

Construction of a Model of Interaction of a Fiber-reinforced Plate with an Elastic Base

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Keywords: fiber-reinforced concrete, fiber concrete, elastic base plate, rigid pavement, modeling of fiber concrete, approximation of experimental data.

Abstract: The article considers a special case of calculation of thin-walled structures, namely plates on an elastic base, as a structure with a fairly significant scope of application in transport construction. The issues related to the construction of a model of deformation of fiber-reinforced plates with an elastic base are considered. In the context of solving the above-mentioned issues, an analysis of the deformation diagrams of fiber-reinforced concrete (fiber concrete), as well as physically nonlinear relations describing the deformations of fiber-reinforced material that variously resists tensile and compression deformations under the plane stress state characteristic of plates, is given.

1 INTRODUCTION

After the beginning of the economic crisis in 2008, transport builders faced a very important problem of reducing the strength of building materials while maintaining their strength properties. Constant changes in prices for materials and labor costs forced Gosstroy to abandon the basic index pricing method (the main method in force since 1991) in 2016 and switch to the resource method. This method has fully justified itself during the crisis caused by sanctions and the coronavirus pandemic (COVID-19). Designers began to regularly, according to Customer requirements, in order to reduce the cost, recalculate the ratio of concrete-reinforcement in building structures (Kokodeev, 2020).

One of the important scientific directions was the development of new materials with predetermined properties, as well as the study of the stress-strain state (SSS) of structures made of them (Varakin, 2020). In the manufacture of thin-walled structures of transport structures, the use of fiber-reinforced concrete (fiber concrete), which has significantly greater resistance to the appearance and growth of cracks compared to conventional concrete, proved to be very effective. This article considers a special case of thin-walled structures, namely plates on an

elastic base, as a structure with a fairly significant scope of application in transport construction. This is a scientific problem, the solution of which has great practical potential.

Within the framework of solving this problem, the article presents an analysis of the state of the issue, the construction of a model of deformation of fiber-reinforced plates on an elastic base, and also provides a method for calculating such structures.

The solution of the problem is reduced to the consideration of such tasks:

1. Analysis of the mechanical properties of fiber-reinforced concrete under various stress conditions.
2. Construction of a model of deformation of fiber-reinforced concrete as a non-linearly deformable material sensitive to the type of stress-strain state.
3. Development of a method for calculating fiber-reinforced plates on an elastic base, which allows analyzing their stress-strain states under different boundary conditions and loading programs.

2 MATERIALS AND METHODS

A large number of researchers have been engaged in increasing the rigidity and crack resistance of structures. One of the promising directions is the use

of composites (materials that are heterogeneous in their composition). Usually, researchers consider polystructural composites, i.e. systems composed of many structures (Selyaev, 1993; 2. Selyaev, 1986). The mechanical characteristics of fiber reinforcement are usually much higher than the mechanical characteristics (Skudra, 1975).

When creating thin-walled structures of transport structures, composite material from a concrete matrix and reinforcing elements should be used, as which either fibers in the form of separate rods, or fine-mesh grids, or sections of steel fibers are used (Kurbatov, 1980; Kurbatov, 1980). These randomly arranged fibers lead to a significant increase in crack resistance, and also improve the resistance of the composite to the action of tensile stresses. Quite a lot of works and publications have been devoted to the study of various issues of the behavior of fiber-reinforced concrete, and, as follows from (Nekrasov, 1925), these issues were investigated by Professor V.P. Nekrasov at the beginning of the XX century.

Over the past three decades, extensive studies have been conducted on the mechanical characteristics of concrete, dispersed reinforced with both steel and synthetic fiber. Professor Stepanova D.S. (Stepanova, 1975), studying fiber concrete, determines the dispersion of the fiber filler (the degree of crushing and dispersal of reinforcement in the structure) through the ratio between the total surface of the fiber reinforcement and the volume of reinforced concrete. And in the work of Tsiskreli G.D. (Tsiskreli, 1954), dispersion refers to the ratio between the percentage of fiber reinforcement and the diameter of fiber reinforcement. Moreover, despite the differences in the terminology used, it was concluded that with an increase in the dispersion index, the work of fiber-reinforced concrete under tension improves, which delays the appearance of cracks.

It is possible to note different directions of research on the work of fiber-reinforced structures:

- study of the adhesion of the filler to concrete depending on its saturation with fibers, (works by Kravinskis V.K., Vylegzhanin V.P. (Vylegzhanin, 1982; Kravinskis, 1979));
- study of structural characteristics of steel fiber concrete, (works of Kopatsky A.V., Lobanov I.A. (Lobanov, 1976; Kopatsky, 1979));
- study of the dependence of the parameters of fiber reinforcement and the properties of the concrete matrix, as well as the selection of the optimal composition of the steel-fiber concrete mixture according to certain parameters (works by Polyakova L.G., Ovchinnikov I.G.,

Rabinovich F.N. (Ovchinnikov, 1990; Rabinovich, 1985));

- study of the location of reinforcing fibers on the properties of fiber-reinforced concrete) works by Browns Ya.A., Nagevich Yu.M., Lagutina G.E., Lavrinev P.G. (Browns, 1986; Lavrinev, 1983; Rodov, 1980)).

Obviously, this is not a complete list of modern research directions for such materials and structures made of them. At the moment, it has been revealed that the presence of fiber slows down the crack opening process from 6 to 20 times, depending on the reinforcement parameters, the loading level compared to traditional reinforced concrete (with the same percentage of reinforcement) (Grigoriev, 1983; Kadysh, 1982; Kurbatov, 1982; Pavlov, 1976; Varakin, 2020).

This leads to the conclusion about the effectiveness of using a composite based on a concrete matrix and steel fiber in the manufacture of thin-walled structures of transport structures (it is characteristic that with thick-walled structures, the advantages of fiber reinforcement are leveled).

In transport construction, flexible pavement has been widely used for quite a long time, which, unfortunately, have a short service life in severe operating conditions. In Russia, many regions are characterized by a sharply continental climate. It is characterized by the freezing of soils to a considerable depth, the presence of permafrost zones. In this case, rigid structures of pavement provide long service life. Building structures in the form of thin-walled plates have become widespread in various industries. Walls of premises, road pavement, airfield coverings, regulatory structures made according to the scheme of rigid plates and slabs are widely used in modern construction.

In transport construction, plates are used as a coating on highways with high traffic intensity, with embankments of poor soils, on urban roads, in areas where heavy machinery is used, as a coating of runways at airfields, etc. Road pavement in the form of plates of fiber-reinforced concrete has a number of significant advantages compared to flexible pavement:

- changes in external temperature influences practically do not affect the stability of mechanical properties;
- the use of such coatings provides a longer service life before major repairs;
- with increasing age of fiber-reinforced concrete, its strength increases;

- the strength and rigidity of fiber-reinforced concrete is significantly greater than that of asphalt concrete;
- stability and its weak dependence on the humidity of the coefficient of adhesion of coatings made of fiber-reinforced concrete with a car wheel.

The above data suggest that the scheme of operation of rigid pavement can be attributed to the operation of plates on an elastic base.

In recent years, a composite based on steel fiber concrete has often been used for reinforced concrete coatings. Composite (composite material) is an artificial material obtained by a volumetric combination of different components performing various functions. One component (matrix) is responsible for plasticity and elasticity, the other (filler) for strength and rigidity. At the same time, the final system has a pronounced emergence (that is, the properties of the composite differ from the properties of the constituent components). In this case, the composite material is continuous and the forces transmitted to it are distributed continuously over its volume.

When constructing a model of deformation of fiber-reinforced plate on an elastic base, we will rely on standard hypotheses, according to which the layers of the plate do not press on each other and the hypothesis of direct normals is valid.

Cut out a rectangular element from the plate $abcd$ with infinitesimally small dimensions dx , dy (Fig. 1) on which a load of intensity q normal to the surface acts from above, and an elastic base reaction of intensity p acts from below. On the face cd act forces Q_x and S_y , tensile force N_x , bending and torques M_x and H . In turn, on the face ab , which is separated from cd by an infinitesimal distance dx , the above forces and moments differ by infinitesimal

quantities $\frac{\partial Q_x}{\partial x} dx$, $\frac{\partial S_y}{\partial x} dx$, $\frac{\partial N_x}{\partial x} dx$, $\frac{\partial M_x}{\partial x} dx$, $\frac{\partial H}{\partial x} dx$.

Similarly, it is possible to obtain both forces and moments on the faces ad and bc .

The equilibrium of this infinitesimal element will be ensured when the conditions of equilibrium of the projections of all forces on the coordinate axis and the equilibrium of bending and torques relative to the axes are met.

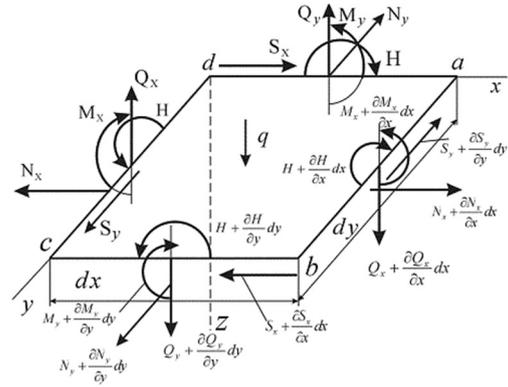


Figure 1: Forces acting on an infinitesimal element.

The projection of all efforts on the Z axis will be recorded (1):

$$\left(Q_x + \frac{\partial Q_x}{\partial x} dx\right) dy - Q_x dy + \left(Q_y + \frac{\partial Q_y}{\partial y} dy\right) dx - Q_y dx + p dx dy = 0 \quad (1)$$

Giving similar terms, we get (2):

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = -p \quad (2)$$

The condition of equilibrium of bending and torques relative to the X axis will be written (3):

$$\begin{aligned} & \left(M_y + \frac{\partial M_y}{\partial y} dy\right) dx - M_y dx + \\ & \left(H + \frac{\partial H}{\partial x} dx\right) dy - H dy + Q_x \frac{dy^2}{2} - \\ & - \left(Q_x + \frac{\partial Q_x}{\partial x} dx\right) \frac{dy^2}{2} - \\ & \left(Q_y + \frac{\partial Q_y}{\partial y} dy\right) dx dy - p dx \frac{dy^2}{2} = 0 \end{aligned} \quad (3)$$

Giving such terms and neglecting the small-order magnitude $\frac{\partial Q_y}{\partial y} \frac{dy}{2} \rightarrow 0$, we write (4):

$$\frac{\partial M_y}{\partial y} + \frac{\partial H}{\partial x} = Q_y \quad (4)$$

The condition of equilibrium of bending and torques relative to the Y axis will be written (5):

$$\frac{\partial M_x}{\partial x} + \frac{\partial H}{\partial y} = Q_x \quad (5)$$

As a result of the transformation of the above formulas, we obtain the following equilibrium equation (6):

$$\frac{\partial^2 M_x}{\partial x^2} + 2 \frac{\partial^2 H}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = -p \quad (6)$$

Analysis of experimental results obtained in the study of the mechanical properties of fiber-reinforced concrete with sufficient justification suggests that the behavior of fiber-reinforced concrete can be described using a model of orthotropic nonlinear multi-resistive material.

In the case of a flat stress state in which a plate of fiber-reinforced concrete is located, the physical relations for an orthotropic nonlinear resistive material take the form (7):

$$\begin{aligned} e_x &= \frac{\sigma_x}{\psi_{xi}} - \frac{\sigma_y}{\psi_{yj}} \nu_{yj} \\ e_y &= \frac{\sigma_y}{\psi_{yj}} - \frac{\sigma_x}{\psi_{xi}} \nu_{xi} \\ e_{xy} &= \frac{\tau_{xy}}{G_j} \end{aligned} \quad (7)$$

where σ ; τ ; e components of the stress tensor and strain tensor, ν - Poisson's ratio, x, y - coordinates, i and j - take into account the direction of deformation, and ($i(j)=1$ - stretching, $i(j)=2$ - compression), G_j - shear modulus, ψ - nonlinear functions that take into account the nonlinearity of the deformation diagram and the resistance of fiber-reinforced concrete (8):

$$\begin{aligned} \psi_{xi} &= \frac{\phi_{xi}(e_u)}{e_u}; \\ \psi_{yj} &= \frac{\phi_{yj}(e_u)}{e_u} \end{aligned} \quad (8)$$

here e_u is the intensity of deformations, ϕ_{xi} ; ϕ_{yj} which quite correctly describe the deformation diagrams of fiber-reinforced concrete, taking into account the direction and type of deformation (9):

$$\begin{aligned} \phi_{xi} &= A_{xi} e_u^{k_{xi}} - B_{xi} e_u^{m_{xi}}; \\ \phi_{yj} &= A_{yj} e_u^{k_{yj}} - B_{yj} e_u^{m_{yj}} \end{aligned} \quad (9)$$

coefficients A_{xi} , A_{yj} , k_{xi} , k_{yj} , m_{xi} , m_{yj} are located in such a way as to provide the best approximation of the deformation diagrams of fiber-reinforced concrete.

The shift modulus G_j is defined through these functions ψ_{xi} , ψ_{yj} and quantities ν_{xi} , ν_{yj} by the following expression (10):

$$\frac{1}{G_j} = 2 \left[\left(\frac{1 + \nu_{xi}}{\psi_{xi}} - \frac{1 + \nu_{yj}}{\psi_{yj}} \right) \left(\frac{\sigma_x}{\sigma_x - \sigma_y} \right) \right] \quad (10)$$

Finding from the physical relations of the voltage, we can write (11):

$$\begin{aligned} \sigma_x &= \frac{\psi_i}{1 - \nu_i \nu_j} (e_x + \nu_j e_y) \\ \sigma_y &= \frac{\psi_j}{1 - \nu_i \nu_j} (e_y + \nu_i e_x) \\ \tau_{xy} &= G_j e_{xy} \end{aligned} \quad (11)$$

The experimental deformation diagram of fiber-reinforced concrete is approximated by the function (12):

$$\sigma = \begin{cases} A_1 \varepsilon - B_1 \varepsilon^3 & \text{for } \sigma \geq 0 \\ A_2 \varepsilon - B_2 \varepsilon^3 & \text{for } \sigma < 0 \end{cases} \quad (12)$$

The coefficients of which can be easily determined from the conditions of the minimum functional (13)

$$I = \sum_{j=1}^n (A \varepsilon_j - B \varepsilon_j^3 - \sigma_j^{\text{acc}}) \rightarrow \min \quad (13)$$

As a result, we get the following expressions for finding them (14):

$$\begin{aligned} A &= \frac{\left(\sum_{j=1}^n \sigma_j \varepsilon_j \right) \left(\sum_{j=1}^n \varepsilon_j^6 \right) - \left(\sum_{j=1}^n \sigma_j \varepsilon_j^3 \right) \left(\sum_{j=1}^n \varepsilon_j^4 \right)}{\left(\sum_{j=1}^n \varepsilon_j^2 \right) \left(\sum_{j=1}^n \varepsilon_j^6 \right) - \left(\sum_{j=1}^n \varepsilon_j^4 \right)^2} \\ B &= \frac{\left(\sum_{j=1}^n \sigma_j \varepsilon_j \right) \left(\sum_{j=1}^n \varepsilon_j^4 \right) - \left(\sum_{j=1}^n \sigma_j \varepsilon_j^3 \right) \left(\sum_{j=1}^n \varepsilon_j^2 \right)}{\left(\sum_{j=1}^n \varepsilon_j^2 \right) \left(\sum_{j=1}^n \varepsilon_j^6 \right) - \left(\sum_{j=1}^n \varepsilon_j^4 \right)^2} \end{aligned} \quad (14)$$

Indexes at coefficients A and B are omitted.

Note that the values of the coefficients A_1, B_1 are based on experimental strain curves under tension, and the coefficients A_2, B_2 are based on experimental data under compression.

If we do not take into account the nonlinearity of deformation of fiber-reinforced concrete, but take into account only its resistance to deformation, then the deformation diagram can be described by the function (15):

$$\sigma_u(\varepsilon_u, c) = E_i \varepsilon_u \quad (15)$$

where $i=1$ corresponds to stretching, and $i=2$ corresponds to compression.

To assess the suitability of the above relations for describing the deformation of fiber concrete, we use the experimental data shown in Fig. 2

In this case, to determine the elastic modulus E , we use the minimum condition of the following functional (16).

$$I = \sum_{j=1}^n (\sigma_i^j - E \varepsilon_i)^2 \rightarrow \min \quad (16)$$

As a result, we get the expression (17):

$$E = \frac{\sum_{i=1}^n \sigma_i \varepsilon_i}{\sum_{i=1}^n \varepsilon_i^2} \quad (17)$$

The results of the description of the deformation curves of fiber concrete and fiber concrete with the

addition of CMID according to various models are given in Table 1 and 2. The deviation of theoretical data from experimental data is also shown there.

As can be seen, when using a nonlinear deformation model to describe experimental results, the results mostly fall into a five percent error corridor. When using a linear model, the approximation error reaches 15-20%.

Let's make up the resolving equation of a plate of fiber-reinforced concrete on an elastic base. Expressions for moments and efforts will be taken in the following form (18):

$$\begin{aligned} M_x^{\phi\delta} &= \int_{-h/2}^{z_x} \sigma_{xj}^{\phi\delta} z dz + \int_{z_x}^{h/2} \sigma_{xi}^{\phi\delta} z dz, \\ M_y^{\phi\delta} &= \int_{-h/2}^{z_y} \sigma_{yj}^{\phi\delta} z dz + \int_{z_y}^{h/2} \sigma_{yi}^{\phi\delta} z dz, \\ H^{\delta\delta} &= \int_{-h/2}^{z_{xy}} \tau_{xyj}^{\delta\delta} z dz + \int_{z_{xy}}^{h/2} \tau_{xyi}^{\delta\delta} z dz, \\ N_x^{\phi\delta} &= \int_{-h/2}^{z_x} \sigma_{xj}^{\phi\delta} dz + \int_{z_x}^{h/2} \sigma_{xi}^{\phi\delta} dz, \\ N_y^{\phi\delta} &= \int_{-h/2}^{z_x} \sigma_{yj}^{\phi\delta} dz + \int_{z_x}^{h/2} \sigma_{yi}^{\phi\delta} dz, \\ S^{\phi\delta} &= \int_{-h/2}^{z_{xy}} \tau_{xyj}^{\phi\delta} dz + \int_{z_{xy}}^{h/2} \tau_{xyi}^{\phi\delta} dz, \end{aligned} \quad (18)$$

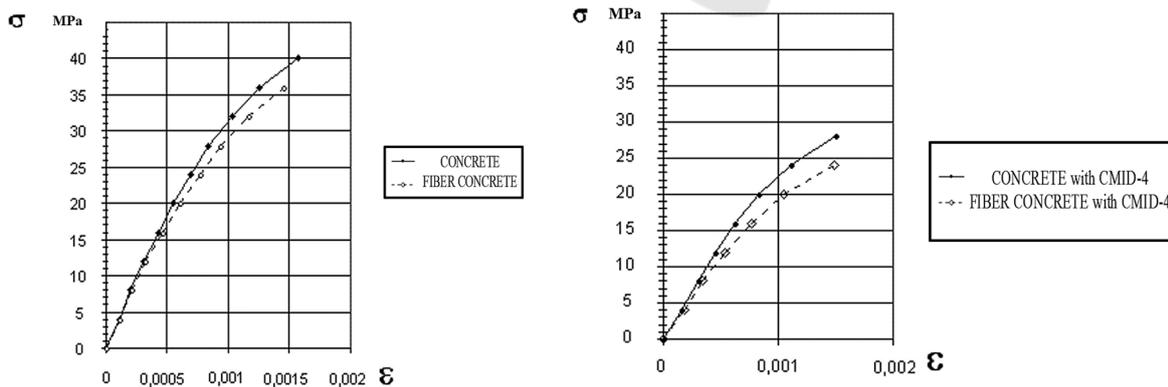


Figure 2: Dependence of σ (under compression) from ε for a- concrete and fiber concrete, b- for concrete and fiber concrete with the addition of CMID-4 (Pestryakov, 2003; Polyakova, 1991).

Table 1: The results of modeling experimental deformation curves of fiber concrete with the addition of CMID under compression, and: $A_2=36.44 \cdot 10^3$ Mpa, $B_2=46.20 \cdot 10^8$ Mpa, $E_2=28.93 \cdot 10^3$ Mpa.

Experimental values of stresses, MPa	Experimental values of deformations, mm	Calculated stresses according to the nonlinear model. MPa	Calculated stresses according to the linear model. MPa	Deviations in the nonlinear model, MPa	Deviations in the linear model, MPa	Error of the nonlinear model %	Error of the linear model %
4,00	0,00011	4,16	3,30	0,16	-0,70	3,90	-17,38
8,00	0,00021	7,76	6,20	-0,24	-1,80	-3,00	-22,54
12,00	0,00032	11,55	9,30	-0,45	-2,70	-3,71	-22,54
16,00	0,00043	15,25	12,39	-0,75	-3,61	-4,70	-22,54
20,00	0,00055	19,38	16,01	-0,62	-3,99	-3,09	-19,96
24,00	0,00070	23,81	20,14	-0,19	-3,86	-0,80	-16,08
28,00	0,00085	28,03	24,48	0,03	-3,52	0,12	-12,58
32,00	0,00104	32,60	29,95	0,60	-2,05	1,87	-6,40
36,00	0,00129	37,02	37,18	1,02	1,18	2,85	3,28
40,00	0,00162	39,39	46,99	-0,61	6,99	-1,53	17,48

Table 2: The results of modeling experimental deformation curves of fiber concrete with the addition of CMID under tension, and: $A_1=30.71 \cdot 10^3$ Mpa, $B_1=11.79 \cdot 10^{10}$ Mpa, $E_1=22.21 \cdot 10^3$ Mpa.

Experimental values of stresses, MPa	Experimental values of deformations, mm	Calculated stresses according to the nonlinear model. MPa	Calculated stresses according to the linear model. MPa	Deviations in the nonlinear model, MPa	Deviations in the linear model, MPa	Error of the nonlinear model %	Error of the linear model %
1,00	0,00003	0,98	0,71	-0,02	-0,29	-1,71	-28,65
2,00	0,00005	1,52	1,11	-0,48	-0,89	-23,99	-44,50
3,00	0,00010	3,05	2,30	0,05	-0,70	1,62	-23,36
4,00	0,00014	3,96	3,09	-0,04	-0,91	-1,06	-22,70
5,00	0,00020	5,20	4,44	0,20	-0,56	3,95	-11,21
6,00	0,00032	5,96	7,14	-0,04	1,14	-0,73	18,92

Here $z_x(x,y)$, $z_y(x,y)$, $z_{xy}(x,y)$ are functions describing the position of neutral surfaces determined from the conditions $\sigma_x=0$, $\sigma_y=0$, $\tau_{xyj}=0$ and separating the stretched zones of the fiber-reinforced plate from the compressed ones, and if the lower zone of the bent plate is stretched, then $j=1$, $i=2$, if the upper zone is stretched and the lower one is compressed, then $j=2$, $i=1$. The functions $z_x(x,y)$, $z_y(x,y)$, $z_{xy}(x,y)$ are expressed in terms of deformation parameters as follows (19):

$$z_x(x,y) = -\frac{\epsilon_x + \nu_i \epsilon_y}{\chi_x + \nu_i \chi_y};$$

$$z_y(x,y) = -\frac{\epsilon_y + \nu_j \epsilon_x}{\chi_y + \nu_j \chi_x}; \quad (19)$$

$$z_{xy}(x,y) = -\frac{\epsilon_{xy}}{2\chi_{xy}}$$

If you use the notation (20):

$$\alpha_i = \frac{\psi_i}{1 - \nu_i \nu_j}, \alpha_j = \frac{\psi_j}{1 - \nu_i \nu_j},$$

$$\frac{1}{\beta_j} = 2 \left[\left(\frac{1 + \nu_{xi}}{\psi_{xi}} - \frac{1 + \nu_{yj}}{\psi_{yj}} \right) \left(\frac{\sigma_x}{\sigma_x - \sigma_y} \right) \right]$$

$$\frac{1}{\beta_i} = 2 \left[\left(\frac{1 + \nu_{yj}}{\psi_{yj}} - \frac{1 + \nu_{xi}}{\psi_{xi}} \right) \left(\frac{\sigma_y}{\sigma_y - \sigma_x} \right) \right]$$

$$\begin{aligned}
 J_k^x &= \int_{-h/2}^{z_x} \alpha_j z^k \partial z + \int_{z_x}^{h/2} \alpha_i z^k \partial z, \text{ for } k=0,1,2 \\
 J_k^y &= \int_{-h/2}^{z_y} \alpha_j z^k \partial z + \int_{z_y}^{h/2} \alpha_i z^k \partial z, \text{ at } k=0,1,2 \quad (20) \\
 I_k^x &= \int_{-h/2}^{z_x} \alpha_j v_j z^k \partial z + \int_{z_x}^{h/2} \alpha_i v_i z^k \partial z, \text{ at } k=0,1,2 \\
 I_k^y &= \int_{-h/2}^{z_y} \alpha_j v_j z^k \partial z + \int_{z_y}^{h/2} \alpha_i v_i z^k \partial z, \text{ at } k=0,1,2 \\
 T_k &= \int_{-h/2}^{z_{xy}} G_j z^k \partial z + \int_{z_{xy}}^{h/2} G_i z^k \partial z, \text{ at } k=0,1,2
 \end{aligned}$$

Then the bending and torques, as well as the forces, can be written as (21):

$$\begin{aligned}
 M_x^{\phi\delta} &= \varepsilon_x J_1^x + \chi_x J_2^x + \varepsilon_y I_1^x + \chi_y I_2^x; \\
 M_y^{\phi\delta} &= \varepsilon_y J_1^y + \chi_y J_2^y + \varepsilon_x I_1^y + \chi_x I_2^y \\
 H^{\phi\delta} &= \varepsilon_{xy} T_1^{\phi\delta} + 2\chi_{xy} T_2^{\phi\delta}; \\
 N_x^{\phi\delta} &= \varepsilon_x J_0^x + \chi_x J_1^x + \varepsilon_y I_0^x + \chi_y I_1^x \quad (21) \\
 N_y^{\phi\delta} &= \varepsilon_y J_0^y + \chi_y J_1^y + \varepsilon_x I_0^y + \chi_x I_1^y; \\
 S^{\phi\delta} &= \varepsilon_{xy} T_0^{\phi\delta} + 2\chi_{xy} T_1^{\phi\delta}
 \end{aligned}$$

If there are no linear normal and horizontal shear forces in the plate sections $N_x=0$, $N_y=0$, $S=0$, then using additional convolution notation (22):

$$\begin{aligned}
 f_2 &= \frac{I_0^x I_1^y - J_0^y J_1^x}{J_0^x J_1^y - I_0^x I_1^y}; \quad f_1 = \frac{I_0^x J_1^y - I_0^x J_1^x}{J_0^x J_1^y - I_0^x I_1^y}, \\
 f_4 &= \frac{I_0^y I_1^x - J_1^y J_0^x}{J_0^x J_1^y - I_0^x I_1^y} \\
 f_3 &= \frac{I_0^y J_1^x - I_1^y J_0^x}{J_0^x J_1^y - I_0^x I_1^y}, \quad (22)
 \end{aligned}$$

we can find fairly simple expressions for deformations ε_x , ε_y , ε_{xy} (23):

$$\begin{aligned}
 \varepsilon_x &= f_2 \chi_x + f_1 \chi_y, \quad \varepsilon_y = f_3 \chi_x + f_4 \chi_y, \\
 \varepsilon_{xy} &= -\left(\frac{2T_1}{T_0}\right) \chi_{xy} \quad (23)
 \end{aligned}$$

Considering additional notations (24):

$$\begin{aligned}
 D_1 &= f_2 J_1^x + f_3 I_1^x + J_2^x; \\
 D_2 &= f_1 J_1^x + f_4 I_1^x + I_2^y; \\
 D_3 &= f_3 J_1^y + f_2 I_1^y + I_2^y; \\
 D_4 &= f_4 J_1^y + f_1 I_1^y + J_2^y; \quad (24) \\
 D_5 &= 2T_2 - 2\frac{(T_1)^2}{T_0},
 \end{aligned}$$

we get (25):

$$\begin{aligned}
 M_x &= D_1 \chi_x + D_2 \chi_y, \quad M_y = D_3 \chi_x + D_4 \chi_y, \\
 H &= D_5 \chi_{xy} \quad (25)
 \end{aligned}$$

Using the expressions for bending and torques M_x , M_y , H , we transform the initial equilibrium equation into the differential equation of bending of a plate made of fiber-reinforced concrete interacting with an elastic base (26):

$$\begin{aligned}
 \frac{\partial^2}{\partial x^2} \left(D_1 \frac{\partial^2 w}{\partial x^2} \right) + \frac{\partial^2}{\partial x^2} \left(D_2 \frac{\partial^2 w}{\partial y^2} \right) + \\
 2 \frac{\partial^2}{\partial x \partial y} \left(D_5 \frac{\partial^2 w}{\partial x \partial y} \right) + \frac{\partial^2}{\partial y^2} \left(D_3 \frac{\partial^2 w}{\partial x^2} \right) + \\
 + \frac{\partial^2}{\partial y^2} \left(D_4 \frac{\partial^2 w}{\partial y^2} \right) = p(x, y) - q(x, y) \quad (26)
 \end{aligned}$$

3 RESULTS AND DISCUSSION

In the article, models of deformation of an orthotropic nonlinear multi-modulus fiber-reinforced structure under the conditions of a plane stress state characteristic of bent plates are constructed. As a result of the identification of models of deformation of the material using experimental data on the stretching and compression of fiber concrete, the coefficients of the models were determined. Using the proposed nonlinear orthotropic multi-resistive model of deformation of fiber-reinforced concrete, differential equations are obtained that are a model of deformation of a plate of fiber-reinforced concrete interacting with an elastic base. This model describes the deformation of a fiber-reinforced plate on an elastic base under various boundary conditions on the contour. The work of the elastic base can be taken into account according to one or another

model by specifying an expression for the rebuff of the elastic base p .

4 CONCLUSIONS

A fairly wide range of scientific methods were used in the work. Experimental data were collected and analyzed, mathematical modeling was performed, and numerical simulation results were compared with experimental data. It is shown that the nonlinear multi-modulus model of fiber concrete deformation shows a much better approximation of experimental data than the linear, albeit multi-modulus model. All this in general allowed us to draw a conclusion about the reliability of the results. The practical significance of the article consists in constructing models of deformation of a fiber-reinforced plate interacting with an elastic base, taking into account the nonlinearity and diversity of the plate material. The results of the work are used in the educational process in the training of civil engineers when presenting the issues of calculating structures interacting with an elastic base, taking into account the real conditions of the properties of fiber-reinforced concrete from which the structures are made. It should also be noted that the obtained model of deformation of a fiber-reinforced concrete plate operating on an elastic base can be effectively used in the calculation of roadway plates on bridge structures, in the calculation of rigid road pavement made of fiber-reinforced concrete arranged on highways.

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