenomenological formulas for stiffness characteristics of this theory were first derived by Botkin: for modulus of volume change



(bulk modulus)  $K = \sigma / \varepsilon = \sigma^{1-\alpha} / A_0$  and for modulus of change of shape (shear modulus)  $G = \sigma_i / \varepsilon_i = (\sigma_u - \sigma_i) / B = (A\sigma + C) / (B + \sigma_u)$  (Here  $\sigma_u = A\sigma + C$  – strength condition for soil according to Mises-Botkin version). Thus, for soils, it is incorrect to determine values of the parameters of the physically linear model  $E$  (deformation modulus), which are different at different points of the soil base and  $\nu$  (Poisson's ratio, which, by the way, for soils is almost impossible to determine by direct measurements, but it is necessary to determine values of parameters-constants of a physically nonlinear model (for example, Botkin's model:  $A_0, \alpha, A, B, C$ . Since values of these parameters, as well as the parameters of any soil model (and now there are quite a lot of them, depending on the problems to be solved (Fedorovsky, 1985)), depend on the natural state of soil, including its natural stress state, then their true significations must be determined from the results of in-situ static tests with the simplest scheme and the least disturbing the natural state of soil. Brief mathematical description of the algorithm of the method (Fedorovsky, 1985) for Botkin soil model, as the most verified in accordance with paragraph 5.1.12 of SP 22.13330.2016 ("Soil bases of buildings and structures") (Gostroyizdat of Russia, 2017) Using true values of parameter stresses and strains of soil masses and corresponding displacements of foundations are determined. For rigid foundations of columns and bridges supports, as well as for rigid slab foundations, this problem has been completely solved; it was also solved to determine the deformations and stresses under the embankment,

which is important for analysis of karst problem. At present, post-graduate students of Bridges and Transport Tunnels Department are solving a similar, but at the same time, due to computational features, most difficult problem for slab foundations of any rigidity. Comparison of results of geotechnical calculations for bridge foundation using physically nonlinear and physically linear (Hooke's) deformation theories showed that at a pressure on soil base of 400 kPa (40 t/m<sup>2</sup>), for first case average settlement of foundation is 6.3 cm, and for second case – 7.7 cm. Thus, settlement according to physically linear theory exceeded settlement according to physically nonlinear theory by 20%, which is explained by fact that at physically linear case increase of soil stiffness with increasing depth is not taken into account. Taking into account the still unexplored uncertainty for soil on replacement of granular soil medium to ideally continuous (according to some data, it can be up to + 25%, and in the case of using a linear model that is inadequate to the soil, naturally more), the total difference in the calculated settlements can reach 50%. The nature of the distribution of contact pressures under foundation, as well as in the soil mass differ significantly in physically nonlinear and physically linear calculations, and in both cases, they are uneven and not equal to average pressure under bridge support (see Fig. 2). unequal to average pressure from the bridge (see Fig.2). The unevenness of diagram at nonlinear case is explained by the dependence of soil stiffness on stress state, which is adequately reflected by physically nonlinear model. Nonlinearity of the

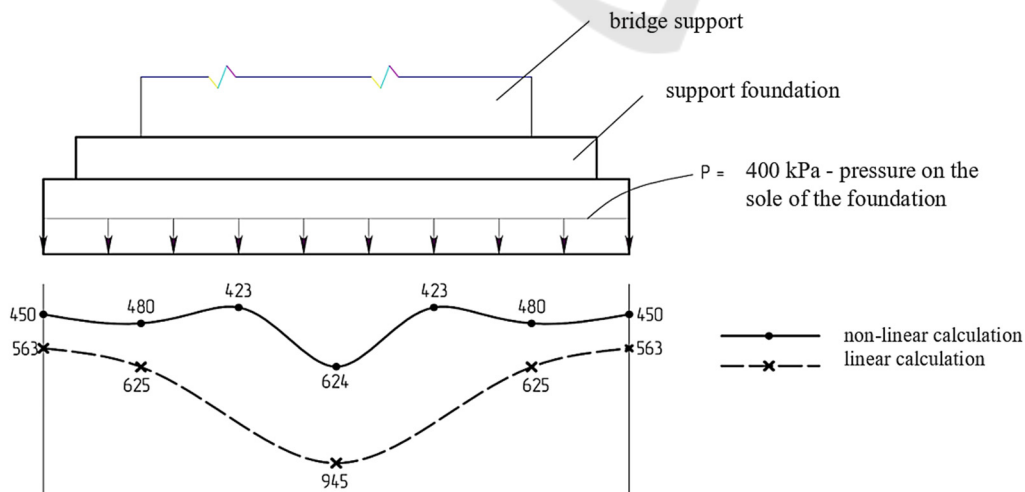


Figure 2: Diagrams of contact pressures  $P_k$  under the base of the bridge foundation according to nonlinear (solid line) and linear (dashed line) calculations at ground pressure  $P = 400$  kPa.

diagram at linear case is at more significant values of pressure along the edges than in the nonlinear case is explained by increased distribution ability of linear model, inadequate to soil, in entire space of acting forces compared to nonlinear one (for analogy, one can compare distribution ability of rubber and plasticine). The unevenness of contact pressures seriously affects at the roll of bridge supports, which is sensitive for bridge spans, therefore, the most accurate determination of contact pressures is extremely important for design of bridges, especially railway ones. At the same time, it must be mind that it is the physically nonlinear deformation that is main mechanical feature of non-rocky soils.

### 3 CONCLUSIONS

To improve traffic safety deformations of the soil base of bridge supports must be calculated using physically and geometrically nonlinear soil model, as required by Federal Law № 384-FZ (Technical Regulations on the Safety of Buildings and Structures).

The use of a physically and geometrically nonlinear soil model makes it possible to obtain the most reliable prediction of deformation of ground embankments and its soil bases in occurrence of karst cavities and other defects in them.

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