The Assessment of the Condition of the Variable Stiffness Sections by Determining the Frequency of Natural Vibrations

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Abstract: The presence of "barrier places" significantly limits the traffic capacity of railway lines. This situation can be improved by increasing speeds and tonnage, but these measures require preliminary strengthening of the existing track infrastructure. The increase in the efficiency of the infrastructure operation depends on the improvement of infrastructure monitoring and diagnostics systems based on modern approaches, which allow to predict the development of pre-failure conditions. This study is devoted to the analysis of the monitoring peculiarities of variable stiffness sections, which are located in front of engineering structures, meanwhile the formulation of the problem allows to take into account the possibility of defects at various stages of the life cycle. The proposed method can be applied to the development of algorithms for monitoring and diagnostics of transitional sections, which is relevant due to the difficulties of visual inspection of such structures and a large range of dynamic loads and stiffness parameters in different seasons.

1 INTRODUCTION

Railway transportation safety is the main requirement in the work of JSC "Russian Railways", which is especially important when there is an increase in train speeds, train weight, axle load (Hess, 2009). Unfortunately, without any significant repairing works, the current condition of the transportation infrastructure does not allow to carry out the measures which increase the traffic capacity of the lines. The length of such sections is more than 10 thousand km (11% of the total length of the railway network of the Russian Federation). The existing regulatory and technical framework contemplates to bring the track infrastructure into the condition of the required level, using modern track structures, which will significantly increase the operational life and reduce the life cycle cost on the sections with high-speed, heavy haul and especially heavy traffic due to the overall repair of the first level tracks (Organization of Railway Cooperation, 2016; Technical standards and requirements for design and construction, 2014; Ivanchenko, 2011).

The main reasons, causing the track deterioration, are the increased dynamic impact on the railway track. This problem is especially acute in the zone of the so-called areas of variable stiffness, where track deformations in the form of shaped depressions occur not only due to over-compaction of the ballast, but of the subgrade as well, and the depth of this embankment layer can reach 2-3 m (Organization of Railway Cooperation, 2016). These areas may also include railway track sections on approaches to engineering structures (bridge crossings, tunnels, through bridges), boundary sections with various track structures (ballast and ballastless) where there are various vertical stiffness parameters of the rail seat. The increase in train speeds and their weight significantly affects the length of deformations along such sections and the intensity of their manifestation. Another negative factor, causing the increased track deformability in the transition zone of bridge crossings is the increased moistening of the floodplain embankment soil, induced by the hydrological regime of rivers, the increase in the groundwater level, poor drainage on the bridge structure (Figure 1) (Matsumoto, 2009; Fryba, 1996; Poliakov, 2017).

Monitoring of the variable stiffness section technical condition plays an important role in the extinction of the entire engineering structure life cycle. A distinctive feature of the current stage in the development of transportation routes is that the improvement of regulatory and legal documentation, which is supposed to extend the life cycle of the entire

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structure and its individual elements, does not often keep up with the change in speed modes and mass-dimensional characteristics of vehicles (Barchenkov, 1976; Bolotin; Kogan, 1997).

2 METHODS

This study is aimed at the development of the mathematical models of the variable stiffness sections and their adaptation to diagnostic needs without imposing restrictions on the facility operation.

The improvement of diagnostic algorithms will minimize the possible rapid development of significant defects on the approaches to the engineering structure and on the structure itself (Chen, 2000; Kawatani, 2001; Pan, 2002), which will optimize repair and restoration processes. This research work proposes to consider the behavior of a variable stiffness section simulated by a plate element and to consider it with the respect to its interaction with other elements of the structure and the environment, as the characteristic, the change of which takes into account the change in the transition section condition (Kou, 1997; Kurbatsky, 2018). It is proposed to use the frequency of natural vibrations. The determination of the structure vibration frequency will allow to identify defects, damages and deviations from the project, to assess the strength and durability (Fryba, 1996; Kurihara, 1978; Loktev, 2012).

The used mathematical model of a flat element allows to take into account the anisotropic properties of the variable stiffness section and to present the design scheme in the form of a plate (Loktev, 2011; Glusberg, 2020; Vinogradov, 2018; Chistyy, 2018), lying on the deformable roadbed, two edges of which are pivotally supported, and the other two edges are rigidly fixed.

3 RESULTS AND DISCUSSION

According to the proposed model, free vibrations of a transversally isotropic plate of the constant thickness, characterized by boundary parameters in a non-deformable condition within limits $\{0 \le x \le l_1; 0 \le y \le l_2; -h \le z \le h\}$.

The approximate equation of transverse vibrations of such a flat element in partial derivatives of the fourth order has the form:

$$A_1 \frac{\partial^2 W}{\partial t^2} + A_2 \frac{\partial^4 W}{\partial t^4} - A_3 \frac{\partial^2}{\partial t^2} \Delta W + A_4 \Delta^2 W + P(W) = 0, \quad (1)$$

where W – the value of the transverse displacement of points in the plate median plane;

Δ – the Laplace operator.

The functional coefficients have the following form

a) b)

Figure 1: Engineering structures: a) a bridge of the combined design scheme, b) an elevated structure for the urban rail transport.

$$\begin{aligned}
&A_1 = \rho_1; A_2 = \rho_1^2\left(A_{33}^{-1} + 3A_{44}^{-1}\right)\frac{h^2}{b};\\
&A_3 = \left\{-\rho_1\left[2 - 2A_{11}A_{33}^{-1} - 3\left(A_{13}^2 - A_{11}A_{33}\right)A_{33}^{-1}A_{44}^{-1}\right]\right\}\frac{h^2}{b};\\
&A_4 = 2A_{33}^{-1}\left(A_{11}A_{33} - A_{13}^2\right)\frac{h^2}{b};\\
&A_5 = \frac{S}{2h}\rho_1; A_6 = \frac{S}{2h}\rho_1\frac{h^2}{2}\left(\rho_1 A_{44}^{-1} + 3A_{33}^{-1}\right);\\
&A_7 = -4\frac{S}{2h}\rho_1 A_{11}A_{33}^{-1};
\end{aligned} \qquad (2)$$

where

$P(W) = A_5\dfrac{\partial W}{\partial t} + A_6\dfrac{\partial^3 W}{\partial t^3} + A_7\Delta\dfrac{\partial W}{\partial t}$ - the support reaction from the subgrade soil;

ρ - the density of the span material,

b – the velocity of the transverse (shear) wave,

$A_{11} = A_{13} = \ldots = A_{nm}$ – anisotropy coefficients (Loktev, 2012; Loktev, 2020; Lyudagovsky, 2018).

The boundary conditions for the problem in such a formulation take the form:

$$\begin{aligned}
&W = \frac{\partial^2 W}{\partial x^2} = 0; x = 0, l_1\\
&W = \frac{\partial W}{\partial y} = 0; y = 0, l_2
\end{aligned} \qquad (3)$$

The solution of the homogeneous equation is presented in the following form:

$$W(x,y,t) = W(x,y)\exp\left(\xi\frac{bt}{h}\right), \qquad (4)$$

here $\xi -$ the desired frequency of the plate natural vibrations

With respect to (4) the equation (1) may be presented in the following form

$$W(x,y)\left(\Delta^2 + B_1\Delta + B_2\right) = 0, \qquad (5)$$

Here the following notations were adopted

$$B_1 = \frac{1}{A_4}\left(-A_3\left(\frac{b\xi}{h}\right)^3 + A_7\left(\frac{b\xi}{h}\right)\right);$$

$$B_2 = \frac{1}{A_4}\left(A_1\left(\frac{b\xi}{h}\right)^2 + A_2\left(\frac{b\xi}{h}\right)^4 + A_5\left(\frac{b\xi}{h}\right) + A_6\left(\frac{b\xi}{h}\right)^3\right)$$

Considering dimensionless coordinates

$$x = \frac{l_1}{\pi}\alpha; \quad y = \frac{l_2}{\pi}\beta; \quad W(x,y) = \frac{l_1^4}{\pi^4}V(\alpha,\beta),$$

$$\eta = \frac{l_1}{l_2};$$

From the equation (5) we obtain the following functional relation

$$V(\alpha,\beta)\left(\begin{aligned}&\left(\frac{\partial^4}{\partial\alpha^4} + 2\eta^2\frac{\partial^4}{\partial\alpha^2\partial\beta^2} + \eta^4\frac{\partial^4}{\partial\beta^4}\right)\\ &+B_1\frac{l_1^2}{\pi^2}\left(\frac{\partial^2}{\partial\alpha^2} + \eta^2\frac{\partial^2}{\partial\beta^2}\right) + B_2\frac{l_1^4}{\pi^4}\end{aligned}\right) = 0; \qquad .(6)$$

The equation (6) can be represented as a set of three auxiliary tasks, each of which has its own relevant function $V(\alpha, \beta)$,

$$\frac{\partial^4 V_1}{\partial\alpha^4} = f_1(\alpha,\beta) \qquad V_1 = \frac{\partial V_1}{\partial\alpha} = 0$$
$$\alpha = 0, \pi$$

$$\eta^4\frac{\partial^4 V_2}{\partial\beta^4} = f_2(\alpha,\beta)$$
$$V_2 = \frac{\partial V_2}{\partial\beta} = 0 \qquad \beta = 0, \pi \qquad (7)$$

$$\left[2\eta^2\frac{\partial^4}{\partial\alpha^2\partial\beta^2} + B_1\frac{l_1^2}{\pi^2}\left(\frac{\partial^2}{\partial\alpha^2} + \eta^2\frac{\partial^2}{\partial\beta^2}\right) + B_2\frac{l_1^4}{\pi^4}\right]V_3$$
$$+ f_1 + f_2 = 0;$$

Due to the applied decomposition method, it can be clearly seen that the following relations are valid for the given points of a flat element

$$V_1 \cong V_2; V_3 = \frac{1}{2}\left(V_1 + V_2\right), \qquad (8)$$

$$\begin{aligned}
&V_1(\alpha,\beta) = \sum_{n,m=1}^{\infty}\frac{a_{n,m}^{(1)}}{n^4}\sin(n\alpha)\sin(m\beta) +\\
&\frac{\alpha^3}{6}\psi_1(\beta) + \frac{\alpha^2}{2}\psi_2(\beta) + \alpha\psi_3(\beta) + \psi_4(\beta);\\
&V_2(\alpha,\beta) = \sum_{n,m=1}^{\infty}\frac{a_{n,m}^{(2)}}{\eta^4 m^4}\sin(n\alpha)\sin(m\beta) +\\
&\frac{\beta^3}{6}\phi_1(\alpha) + \frac{\beta^2}{2}\phi_2(\alpha) + \beta\phi_3(\alpha) + \phi_4(\alpha);
\end{aligned} \qquad (9)$$

here $\psi_i(\beta)$ и $\varphi_i(\alpha)$ arbitrary functions, and $a_{n,m}^{(i)}$ - arbitrary constants, $i = 1,2$.

Substituting the possible solutions to the auxiliary tasks into the expressions of boundary conditions (3), we determine the functions $\psi_i(\beta)$ and $\varphi_i(\alpha)$. After that, the determining relations for auxiliary tasks can be reduced to the system of equations (Matsumoto, 2009; Lyudagovsky, 2018)

$$\begin{cases} c_{11}a_{11}^{(1)} + c_{12}a_{11}^{(2)} = 0 \\ c_{21}a_{11}^{(1)} + c_{22}a_{11}^{(2)} = 0 \end{cases} \qquad (10)$$

in which the following notations are adopted

$$c_{11} = -B_1 \frac{l_1^2}{2\pi^2}\left[1+\eta^2\right] + B_2 \frac{l_1^4}{2\pi^4} + \eta^2 + 1,$$

$$c_{12} = B_1 \frac{l_1^2}{2\pi^2\eta^2}\left[-1+\frac{2}{\pi}-\frac{1}{\eta^2}\left(1-\frac{\pi}{4}\right)\right] + B_2 \frac{l_1^4}{2\pi^4}\frac{1}{\eta^4}\left(1-\frac{\pi}{4}\right) - \frac{1}{\eta^2}\left(-1+\frac{2}{\pi}\right) + 1,$$

$$c_{21} = 1,$$

$$c_{22} = -\frac{1}{\eta^4}\left(1-\frac{\pi}{4}\right).$$

The determining system of equations (10) has a non-zero solution only when the main determinant is equal to zero, revealing which we obtain a characteristic equation in reference to the frequencies of plate natural vibrations

$$d_1\xi^4 + d_2\xi^3 + d_3\xi^2 + d_4\xi + d_5 = 0, \qquad (11)$$

here

$$d_1 = -\frac{A_2}{A_4}\frac{l_1^4}{\pi^4\eta^4}\left(1-\frac{\pi}{4}\right)\left(\frac{b}{h}\right)^4$$

$$d_2 = -\frac{A_6}{A_4}\frac{l_1^4}{\pi^4\eta^4}\left(1-\frac{\pi}{4}\right)\left(\frac{b}{h}\right)^3$$

$$d_3 = \frac{l_1^2}{A_4\pi^2\eta^2}\left\{ A_3\left[\left(\frac{1}{\eta^2}+\frac{1}{2}\right)\left(1-\frac{\pi}{4}\right)+\frac{1}{2}-\frac{1}{\pi}\right] - A_1\frac{l_1^2}{\pi^2\eta^2}\left(1-\frac{\pi}{4}\right)\right\}\left(\frac{b}{h}\right)^2$$

$$d_4 = \frac{l_1^2}{A_4\pi^2\eta^2}\left\{ \frac{A_7}{2}\left[\frac{1}{2}-\frac{1}{\pi}+\frac{3}{2}\left(1-\frac{\pi}{4}\right)\right] - A_5\frac{l_1^2}{\pi^2\eta^2}\left(1-\frac{\pi}{4}\right)\right\}\left(\frac{b}{h}\right)$$

$$d_5 = \frac{1}{\eta^2}\left[\left(-1+\frac{\pi}{4}\right)\left(1+\frac{1}{\eta^2}\right)-1+\frac{2}{\pi}\right]-1$$

When solving the characteristic equation (11), it is possible to obtain natural frequencies, with respect to different values of the mechanical characteristics of the variable stiffness section structures.

4 CONCLUSIONS

The proposed models of a plate element on the soil subgrade can be used for the vibration diagnostics systems of both railways and highways on approaches to engineering structures. They will allow not only to detect and identify defects and deviations from the project, but also to predict the dynamics of changes in the major characteristics, affecting traffic safety, as well as considering the changes in the properties of the subgrade soil due to moistening.

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