Enhancing the Geometry of Curvilinear Sections of the Track Plan for High-speed Traffic

Sergey Shkurnikov[1][a], Sergey Kosenko[2][b] and Olga Bulakaeva[1][c]
[1]*Emperor Alexander I St. Petersburg State Transport University, St. Petersburg, Russian Federation*
[2]*Siberian Transport University, Novosibirsk, Russian Federation*

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Abstract: The paper outlines the key trends in the current science of high-speed railways layout and profile design. The employment of sophisticated laser scanning systems and satellite navigation systems simplifies the task of constructing and maintaining complex spatial curves - areas of overlapping vertical curves in the profile and transitional curves in the plan, sinusoidal or polynomial curves. As part of this research, a mathematical model has been developed to describe different transition curves with non-linear curvature and elevation characteristics of the outer rail depending on the number of zero derivatives required at the ends of the transition curves. The shape influence (character of curvature function change within the transition curve) of curvilinear sections of high-speed railway track plan on traffic dynamics is investigated. It has been found that the best dynamic performance is in the curved section of the alignment equipped with transition curves whose curvature function is represented by a 5th degree polynomial. There may also be no circular curve within the plot in question. The paper examines the construction and mutual arrangement of differently shaped curved sections and identifies their advantages and disadvantages.

1 INTRODUCTION

The requirements of the present development and implementation phase of high-speed rail traffic in the Russian Federation are shaping new approaches to the design of the track plan and profile. As a result of developments in the railway industry some provisions previously considered fundamental and indisputable are subject to critical rethinking.

The permissibility of overlapping transition and vertical curves in the same alignment is limited by the requirements of the existing regulatory framework which has not lost its relevance for decades. When it was formed in the 1960s this prohibition was justified by the difficulty of maintaining a complex spatial curve. However, the application of modern laser scanning systems, GPS and GLONASS makes this task easier. Compliance with the requirement for non-conformity of vertical curves in the plan and transition curves in the profile leads to an increase (up to 33.8 % (Akkerman, 2017) in the construction cost of high-speed railways (HSR). A maximum speed of 400 km/h can be realised by using large radius vertical curves (30-40 km). The vertical curve length is proportional to the radius, therefore longer longitudinal profile elements are required to accommodate vertical curves outside the transition line which causes difficulties in routing and increases the cost of highway construction (EN 13803-1:2010).

In order to improve the smooth running of trains studies are being carried out on the possibility of constructing a curve by interfacing two clothoids both in plan and in track profile. The need for a straight insertion between two adjacent curves in the plan, assuming a "clothoidal" coupling, is also not obvious, according to (Akkerman, 2017).

The regulatory value of unaccelerated speed ensures passenger comfort when running a high-speed train and indirectly determines the level of permissible lateral impact of the rolling stock on the track. In Russia the unaccelerated speed of 400 km/h is 0,4 m/s^2. This value is generally commensurate

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[a] https://orcid.org/0000-0002-4273-389X
[b] https://orcid.org/0000-0003-2987-1435
[c] https://orcid.org/0000-0003-0982-5183

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with the requirements of European standards (Lindahl, 2001; Sirong, 2018). Nevertheless, Russian research on the basis of train tests has established an unaccelerated speed limit of 1,0 m/s^2 for high-speed and high-speed traffic due to the improved dynamic qualities of high-speed rolling stock. An increase in the regulatory value of unaccelerated acceleration up to 0.75 m/s^2 has also been noted on PRC high-speed roads (Sirong, 2018). Increasing the allowable rates of unaccelerated speed leads to a reduction in the required curve radius in the plan. On the one hand, this ensures a better alignment with the terrain, reduces the construction and maintenance costs of very gentle curves and on the other hand it affects the comfort of the journey and the quality of the service provided.

A separate area of improvement in the geometry of the HSR route is the definition of a rational shape of the transition curves due to the nature of the spatial variation in curvature within its length. In addition to the transition curves with linearly varying curvature (clothoids) traditional for the Russian Federation international experience distinguishes sinusoidal (Sine (EN 13803-1:2010), cosine (Cosine (EN 13803-1:2010) and polynomial (Wiener Bogen, Bloss) transition curves (Hasslinger, 2005; Wojtczak, 2018; Velichko, 2020). The main disadvantage of clothoidal transition curves is that there is significant rolling stock oscillation when traversing curved sections of the alignment at the junctions of straight lines with transition curves and transition curves with circular curves (Xiaoyan, 2017; Morozova, 2020). The cause of oscillation, in terms of the laws of mechanics, is the piecewise linear description of the curvature function within the length of the section in question. Negative dynamic impacts are considered to be compensated for by rational assignment of transition and circular curve lengths. However, the increased demands on traffic dynamics on the HSR lead to the need to select a rational transition curve shape and to determine the conditions for their applicability.

2 MAIN TEXT

2.1 Materials and Methods

The design of transition curves on railways has been a major focus of domestic and foreign specialists since the beginning of the twentieth century. Proposals G. Schramm (1931) consisted in using some composite curves as transition curves one of which includes two centrally symmetric segments of a 3rd-degree parabola (Helmert curve (EN 13803-1:2010). In the case of curves of small radii, B.N. Vedenisov suggested using only transition curves in the form of transformed clothoids in the absence of circular curves. Professor G.M. Shahunyants based on the provisions of classical mechanics analysed the change in progressive and rotational accelerations arising from the movement of rolling stock in a curve. Based on this analysis, G. M. Shahunyants developed a transition curve whose 2nd derivative curvature varies according to a sinusoidal law. A higher requirement for smoothness of the curvature function is suggested by V.P. Minorski. The curvature function of the transition curve must comply with four boundary conditions ensuring that the 1st, 2nd, 3rd and 4th derivatives of the curvature at the start and end of the transition curve are zero.

As mathematical basis for the transition curves considered in the present study we propose a polynomial B_0, defined on the interval [0; L_{TC}], having at least one zero derivative on the ends of the interval. It is assumed to grow monotonically and to take $B_{\min}=0$ и $B_{\max}=1$ at its beginning and end respectively calculated by the formula:

$$B_0=\left(\frac{l}{L_{TC}}\right)^a\cdot B, \tag{1}$$

where B is a degree polynomial $a-1$ determined from the condition:

$$B\left(\frac{l}{L_{TC}}\right)=1+\sum_{n=1}^{k}\left((-1)^n\frac{\left(\frac{l}{L_{TC}}-1\right)^n\prod\limits_{b=0}^{n-1}(a+b)}{n!}\right). \tag{2}$$

In equations (1) and (2) a is the multiplicity of the node points which determines the degree of the polynomial sought and is numerically equal to the value k+1, k is the number of derivatives at the ends of the segment set as zero; l is the current value of the length of the transition curve and L_{TC} is the total length of the transition curve (if k=0, then the transition curve is represented as a clothoid).

The mathematical description of the function B_0 is based on the theory of boundary conditions by V.P. Minorsky assuming that the zero derivatives at the ends of the segment can be more than 4. The formation of the polynomial B_0 is realised by means of an hermitian interpolation method.

The correlation between the function B_0 and the curvature function k is provided by the condition:

$$k = \frac{B_0}{R} \tag{3}$$

The curvature and elevation of the outer rail must coincide (proportionality):

$$h = \frac{B_0}{H}\,. \tag{4}$$

In formulae (3) and (4) R is the radius of the circular curve, H is the elevation of the outer rail in the circular curve, k is the current curvature within the transition curve, h is the current elevation of the outer rail within the transition curve.

Transition curve lengths whose mathematical description is based on the application of the B_0 function must comply with the conditions that the rate of wheel lift [f] and the rate of increase of unaccelerated acceleration [Ψ] do not exceed their maximum values laid down in the regulations. It should be noted that as the polynomial degree B_0 increases a lengthening of the transition curve is required due to the magnitude of the curvature growth rate (1st derivative of curvature) in the middle of the transition curve. The dependence of the required transition curve length L_{TC}on the polynomial degree B_0 is shown in Figure 1.

In terms of the motion kinematics of a material point on a curvilinear element the condition that the first derivative of the curvature is zero (the third derivative of the coordinate) at the start and end of the transition curve is sufficient. This condition ensures the continuity of the 'jerk' (the value ofψ on railways) responsible for the reaction of passengers to changes in acceleration and the safety of fragile goods. Large-order derivatives are used quite rarely and do not even have an approved name.

However, the movement of a railway carriage within a curved section takes place simultaneously in two planes: the horizontal transverse. The translational motion and the vertical transverse - the inclination of the carriage resulting from the ascent to and descent from the elevation. The coordinates of the vertical plane are proportional to the curvature. This requirement allows the physical meaning to be interpreted up to the third curvature derivative (the fifth coordinate derivative):

- the first derivative of the curvature along the length taking a zero value at the start and end of the transition curve ensures continuity of the additional force transient factors i, f и ψ, as well as the angular velocity of the crew.
- the second deviation of the transition curve turning to zero at the conjunctions with the straight track and the circular curve ensuring continuity of the angular acceleration.
- the third derivative of the curvature continuity at the start and end of the transition curve ensures that the angular acceleration rate function or "jerk" in the vertical plane is smooth.

2.2 Results and Discussion

2.2.1 Impact Studies of the Curved Track Plan Shape on the Traffic Dynamics of Highspeed Rolling Stock

The rolling stock dynamics during movement in transition curves was investigated using a simulation model of a high-speed vehicle (Wang, 2014) adapted for the tasks of selecting design plan and route profile parameters of high-speed railways using the "Universal Mechanism" program.

A rolling stock simulation at 400 km/h was carried out within a curved track element consisting of a transition curve and a circular curve (R = 9400 m). The degrees of the polynomial B_0, used to describe the transition curves were varied from 1 to 5 within the framework of the numerical experiment conducted. The experimental findings are shown in Figure 2 in the form of oscillograms of unrecovered transverse accelerations determined at the level of the wheelset axle.

An examination of the results shows that the maximum value of the unsuppressed transverse acceleration (Zolotas, 2007) occurs at the interface between the circular curve and the transition curve

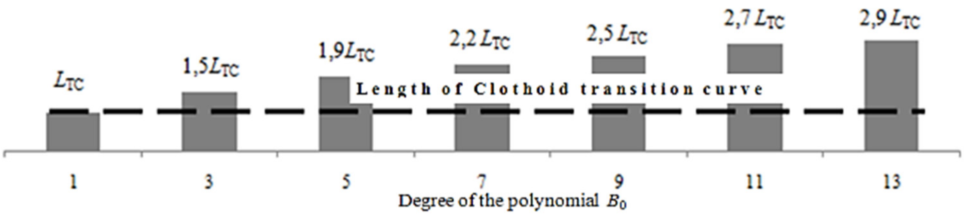
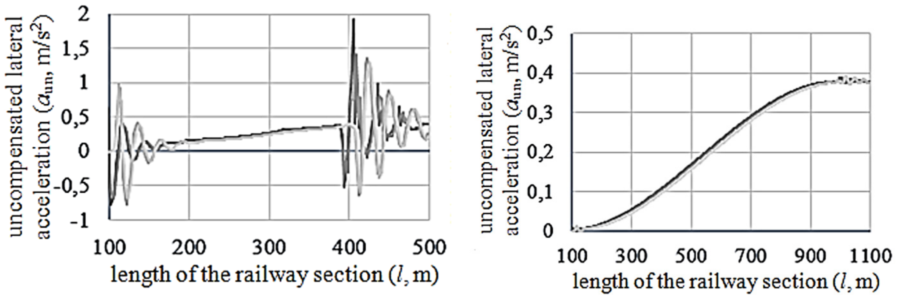


Figure 1: Dependence of the required transition curve length on the degree of polynomial function B_0.

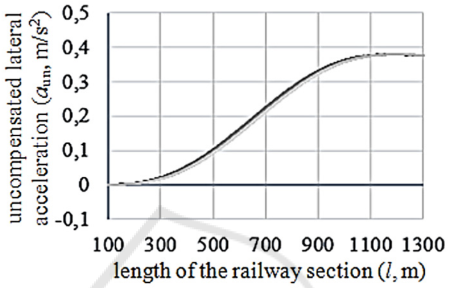


the 1-st degree of the polynomial B_0

a)

the 3-st degree of the polynomial B_0

b)



the 5-st degree of the polynomial B_0

c)

Figure 2: Oscillograms of lateral acceleration during high-speed rolling stock movement within differently shaped transition curves.

whose curvature function is represented by a polynomial of degree 1. The maximum actual value of the unrelieved lateral acceleration $a_{\text{unv.max}}^{\text{act.}}$ is 2.0 m/s^2 and is several times greater than the calculated value $a_{\text{unv.max}}^{\text{calc.}}$ (0.4 m/s^2 at a circular curve radius of 9400 m). Transition curves whose curvature function is described by 3rd- and 5th-degree polynomials are overcome without additional rolling stock oscillations ($a_{\text{unv.max}}^{\text{act.}} \approx a_{\text{unv.max}}^{\text{calc.}}$).

The curved alignment shape is determined not only by the degree of the transition curve polynomial used to describe the curvature function but also by the presence or absence of a circular curve. Figure 3 shows the simulation results for movement within differently shaped curved sections.

According to the results obtained (Fig. 3 and Fig. 4) when organising high-speed rail traffic at 400 km/h the best dynamic performance is in the curvilinear section of the route plan equipped with transition curves whose curvature function is represented by a polynomial of degree 5. There may also be no circular curve within the plot in question.

2.2.2 Features of the Arrangement and Relative Positioning of Differently Shaped Curvilinear Sections

A pie chart is developed to compare the device requirements for non-linear curvature sections arranged in series and traditional curvature:

- the minimum rotation angle $\alpha_{\min}$ required to construct a curved section of radius$R$ (sequence of steps 1-2-3 in Figure 4);

$$\alpha_{\min} = \frac{\alpha_0 R_0}{R} \tag{5}$$

- Determination of the minimum radius $R_{\min}$ required to construct a curved section at a given α (sequence of steps 4-5-6 in Figure 4);

$$R_{\min} = \frac{\alpha_0 R_0}{\alpha} \tag{6}$$

On the presented diagram Table 1 is filled in to clearly evaluate the advantages and disadvantages of the different shapes of the curved sections of the alignment plan.

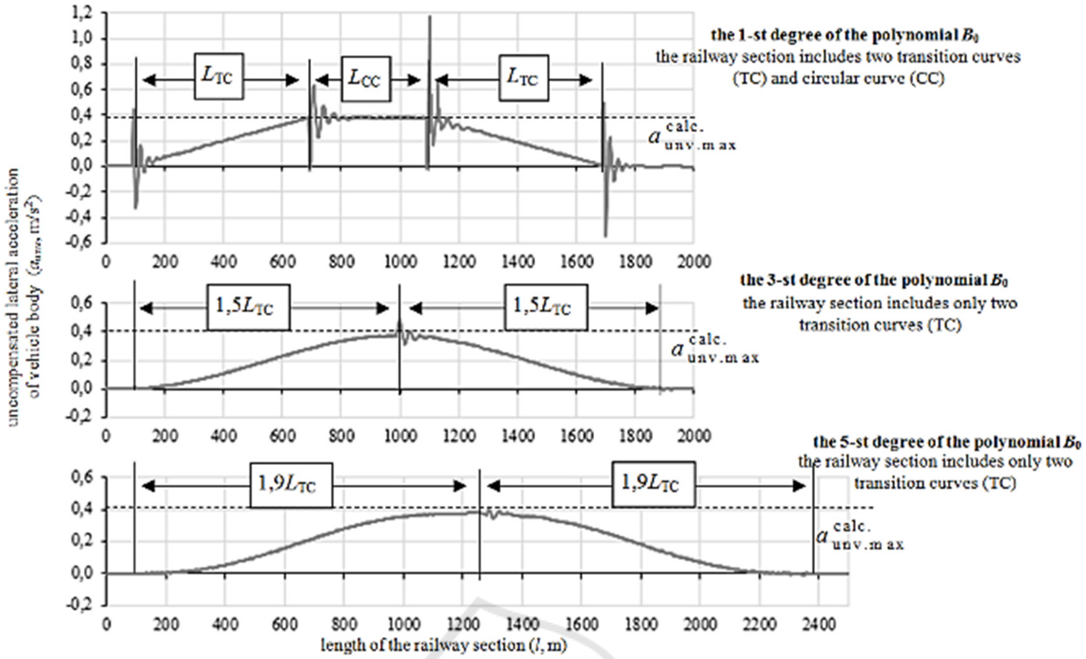


Figure 3: Magnitudes of unaccelerated acceleration in the crew bed occurring when driving within curved sections of different shapes.

Table 1: Curved sections of different shapes.

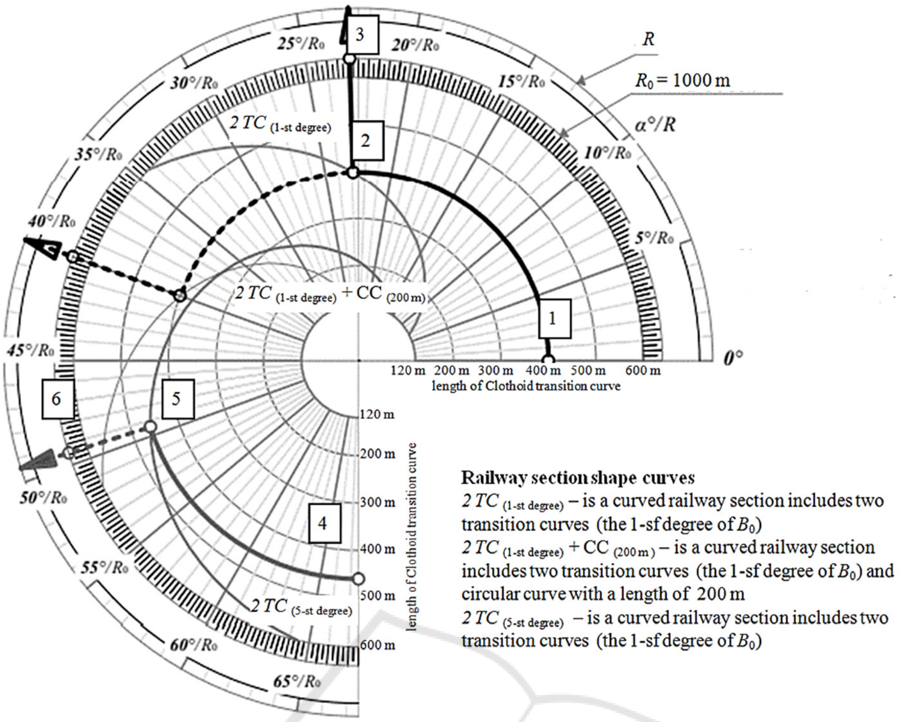
Comparative characteristics	Swivel angle 10°	
	B_0polynomial of degree 1	B_0polynomial of degree 5
Set / minimum radius of curved section, m	5000	5400
Length of transition curve, m	500	940
Total plot length, m	1373	1876
Length of constant curvature section, m	373	–
Bisector of the curved section, m	21	24

An obvious disadvantage of curvilinear sections that include transition curves with a curvature function described by a 5th degree polynomial is that their length increases (1.36 times in the case in question). However, for small alignment angles ($\alpha<\alpha_{min}$) the required radius of the curved section consisting of two transitional curves of non-linear curvature increases thereby ensuring a reduction in operating costs in the curve.

3 CONCLUSIONS

The result of this study allows us to change the usual perceptions of the curved section of the high-speed rail track plan. The best shape in terms of driving dynamics is the curvilinear section within which the additional oscillations of the crew tend to zero. Equipping a curved section with transition curves whose curvature and elevation divergence functions are represented by5th degree polynomials meets these requirements. The circular curve may not be present at all in the section or may be smaller than the standard value.

Despite better dynamic performance the length of these transition curves is almost twice as long. It must be taken into account that the permissible standard values [*f*] and [Ψ] have been determined for clothoidal transition curves. It should be taking into account the significant oscillations of the rolling stock when passing through curved sections of the alignment at the junctions of straight, transition and circular curves. The implementation of transient curves with non-linear curvature leads to the complete damping of this kind of oscillation which theoretically leads to an increase in the tolerance rates [*f*] и [Ψ]. Increasing these values contributes to reducing the required length of the transition curves.



Railway section shape curves

$2\,TC_{\text{(1-st degree)}}$ – is a curved railway section includes two transition curves (the 1-sf degree of B_0)

$2\,TC_{\text{(1-st degree)}} + CC_{\text{(200 m)}}$ – is a curved railway section includes two transition curves (the 1-sf degree of B_0) and circular curve with a length of 200 m

$2\,TC_{\text{(5-st degree)}}$ – is a curved railway section includes two transition curves (the 1-sf degree of B_0)

Diagramme guide

Step 1, step 4 – is the detetmination of the length of Clothoid transiton curve (L_{TC})

Step 2, step 5 – is the finding the intersection of the concentric circle corresponding to L_{TC} with one of the railway section shape curves

Step 3, step 6 – is the detetmination α_0 angle

Figure 4: Pie chart for determining curvilinear section design features.

At the present stage of the study in the curvature design of the high-speed railway alignment plan it is recommended to replace the traditional curvature sections with polynomial (B_0 polynomial of degree 5) transition curves, conjugate without circular curve in cases of small alignment angles ($\alpha<\alpha_{\min}$). For an unbiased assessment of other design cases ($\alpha_{\min}>\alpha$) the installation of an experimental curvilinear section and a train test to assess the possibility of increasing the tolerances [f] and [Ψ] is necessary.

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