Mathematical Modeling of the Management of the Processes of Training Specialists of Transport Universities

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Abstract: The article presents scientific research in the field of mathematical modeling, namely the construction of a mathematical model for managing the processes of training specialists. When formalizing this complex process, problems arise that are explained by the presence of a large number of factors that ensure a high-quality educational process. Therefore, great importance is attached to the use of the mathematical apparatus in the implementation of quality management of the educational process in higher educational institutions. The article presents the construction of mathematical models describing the most important links between the characteristics of the educational process at the university. The considered mathematical models contribute to the development and adoption of managerial decisions on the strategy of managing the processes of training specialists.

1 INTRODUCTION

At the present stage of the socio-economic development of Russia, the development of the higher education system, the training of highly qualified specialists depends on the achievements in all areas of activity of each university in the country. This requires: improving the system of effective management of the processes of the university, the introduction into the practice of developing universities of new scientific and pedagogical achievements that correspond to the subject of management and the requirements of modern education (Ansoff, 1989). To do this, it is necessary to solve the following tasks:

1. Development and implementation of methods and means of planning and management as a complex of components of effective management of high-quality training of specialists.
2. Provision of qualified personnel: managerial, managerial, scientific, pedagogical; including their training, systematic professional development and the effectiveness of forms of work organization.
3. Development of methods and means for the effective use of material and technical resources in the organ-ization of the educational process, ensuring the completeness and reliability of resource support.
4. The use of modern methods of systems analysis, strategic forecasting, effective management and mathematical modeling.

Modeling as a universal method of cognition of social and pedagogical problems, the tasks of didactics today is an integral part of the study of the educational process. The beginning of modeling is a goal setting that takes into account modern educational problems, the complexity of which determines their complex nature. The key task of building and applying a mathematical model is to prepare a competent, qualified specialist.

The constructed mathematical model will allow choosing the optimal control actions, as well as making an objective forecast of the state of the control system (Samarsky, Mikhailov, 2005; Neymark, Yu. I., 2010).

The article (Solodova, 2005) considers a rather simple from a mathematical point of view and a more

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general conceptual model developed by the doctor of philosophical sciences A.S. Panarin.

$$\frac{dy_{межотр}}{dt} > \frac{dy_{отрасл}}{dt},$$

$$\frac{dy_{фундам}}{dt} > \frac{dy_{приклад}}{dt},$$

$$\frac{dy_{учебы}}{dt} > \frac{dy_{работы}}{dt} \quad (1)$$

These inequalities (1) can be interpreted as follows: the first inequality indicates a higher growth rate of cross-sectoral knowledge compared to sectoral knowledge. The second inequality indicates an excess of the growth rate of fundamental knowledge over the growth rate of applied knowledge. In the third inequality: the growth rate of study time must be greater than the growth rate of working time (Gianetto, Wheeler, 2005).

Projecting system (1) onto the educational process of the university, the following conclusions can be drawn: inequalities 1 and 2 determine the content of education, thus, these inequalities organize the activities of the university to develop qualification requirements, curriculum, curriculum and thematic plan. university plans; Inequality 3 expresses the principle of lifelong education, the principle of self-development, formalizes the system of additional education.

The basis for the development of these management models is the state standard, which not only fixes the subject area, but also formulates the learning objectives. Gosstandart sets the legal and substantive basis for more detailed knowledge models designed for a specific training course or part of it (module), or sets the search vector for the optimal knowledge acquisition process.

For an adequate choice of a mathematical analytical model, it is necessary to formalize its parameters:

– to consider educational texts of a textbook, lecture notes, records of problem solutions, etc. as information flows of a certain finite amount of knowledge, taking into account the sequential dynamics of their development and memorization;
– to build and investigate the information flow as a model of knowledge transfer "teacher-learners" in order to determine the main characteristics of their interactions, connections, to identify the effectiveness of this model in terms of adequate management of the individual process of mastering professional knowledge by each student, group, course;
– the mathematical model should form a finite set of control parameters (components), with the help of which control decisions can be made for the further improvement of the educational process at the university. The managerial function of the regularity of the process, contained in mathematical models, can help management departments to make scientifically sound decisions to improve it.

The problems of constructing mathematical models of optimal control of the processes of training qualified specialists have been considered by many authors, for example (Vasiliev, 1997; Avetisov, 1998).

As you know, one of the management concepts that emerged in the 80s of the last century is the process approach. In accordance with this concept, the entire activity of a higher educational institution is a set of sequential and interrelated processes. The process approach is one of the key elements of improving the quality of training, therefore, effective quality management is impossible without replacing subjective descriptions with objective assessments of the learning process by building appropriate mathematical models (Kalganova, 2021; Golubeva, 2016).

1.1 Building a Mathematical Model for Managing the Processes of Training Specialists

Studying the content of the educational process, analyzing statistical data, establishing cause-and-effect relationships between the elements of the process being studied, which can be described quantitatively. This made it possible to formulate the relationship between the parameters and build a mathematical model in the form of equations between the main objects of the model (Stepanov, 2006).

Let x(t) be the amount of knowledge accumulated by the student at a certain point in time t, including: the ability to reason, solve problems, understand the material presented by the teacher. The unit is important here. As a measure for x(t), you can enter the sum of the exam grades, the number of successfully passed credits, etc.

In this study, it is logical to consider the functions introduced to build a mathematical model as dimensionless quantities. Thus, the conventional units (AU) act as a fixed number of credit units. A set of conventional units is a system of theoretical knowledge and practical skills formed in the process of implementing the educational process at a university. Based on the results of examinations for the differentiated assessment of student performance in the learning process, it is possible to determine x1 – the minimum value of CU, but sufficient to complete the training, x2 – the maximum value of CU, for graduates who have received honors. degree after graduation.

The task of constructing a mathematical model, thus, acquires an additional condition: to find the law of assimilation of knowledge during the period of study by each student who has the initial amount of knowledge upon entering the university x0 and who completed training with the amount of knowledge x ≥ x2.

As a result, the inequality holds for the function x(t):

$$x1 \le x(t) \le x2+d.$$

where d – is additional knowledge to the main program (electives, courses, self-education according to interests, etc.).

The model in this case will have the form:

$$\frac{dx(t)}{dt} = \alpha_1 x(t)\big(x_2 - x(t)\big), \qquad (2)$$

where dx(t)/dt – the speed of mastering conventional units;

α1 – proportionality coefficient;

x(t) – the number of conventional units mastered by students at a time t;

x2 – x(t) – the amount of knowledge on the program required for successful completion of the training.

The coefficient α1>0 depends on the individual abilities of the student, his attitude to educational work and the level of the previous one, up to the moment t, of the qualitative development of the educational material. The right side of equation (1) depends only on previously acquired knowledge. Please note that the value of x(t) with proper self-organization of the student, tends to only increase; therefore, it makes sense to apply model (1) on time intervals: one semester or an academic year, a four-year study for bachelors or a five-year period for specialists..

Equation (1), known as the Verhulst equation, originally arose from the study of population change. This mathematical model is widely used not only in various fields of natural science, but also plays an important role in understanding the mechanisms of applying nonlinear dynamics to socio-economic models, in the tasks of introducing technological innovations and developing science (Zhirkov, 2016; Bagrinovsky, 1980).

The dynamics described by the Ferhulst equation is a logistic curve in Figure 1.

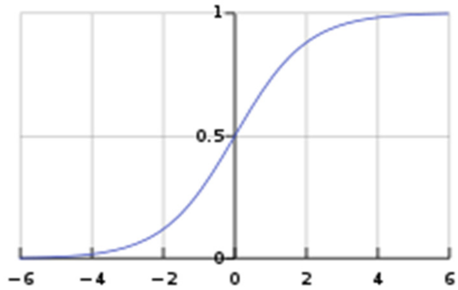

Figure 1: Logistic curve for x_2 = 1, x_0 = 0,5, α_1= 1.

The completeness and target volume of knowledge necessary for the successful assimilation of educational material during the period of study at a university is a key component of the quality of education, therefore, to study their dynamics, it is advisable to use a logistic equation. However, other parameters (directions) of the university's activity also affect the successful assimilation of educational-theoretical and practical material when studying at a university.

Two important components are considered that directly affect the quality of training of specialists. This is the qualitative composition of the teaching staff of the university and the presence of a modern and fully material and technical base (Kalganova, 2020).

Let's consider the first component of the pedagogical process and its influence on the quality of training of specialists. The pedagogical activity of the teacher is united by the main components of this activity:

– transfer of knowledge through direct interaction «teacher - student»;
– methodical work;
– scientific work;
– educational work.

In order to formalize a rather complex, multifaceted activity of a teacher and reflect it in the form of quantitative characteristics of numerical assessments, it is necessary to find the criteria of pedagogical work and the value of such assessments; the methodology for calculating rating indicators is partially presented in Table 1.

Thus, using the methods of organizing ratings at the university using quantitative and qualitative indicators of teachers' academic performance in points, it is possible to introduce a numerical, functional relationship with other parameters of the mathematical model being developed.

Table 1: Methodology for calculating the rating indicators of teachers.

Indicators	The value of the indicator in points
Educational activities	
Innovative pedagogical activity (online courses; development of practice-oriented technologies; tests)	0-3
Students' assessment of the teacher's activities by factors: the availability of material, the development of work programs by disciplines, practices, - the quality of classes	2-5
Professional development over the last year	0-2
Availability of developed educational and methodological documentation for training courses	0-3
Preparation of students for international and (or) republican Olympiads, competitions	0-2
Research work	
Participation in research, research and development work, in obtaining grants, fulfillment of business contracts	0-3
Obtaining patents/certificates issued to the university	0-3
Scientific publications	0-2
References to the author's works received in the reporting year	0-2
Reports at conferences	0-2
Defense of dissertations	0-2

Let us introduce the function y(t) – the rating indicators of the teacher, as a measure of the effectiveness of the influence of the teaching staff on the quality of teaching. The numerical value of the overall grade for each teacher will satisfy the inequality:

$$y1 \le y(t) \le y2,$$

where y1 – the lowest rating value in conventional units (points), y2 – maximum rating value.

The function y(t) will be considered dimensionless, which characterizes the direct effect on the increase in the growth rate of the volume of students' knowledge with the coefficient β1.

Coefficient β1 there is a function depending on β11 – staffing with teaching staff, β12 – optimal ratio of teachers with academic degrees of candidate and doctor of sciences, β13 – the level of scientific and scientific-methodical work of each teacher, β14 – regular completion of refresher courses and other ranking indicators.

Let us introduce the function z(t) – the relative total cost of all scientific and educational equipment

The function z(t) will be considered dimensionless in conventional units and will have a direct impact on the increase in the growth rate of students' knowledge with a coefficient of y1. The effectiveness of the influence of educational and material resources of the university on the quality of training of specialists can be expressed using the coefficient y1. This coefficient represents a certain function depending on y11 – the full staffing of the educational and material base of the university, y12 – the optimal use of scientific and educational laboratories in the educational process, y13 – compliance with the modern level of scientific and educational laboratories, y14 – the availability of testing and training grounds, etc. Taking into account the above, we finally come to the construction of the following mathematical model for managing the processes of training specialists.

$$\frac{dx(t)}{dt} = \alpha_1 x(t)(x_2 - x(t)) + \beta_1 y(t) + \gamma_1 z(t) + f_1(t)$$

$$\frac{dy(t)}{dt} = \beta_2 y(t) + \gamma_2 z(t) + f_2(t)$$

$$\frac{dy(t)}{dt} = \beta_2 y(t) + \gamma_2 z(t) + f_2(t) \qquad (2)$$

where the functions fi(t) i = 1,2,3, can act as additional conditions due to additional sources of knowledge, lectures by foreign scientists, the use of research and testing laboratories of research institutes, enterprises.

A preliminary analysis of the relationship between the parameters included in the construction of a mathematical model allows us to conclude that it is possible to build a mathematical model for managing the processes of training specialists using systems of three linear differential equations.

Such a system, composed according to the same principle as system (2), will have the following form.

$$\frac{dx(t)}{dt} = \alpha_1 x(t) + \beta_1 y(t) + \gamma_1 z(t) + f_1(t)$$

$$\frac{dy(t)}{dt} = \beta_2 y(t) + \gamma_2 z(t) + f_2(t)$$

$$\frac{dz(t)}{dt} = \gamma_3 z(t) + f_2(t) \qquad (3)$$

We introduce the matrix A of coefficients of the corresponding system of linear differential equations (3):

$$A = \begin{pmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ 0 & \beta_2 & \gamma_2 \\ 0 & 0 & \gamma_3 \end{pmatrix}. \qquad (4)$$

Additionally, we introduce two matrices:

- matrix – a column of unknown functions

$$X = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix}; \quad (5)$$

- matrix – a column of pivot members

$$F = \begin{pmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \end{pmatrix}. \quad (6)$$

Using the introduced matrices, system (3) can be briefly written in matrix form:

$$\frac{d}{dt} X = AX + F. \quad (7)$$

Please note that system (2) is nonlinear, and an analytical solution in the form of a solution formula is not possible. To study the solutions of this system, the use of approximate or numerical methods is required.

Therefore, let us consider in more detail the solution of the linear system (3).

For systems (3), (7), we formulate the Cauchy problem or the problem with initial conditions for t = 0:

$$x(0) = x_0,\ y(0) = y_0,\ z(0) = z_0 \quad (8)$$

1.2 Study of the Mathematical Model and Solution of the Cauchy Problem for Systems of Differential Equations (3)

Let us prove the existence and uniqueness theorem for the solution of the Cauchy problem for system (3) of differential equations with the initial conditions (8).

Theorem.

Let the functions f1(t), i = 1, 2, 3 be continuous on some interval t∈[0,T] then the Cauchy problem for the inhomogeneous system of first-order differential equations (3), (7), (8) with constants coefficients will have the only solution.

Proof.

The third differential equation of system (3) is a first-order linear equation with a solution (Stepanov, 2006):

$$z(t) = e^{\gamma_3 t}\left[z_0 + \int_0^t f_3(\tau)e^{-\gamma_3\tau}d\tau\right] = \varphi_1(t), \quad (9)$$

where the function $\varphi_1(t)$ denotes the solution to the third equation of the system (3).

Let us write the second equation taking into account (9) as follows:

$$\frac{y(t)}{dt} = \beta_2 y(t) + \gamma_2\varphi_1(t) + f_2(t) \quad (10)$$

the resulting differential equation is also a first-order linear equation, and its solution $\varphi_2(t)$ will have the form:

$$y(t) = e^{\beta_2 t}\left[y_0 + \int_0^t(\gamma_2\varphi_1(\tau) + f_2(\tau))e^{-\beta_2\tau}d\tau\right] = \varphi_2(t), \quad (11)$$

We continue to consistently perform similar actions, we find the solution to the first equation:

$$x(t) = e^{\alpha_1 t}\left[x_0 + \int_0^t(\beta_2\varphi_2(\tau) + f_1(\tau))e^{-\alpha_1\tau}d\tau\right] = \varphi_3(t). \quad (12)$$

Suppose that the functions $f(t)$, i=1, 2, 3 are bounded,

$$|f_i(t)| < B, \qquad i = 1,2,3.$$

Then for all $t \in [0,T]$ we obtain the inequalities:

$$|z(t)| = |\ell^{\gamma_3 t}[z_0 + \int_0^t f_3(\tau)\ell^{-\gamma_3\tau}d\tau]| \leq |\ell^{\gamma_3 t}||z_0 + \frac{B}{\gamma_3}\ell^{-\gamma_3 t}| =$$

$$= \ell^{\gamma_3 t}x_0 + \frac{B}{\gamma_3} \leq x_0\ell^{\gamma_3 T} + \frac{B}{\gamma_3} < D = const, t \in [0.T]$$

Finally: $|z(t)| < D$

Similar inequalities can be obtained for all subsequent solutions of the systems of equations (3) and (7).

Assume the existence of two $x_1(t)$ and $x_2(t)$ solutions of system (3), (7) with initial conditions (8), then the function $X(t)=X_2(t) - \overline{X}_1(t)$ will satisfy a homogeneous equation fo $f_1 = f_2 = f_3 = 0$ and zero initial conditions.

Then from formula (9) follows $\bar{z}(t)=0$, similarly, formula (11) follows $\bar{y}(t)$=0 and $\bar{x}(t)$=0 Hence the equality $x_1(t) = x_2(t)$.

Thus, there is only one solution for problem (9) – (10).

The theorem of existence and uniqueness of the solution of the Cauchy problem for systems of differential equations (3) and (7) with initial conditions (8) is proved.

Based on the obtained solution of the Cauchy problem for systems (2), (3), (7). it is possible to develop proposals for managing the learning process by choosing the values of the initial valuesx_0, y_0, z_0 and positive coefficients α_1, β_1, β_2, y_1, y_2, y_3 of the matrices A and F, the elements of which are a set of tunable parameters affecting the optimal control of solutions of systems ordinary differential equations (2), (3) or (7). By varying the elements of the matrix A (coefficients of the system) and the function $fi(t)$ (free terms of the system), one can achieve an

increase or decrease in the rate of change of $x(t)$ – the amount of accumulated knowledge by a student.
Let's introduce a definition. The set of coefficients is called α_1, β_1, β_2, y_1, y_2, y_3 and functions $fi(t)$ of system (2), matrices (4), (6) of systems (3), (7) control matrices for solving systems of ordinary differential equations (2), (4) or (6), i.e. this is a set of control parameters from a set of matrix elements A and the time functions of the matrix F, which have a direct impact on the solutions of systems (2), (4), (6) – systems that model the management of the future development of the educational system of the university and the processes of training specialists.

1.3 Verification of the Mathematical Model

We will check the verification of mathematical models, that is, the correspondence of the solutions of systems of differential equations to the real educational processes of the university, the optimal control of the processes of training specialists.

Analysis of the construction of models, for example, system (2), shows that in the process of building the target value x_2 – the full amount of knowledge for the formation of a specialist, is an integral part of the system to which the function aspires $x(t)$ при $t \to \infty$.

The members $\beta_1 y(t)$, $y_1 z(t)$, $f_1 z(t)$, system (3) only accelerate the achievement of the target value in a limited time interval. This solution shows the possibility of organizing training that is optimal for all students at the same time. In this case, the achievement of the target results depends on the initial position and control matrices for solving the systems of ordinary differential equations.

Let us give examples showing the role of the control matrix A (4) for achieving target values in managing the process of training specialists.

Example 1.

Consider a system of three equations and use the MathCAD 15 software package (Makarov, 2009) to obtain a numerical-graphic solution.

Given the system: (Given)

$$\frac{d}{dt}x(t) = 0.1\cdot x(t) + 0.008\cdot y(t) - 0.034\cdot z(t)$$

$$\frac{d}{dt}y(t) = 0.27\cdot y(t) - 0.028\cdot x(t) + 0.024\cdot z(t)$$

$$\frac{d}{dt}z(t) = -0.67\cdot z(t)$$

Here the steering matrix has the form:

$$A = \begin{pmatrix} 0.1 & 0.008 & 0.034 \\ 0.027 & 0.27 & 0.024 \\ 0 & 0 & 0.67 \end{pmatrix}$$

$x(0) = 7.0$

$y(0) = 2.6$

$z(0) = 1.0$

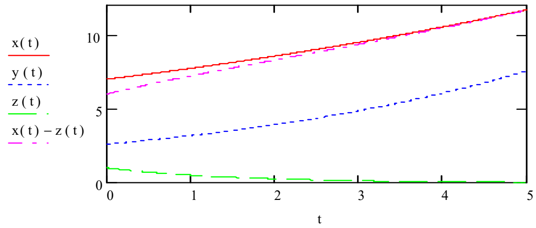

Figure: 2: Graphs of functions x(t),y(t), z(t) and x(t)-z(t).

$x(0) = 7.0$	$x(5) = 11.698$
$y(0) = 2.6$	$y(5) = 7.572$
$z(0) = 1$	$z(5) = 0.035$

Consider the case when some coefficients are zero

$$\frac{d}{dt}x(t) = 0.1\cdot x(t) + 0.08\cdot y(t) - 0.034\cdot z(t)$$

$$\frac{d}{dt}y(t) = 0.35\cdot y(t) + 0.2$$

$$\frac{d}{dt}z(t) = -0.67\cdot z(t)$$

$x(0) = 7.0$
$y(0) = 0.6$
$z(0) = 1$

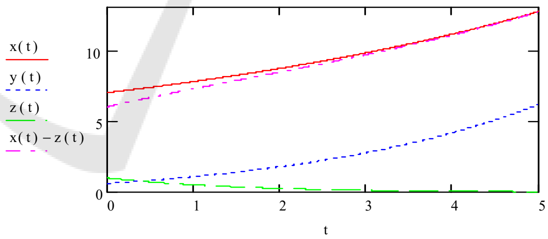

Figure: 3: Graphs of functions $x(t)$, $y(t)$ and $z(t)$.

Let's make calculations

$x(0) = 7$	$x(0) = 0,6$	$z(0) = 1$
$x(5) = 12,712$	$y(1) = 1,091$	$z(5) = 0,035$
$y(1) - 0,6 = 0,491$	$y(2) = 1,788$	
$y(2) - y(1) = 0,697$	$y(3) = 2,776$	
$y(3) - y(2) = 0,989$	$y(4) = 4,179$	
$y(4) - y(3) = 1,403$	$y(5) = 6,17$	
$y(5) - y(4) = 1,991$		

The graphs in Figures 2, 3 show an increase in the volume of students' knowledge while improving the quality of the educational process and educational and material support.

Indeed, the results of analytical studies, the examples given show that it is possible to develop proposals for managing the learning process based on the solutions of systems (3), (7), choosing for each t the values of the initial values x_0, y_0, z_0 the values of the coefficients as a set of adjustable parameters affecting the optimal control of the solution of the systems of ordinary differential equations (3) or (7), as well as the achievement of target values.

Thus, the models of the educational process management process (2), (3) and (7) made it possible to obtain a number of practical recommendations expressed in numerical form. At the same time, there was no need to clarify methods for measuring the amount of knowledge available to students. It is sufficient that these quantities satisfy the qualitative relations leading to the systems of equations (2), (3), and (7).

2 CONCLUSIONS

The article discusses mathematical models that contribute to making managerial decisions about an alternative choice of strategy for managing the processes of training specialists.

The first mathematical model (2) allows you to build an optimal acquisition strategy, taking into account the initial state and the formulated target value at the end of the training period (bachelor's, specialist, master's degree) based on the logistic function. obtained using a differential equation. This model makes it possible to predict and implement strategic programs of the educational process of the university for a longer period.

Based on the analysis of the relationship between the parameters included in the construction of a mathematical model, a mathematical model for managing the processes of training specialists using systems of three linear differential equations with constant coefficients has been built.

The matrices A and F of the coefficients of the corresponding system of linear differential equations and their free terms determine the control matrices for the solution of the systems of ordinary differential equations (3) and (7), which have a direct impact on the process of managing the training of specialists.

The article conducts research on the verification of mathematical models based on the consideration of control matrices and the multivariance of parameters included in the solution of differential equations that form the basis of the mathematical models under consideration.

REFERENCES

Ansoff, I., 1989. *Strategic management.* p. 358.

Samarsky, A. A., Mikhailov, A. P., 2005. *Mathematical modeling*. p. 320.

Neymark, Yu. I., 2010. *Mathematical modeling as a science and art*. p. 420.

Solodova, E. A., Antonov, Yu. P., 2005. *Mathematical modeling of pedagogical systems.* p. 332.

Vasiliev, V. N., et al. 1997. *On mathematical models of optimal control of the system of training specialists.* p. 136.

Avetisov, A. A., Kamyshnikova, T. V., 1998. *Optimization model for assessing and managing the quality of student training at a university*. pp. 105-109.

Kalganova, N. V., 2021. *Mathematical modeling of the process of sustainable development of the personnel potential of industry universities.* E3S Web of Conferences. p. 296.

Golubeva, N. V., 2016. *Mathematical modeling of systems and processes.* p. 192.

Zhirkov, A. M., 2016. *Mathematical modeling of systems and processes.* p. 192.

Bagrinovsky, K. A., Busygin, V. P., 1980. *Mathematics of planning decisions.* p. 224.

Kalganova, N. V., 2020. Mathematical models of optimal management of the material support of educational and scientific activities of industry universities. *Bulletin of USTU.* 4, pp. 120-126.

Gianetto, K., Wheeler, E., 2005. *Knowledge Management.* p. 192.

Stepanov, V. V., 2006. *Course of differential equations.* p. 472.

Makarov, E. G., 2009. *MathCAD: training course.* p. 384.