A Game-theoretical Model of Buy-back Contracts in Assembly Systems with Uncertain Demand

Zhiqi Lei
Finance major, School of International Education, Wuhan University of Technology, Wuhan, Hubei Province, China

Keywords: Buy-Back Contract, Assembly System, Uncertain Demand, System Coordination.

Abstract: An assembler needs to purchase complete sets of components, each component produced by different suppliers. Each party in the assembly system shares the risks of demand uncertainty through a buy-back contract. In this paper, a game-theoretical model is established to analyze how the buy-back contract is developed under two different mechanisms. The first is one where the assembler sets the buy-back price of each component, and the second is one where the suppliers set the buy-back prices of their own components. In both cases, by backward induction, the decision problem is formulated as a constrained optimization problem. The optimal order quantity is derived. Under the first setting, the system performance is demonstrated to be increasing in the assembler's share of the profit per unit of product. Under the second setting, the system performance is demonstrated to be decreasing in the assembler's share of the profit per unit of product. By comparing the first-order conditions, it is shown that the system performances under the two settings are equal if the assembler's share of profit is larger than the reciprocal of the number of parties in the system. Finally, numerical examples are provided to illustrate some of the main results.

1 INTRODUCTION

The competition between supply chains has become the main mode of market competition in the 21st century. Enterprises in supply chains should cooperate to maximize the overall benefit. Compared with the single enterprise, supply chains are usually faced more demand uncertainties and information asymmetry, which may lead to the inefficiency of supply chain. At the same time, each enterprise in the supply chain pursues the maximization of their own interests, which may conflict with the overall goal of the supply chain in the operation process. It has become a popular research topic to coordinate the supply chain and improve the profit of the whole supply chain through well-designed contracts. Supply chain management is an idea of integration, which emphasizes the close cooperation of the supply chain members. However, the supply chain consists of relatively independent members, whose decision-making power is decentralized and there are conflicts of interest among them. The buy-back contract is a typical coordination mechanism which is widely used in practice to alleviate the inefficiency of decentralized supply chain.

In a buy-back contract, suppliers purchase the product that retailers have not sold out at a specified buy-back price after the selling season. By implementing such a contract, the supplier can provide the retailer a protection, so as to induce the retailer to increase the order quantity. As the risk of demand uncertainty is shared by suppliers and retailers, their revenue and cost are balanced better. While the retailer can benefit from the buy-back mechanism, the supplier can also obtain higher profit from a higher order quantity. Thus, a win-win goal can be achieved.

In the 1980s, some researchers began to study the buy-back contract in the supply chain. Paternackde (Pasternack 1985) studied the coordination of supply chain where a single supplier and a single retailer sell a product, focused on the buy-back contract in the common sales channel, and analyzed the potential inefficiency of operation due to the influence of marginal benefit. Under the assumption that the return price is less than the wholesale price, it is proved that the total profit of the distribution channel is similar to that of the vertically integrated supply chain. His research shows that neither Full Returns policy nor No Returns policy is effective. A compromise buy-back contract can promote supply

Lei, Z.
A Game-theoretical Model of Buy-back Contracts in Assembly Systems with Uncertain Demand.
DOI: 10.5220/0011165400003440
In *Proceedings of the International Conference on Big Data Economy and Digital Management (BDEDM 2022)*, pages 137-144
ISBN: 978-989-758-593-7

chain collaboration and improve collaboration efficiency through Pareto optimization. Emmos and Gilbert (Emmos 1998) and Donohue (Donohue 2000) pointed out that it is beneficial for suppliers and retailers to sign buy-back contracts for their trade. Gong (Gong 2008) showed that the optimal buy-back contract between suppliers and retailers could not always be realized under the condition of information symmetry. In recent years, buy-back contracts have attracted extensive attention from the academic community. The effects and design of buy-back contracts have been studied from different perspectives (Das, 2017, Giri, 2014, Lau, 1999, Padmanabhan, 1995, Webster, 2000, Zhang, 2016, Hou, 2009, Wang, 2008, Xiao, 2008, Xu, 2008, Xu, 2008).

Assembly system is a common operation mode in modern manufacturing industry (Wang 2003). With the popularity of industry subdivision and outsourcing, the supply of components for assembly lines is often not controlled by itself. Thus, it is necessary to coordinate the relationship with suppliers. Due to the characteristics of the assembly line, the components of the final product are complementary. It is necessary to coordinate and manage multiple suppliers at the same time, which may be difficult. However, few papers have studied the buy-back contract in an assembly system.

In the context of uncertain demand, this paper establishes a Stackelberg game-theoretical model for the buy-back contract of assembly system, studies the decisions of all supply chain parties, and analyzes the influence of various parameters on the buy-back contract. By comparing the system performance under two different mechanisms, some managerial insights about the supply chain structure are provided. In order to facilitate the presentation and improve the readability of this paper, the following section first introduces and analyzes the buy-back contract model in a simple one-to-one supply chain.

2 BUY-BACK CONTRACT IN ONE-TO-ONE SYSTEM

Consider the following basic case: the demand for the final product is uncertain, but the probability distribution is known. The price of the product is constant. The product is assembled from a set of components. In order to produce and sell a product, the order quantities of all components need to be determined before the demand is realized.

There is only one supplier and one retailer in the system, and the demand of the product is the random variable D, with probability distribution function $F(\cdot)$, and probability density function $f(\cdot)$. The supplier and retailer use the common demand distribution. The retailer has to order the product before the selling season. There is only one chance to place an order. The unit cost of the product produced by the supplier is c, the wholesale price of the product sold to the retailer is w. The price of the product is p. Only when the profit is positive, the supplier and the retailer can be willing to participate in the game. Thus, $p > w > c$ should be satisfied. After the selling season, the retailer can sell the leftover product to the supplier at a price of b, and one unit of product can be salvaged at a value v by the supplier. Obviously, $v < c$ should hold.

The decision variable is the buy-back price b, then the supplier decides its optimal supply quantity Q_1, and the retailer decides its optimal order quantity Q_2. When the buy-back price b is determined, the optimal order quantity of the retailer and the optimal supply quantity of the supplier can be calculated respectively from the classical newsvendor model (at this time, the case that the optimal supply quantity is infinite can be ignored. The detailed analysis is given in the later part of this paper). Obviously, the actual quantity of the product is the minimum of the optimal supply quantity Q_1 and the optimal order quantity Q_2.

For the supplier, the under-storage cost is $w - c$, and the over-storage cost is $b + c - w - v$. Then the optimal order quantity of the supplier satisfies

$$F(Q_1) = \frac{w-c}{b-v}. \tag{1}$$

As $F(\cdot)$ is an increasing function, both the supplier's expected profit and its optimal supply quantity Q_1 are decreasing in the buy-back price b.

For the retailer, the under-storage cost is $p - w$, and the over-storage cost is $w - b$. Then the retailer's optimal order quantity satisfies

$$F(Q_2) = \frac{p-w}{p-b}. \tag{2}$$

As $F(\cdot)$ is an increasing function, both the retailer's expected profit and its optimal order quantity Q_2 are increasing in the buy-back price b.

2.1 The Retailer's Decision on the Buy-back Price

Now, analyze the principles of buy-back pricing from the perspective of the retailer. Consider the following setting: The retailer decides the buy-back price of the product, and its objective is to maximize its own expected profit.

No matter how the buy-back price is set, the supplier's optimal supply quantity and the retailer's optimal order quantity cannot be infinite at the same time, and at least one of them is finite positive. The actual quantity of the product is the minimum value between the optimal supply quantity Q_1 and the optimal order quantity Q_2. If $Q_2 < Q_1$, it means that the retailer has made a lower buy-back price, but it is not conducive to increase the actual quantity of the product. It can be inferred that the optimal policy of the retailer must satisfy $Q_2 \geq Q_1$. In addition, an intuitive conclusion is $b + c \geq w + v$, because when $b + c < w + v$, the supplier's optimal quantity is infinite.

In short, the supplier will become the bottleneck of both sides, and the final order quantity is decided by the supplier. Then this process can be summarized as follows: The buy-back price is set by the retailer, and the final order quantity is determined by the supplier. From the above discussion, the following conditions can be obtained:

$$\frac{p-w}{p-b} \geq \frac{w-c}{b-v},\ b + c \geq w + v. \tag{3}$$

The final quantity of the product is Q. As the conditions of $Q = min(Q_1, Q_2) = Q_1$ and $F(Q) = \frac{w-c}{b-v}$ holds, the retailer's profit is

$$\pi(Q) = E[-wp + pmin(Q, D) + b(Q - D)^+]. \tag{4}$$

The above problem can be summarized as a constrained optimization problem as follows:

$$max\pi_1(Q) = (p-w)Q - (p-b)\int_0^Q F(x)dx \tag{5}$$

$$s.t.\begin{cases} F(Q_1) = \dfrac{w-c}{b-v} \\ \dfrac{p-w}{p-b} \geq \dfrac{w-c}{b-v}, b + c \geq w + v \end{cases}$$

The solution and discussion of these equations are carried out in Section 3.

2.2 The Supplier's Decision on the Buy-back Price

Now, analyze the principles of buy-back pricing from the perspective of the supplier. Consider the following setting: The supplier decides the buy-back price of the product to maximize its own expected profit.

The principle is the same as that described in the previous subsection. When the supplier decides the buy-back price, the supplier will set the buy-back price low enough to maximize its own profit. If $Q_2 > Q_1$, the buy-back price is too high, because it is not conducive to increase the retailer's order quantity. In short, the final quantity of the product is decided by the retailer. Then this process can be summarized as follows: The buy-back price is determined by the supplier, and the final quantity of the product is determined by the retailer.

The actual quantity of the product is the minimum between the supplier's optimal supply quantity Q_1 and the retailer's optimal order quantity Q_2. The retailer's optimal order quantity is infinite if $b > w$. From the above discussion, the following conditions can be obtained:

$$\frac{p-w}{p-b} \leq \frac{w-c}{b-v},\ b + c \leq w + v. \tag{6}$$

The final quantity of the product is Q. As $Q = Q_2$, it can be inferred that $F(Q) = \frac{p-w}{p-b}$. The supplier's profit is

$$\pi_2(Q) = E[(w-c)Q - (b-v)(Q-D)^+]. \tag{7}$$

The above problem can be summarized as a constrained optimization problem as follows:

$$max\pi_2(Q) = (w-c)Q - (b-v)\int_0^Q F(x)dx \tag{8}$$

$$s.t.\begin{cases} F(Q) = \dfrac{p-w}{p-b} \\ \dfrac{p-w}{p-b} \leq \dfrac{w-c}{b-v}, \quad b + c \leq w + v \end{cases}$$

The solution and discussion of these equations will also be carried out in Section 3.

3 BUY-BACK CONTRACT FOR ASSEMBLY SYSTEM

The assembler has to buy the components before the actual demand is known. Due to the uncertainty of demand, the assembler need to make decisions based on demand prediction. The output of the assembly system is limited to each link, and the output capacity of the system is equal to the weakest link. In addition, in order to deal with the risk of demand uncertainty, the supply chain members may sign buy-back contracts to share the risk. The benefit of this contract is to reduce the risk downstream of the supply chain, encourage them to increase their order quantities, and thereby increase the overall profit of the system. Due to the complementarity of components, designing the buy-back contract of an assembly system is relatively complicated.

The demand for the final product of the assembly system is a random variable *D*. The probability distribution function of D is $F(\cdot)$, and the probability density function is $f(\cdot)$. The unit price of the product in the market is *p*. The product consists of *n* components. Without loss of generality, suppose that each supplier produces one of these *n* components. For notational convenience, define $N = \{1,2,\dots,n\}$. All supply chain parties have to decide the order quantity or supply quantity before the selling season. Once the components are in place, the demand is realized and the product can be assembled in a short time.

The unit cost of component *i* is c_i. Considering the assembly process, the production of a final product also needs to invest c_0 as the assembly cost. Obviously, $\sum_{i=0}^{n} c_i < p$ is required to make sure that the profit is positive. In fact, c_0 can be set equal to zero, and then the market price of the product can be adjusted to be $p - c_0$. The wholesale price of component i is w_i, then $p - \sum_{i=1}^{n} w_i$ is the assembler's profit from one unit of the product. To ensure that each member does not refuse to participate in the game, $w_i > c_i$ is required. Because the demand is random and the ordering decisions need to be made before demand realization, overstocking or understocking can occur. The unite salvage value of component *i* is $v_i (v_i < c_i)$.

The decision variable is the buy-back price b_i of each component. Each component supplier shall decide its supply quantity Q_i according to the buy-back price, and the assembler shall decide the order quantity Q_0.

The sum of the corresponding parameters is in uppercase letters for marking convenience. For example, define

$$C \equiv \sum_{i=1}^{n} c_i, \qquad B \equiv \sum_{i=1}^{n} b_i, \tag{9}$$

$$W \equiv \sum_{i=1}^{n} w_i, \qquad V \equiv \sum_{i=1}^{n} v_i,$$

Suppliers and retailers use the same demand distribution. When the buy-back price $b_i, i \in N$ is determined, the optimal order quantity (supply quantity) of each supply chain party can be calculated from the classical newsvendor model. The final quantity of the product and components should be as follows: The quantity of each component is the same, and the actual quantity Q of the product is the minimum among the optimal supply quantity $Q_i (i = 1,2,\dots n)$ and the optimal order quantity Q_0. That is to say, $Q = min\{Q_i\}$.

The profit of the system is

$$\pi(Q) = E[-CQ + pmin(Q,D) + V(Q - D)^+], \tag{10}$$

which can be written as

$$\pi(Q) = (p - C)Q - (p - V)\int_0^Q F(x)dx. \tag{11}$$

According to the classical newsvendor model, the above profit function is concave and has a unique optimal solution. This property can help compare the effectiveness of different mechanisms.

3.1 A Contract Model in Which the Assembler Determines the Buy-back Price

Now, from the assembler's point of view to analyze the principle of buy-back price formulation. Consider this situation: the assembler decides the buy-back price of the product, and the assembler's goal in setting the buy-back price is to maximize its own profit.

Facing *n* suppliers, the assembler formulates the buy-back price b_i of each component $i \in N$. The optimal strategy of the assembler to formulate the buy-back price should meet the following conditions:

$$Q_0 \geq Q_1 = Q_2 = \cdots = Q_n;$$

$$F(Q_0) = \frac{p-W}{p-B}, \; b_i + c_i \geq w_i + v_i, i \epsilon N;$$

$$\frac{p-W}{p-B} \geq \frac{W-C}{B-V};$$

The supplier's optimal policy must satisfy $Q_0 \geq Q_i, i \in N$. If there exists i which makes $Q_0 < Q_i$, then it means that the assembler has set a too low buy-back price, which is not conducive to increase the order quantity. In addition, when the buy-back price is determined, the optimal supply quantity of each component can be obtained immediately according to the classical newsvendor model. It is useless for one supplier to have the optimal supply quantity higher than others, which is a waste to the assembler, and the buy-back price of this component must be increased. Thus, the optimal supply quantity of each supplier should be the same. When the sum of the buy-back prices of each supplier is fixed, the optimal policy is the one that maximizes the output of the system. Therefore, the optimal strategy must satisfy $Q_0 \geq Q_1 = Q_2 = \cdots = Q_n$. In order to ensure that the optimal supply quantity of each supplier is limited, $b_i + c_i > w_i + v_i$ should hold for all $i \epsilon N$. The intuitive explanation is that suppliers will pay cost when their production exceeds the actual demand.

According to the above analysis, the final quantity of the product is determined by the supplier. Then the process of the assembler-as-the-leader buy-back contract can be summarized as follows: the buy-back price is determined by the assembler, and then the optimal order quantity is determined by all of them, and finally the output of the system is determined by the suppliers.

For supplier *i*, the under-storage cost is $w_i + c_i$, and the over-storage cost is $b_i + c_i - w_i - v_i$. Then the supplier's optimal order quantity satisfies

$$F(Q_i) = \frac{w_i - c_i}{b_i - v_i}. \tag{12}$$

It follows that

$$\frac{w_1 - c_1}{b_1 - v_1} = \frac{w_2 - c_2}{b_2 - v_2} = \cdots = \frac{w_n - c_n}{b_n - v_n} = \frac{W - C}{B - V} \tag{13}$$

As $F(\cdot)$ is an increasing function, the larger the buy-back price b_i is, the smaller the optimal supply quantity Q_i and the supplier's expected profit are.

For the assembler, the under-storage cost is $p - W$, and the over-storage cost is $W - B$. Then the optimal order quantity of the assembler satisfies

$$F(Q_0) = \frac{p - W}{p - B}. \tag{14}$$

As $F(\cdot)$ is an increasing function, the larger the buy-back price B is, the larger the optimal order quantity Q_0 and the expected profit of the assembler are.

According to the above discussion, the following conditions can be obtained:

$$\frac{p - W}{p - B} \geq \frac{W - C}{B - V}, b_i + c_i > w_i + v_i, i \epsilon N. \tag{15}$$

The final quantity of the product is $Q = min\{Q_i\} = Q_1$, then $F(Q) = \frac{W-C}{B-V}$.

The profit function of assemblers is

$$\pi_0(Q) = E[-WQ + pmin(Q, D) + B(Q - D)^+]. \tag{16}$$

The expression of the profit function is

$$\pi_0(Q) = (p - \mathrm{W})Q - (p - \mathrm{B}) \int_0^Q F(x)dx. \tag{17}$$

where $F(Q) = min\left\{\frac{p-W}{p-B}, \frac{W-C}{B-V}\right\}$.

It follows that $\pi_0(Q)$ is continuously differentiable at the point B=$\frac{(p-V)(W-C)}{p-W} + V$, which is equivalent to $\frac{p-W}{p-B} = \frac{W-C}{B-V}$. However, the previous analysis has shown that the optimal value of B should satisfy $\frac{p-W}{p-B} \geq \frac{W-C}{B-V}$. Thus, the problem can be simplified by narrowing down the feasible region. The above problem can be summarized as a constrained optimization problem:

$$max\pi_0(Q) = (p - W)Q - (p - B) \int_0^Q F(x)dx \tag{18}$$

$$s.t. \begin{cases} F(Q) = \dfrac{W - C}{B - V} \\ \dfrac{p - W}{p - B} \geq \dfrac{W - C}{B - V}, \quad B + C \geq W + V \end{cases}$$

Here, the inequality constraint $B + C \geq W + V$ can be omitted because it can be inferred from the first equality constraint as follows:

$$\frac{W - C}{B - V} = F(Q) \leq 1. \tag{19}$$

The derivative of $\pi_0(Q)$ with respect to Q is

$$\frac{d\pi_0(Q)}{dQ} = p - C - (p - V)F(Q) - (W - C)\varphi(Q). \tag{20}$$

Where

$$\varphi(Q) = \frac{f(Q)}{[F(Q)]^2} \int_0^Q F(x)dx. \tag{21}$$

If $\frac{d\pi_0(Q)}{dQ}$ is a decreasing function, then the profit function $\pi_0(Q)$ of the assembler is concave. The following properties can also be obtained from the above derivative function.

Theorem 1. If the buy-back price is determined by the assembler, then

- The output Q of the system and the overall profit $\pi(Q)$ of the system are not affected by the number of suppliers. If C, W, V are fixed, the specific parameter of each supplier does not affect Q and $\pi(Q)$;
- $\pi(Q)$ and Q are increasing in w_i and v_i, decreasing in c_i;

- When the marginal sales profit $p-C$ of the product is fixed, the system output Q and profit $\pi(Q)$ decrease with $W-C$, the profit obtained by the supplier.

The practical implications of these properties will be discussed in the next section. In addition, it can be inferred that the inequality condition $\frac{p-W}{p-B} \geq \frac{W-C}{B-V}$ can be ignored in the process of finding the optimal solution if $\frac{d\pi_0(Q)}{dQ}$ is a decreasing function, because the stationary point of the objective function must satisfy this inequality. This can help to simplify the process of solving the optimization problem.

3.2 A Contract Model in Which the Suppliers Determine the Buy-back Prices

Now, analyze the principle of buy-back price from the supplier's perspective. Consider this following setting: The suppliers decide the buy-back prices of the components to maximize their own profits.

After the n suppliers determine their buy-back prices respectively, the optimal supply quantity of each supplier is determined. Then the assembler determines the order quantity, which is no higher than each of the supplier's optimal supply quantity. The optimal policy for the suppliers to set the buy-back prices should satisfy the following properties:

- $b_i \leq w_i; i\epsilon N$;
- $Q_0 \leq Q_i; i\epsilon N$;
- $\frac{p-W}{p-B} \leq \frac{W-C}{B-V}, F(Q) = \frac{p-W}{p-B}$.

Each supplier will set a low enough buy-back price so that the assembler's order quantity is no higher than the supplier's optimal supply quantity. That is to say, $Q_0 \leq Q_i, i \in N$. If this condition does not hold, the supplier will decrease the buy-back price so as to reduce its own risk without affecting its supply quantity. This can be summarized as the following conditions:

$$\frac{p-W}{p-B} \leq \frac{w_i - c_i}{b_i - v_i}, i \in N. \tag{22}$$

According to previous analysis, $Q = min\{Q_i\} = Q_0$. The supplier's profit is

$$\pi_i(Q) = E[(w_i - c_i)Q - (b_i - v_i)(Q-D)^+]. \tag{23}$$

Then the problem can be expressed as the following constrained optimization problem:

$$max\pi_1(Q) = (w_i - c_i)Q - (b_i - v_i)\int_0^Q F(x)dx \tag{24}$$

$$s.t. \begin{cases} F(Q) = \dfrac{p-W}{p-B} \\ \dfrac{p-W}{p-B} \leq \dfrac{w_i - c_i}{b_i - v_i}, b_i + c_i \geq w_i + v_i, i \in N \end{cases}$$

The value of Q in the above formula depends on all b_i values, i.e., $F(Q) = \frac{W-C}{B-V} = \frac{W-C}{\sum_{i=1}^n b_i - V}$, which makes the problem difficult to solve. The derivative of the profit function $\pi_i(Q)$ is

$$\frac{d\pi_i(Q)}{dQ} = w_i - c_i - (b_i - v_i)F(Q) - (p-W)\varphi(Q). \tag{25}$$

From the above equation, it can be concluded that the output of the system satisfies

$$p - C - (p-V)F(Q) - n(p-W)\varphi(Q) = 0. \tag{26}$$

Theorem 2. If the buy-back price is determined by the suppliers, then

- When the parameters *C, W, V* are fixed, $p(Q)$ and Q have nothing to do with the specific parameters of each supplier;
- $\pi(Q)$ and Q are increasing in w_i, v_i, but decreasing in c_i and the number of suppliers n;
- When the unit sales profit $p-C$ is fixed, $\pi(Q)$ and Q decrease with $p-W$.

As in the assembler-led case, it can be inferred that if $\frac{d\pi_i(Q)}{dQ}$ is a decreasing function, the optimal solution must satisfy the inequality $\frac{p-W}{p-B} \leq \frac{w_i - c_i}{b_i - v_i}$. Otherwise, it is not optimal. This property can help to simplify the solution process.

The results in Theorem 2 and Theorem 1 are very different. In the case that the assembler decides the buy-back prices, the final quantity of the product has nothing to do with the number of suppliers, and the profit proportion of the assembler plays a positive role in the system performance. In the case that suppliers decide the buy-back prices, both the number of suppliers and the profit proportion of the assembler have negative effects on the system performance.

4 PERFORMANCE ANALYSIS

Sections 2 and 3 discuss two determination mechanisms of buy-back prices in a decentralized assembly system and get some results. This section further compares the system performance (the overall profit of the system) in different mechanisms.

4.1 Performance Analysis of Decentralized and Centralized Systems

Assume that the parameters of an assembly system are given, the profit of the system can be calculated in both decentralized and centralized cases. An intuitive conjecture is that the profits of centralized systems are higher than those of decentralized systems. Next, some analysis is provided to support this conjecture.

From the classical newsvendor model, the optimal output of the centralized system satisfies $F(Q)=\frac{p-C}{p-V}$. It should also be pointed out that the profit function of the classical newsvendor model is concave. Thus, the profit function is increasing on the left side of the optimal solution, and decreasing on the right side of the optimal solution.

In the case that the assembler decides the buy-back price, the constraint condition (18) implies several intuitive facts. At the critical point $\frac{p-W}{p-B}=\frac{W-C}{B-V}$, the system output satisfies $F(Q)=\frac{p-C}{p-V}$, which is the same as that of the centralized system. However, by substituting $\frac{p-W}{p-B}=\frac{W-C}{B-V}$ into the derivative function, it can be obtained that

$$\frac{d\pi_0(Q)}{dQ}=-(W-C)\varphi(Q)<0. \tag{27}$$

The optimality condition of centralized system does not hold. It is easy to know that the system output is lower than the case of centralized system.

In the case where the supplier decides the buy-back prices, the constraint condition (24) implies that

$$F(Q)=\frac{p-W}{p-B}\leq\frac{W-C}{B-V}. \tag{28}$$

At the critical point $\frac{p-W}{p-B}=\frac{W-C}{B-V}$, $F(Q)=\frac{p-C}{p-V}$ holds but the optimality condition (26) is violated.

According to the concavity of the profit function, it can be known that the system performance of the two decentralized system is lower than that of the centralized system.

4.2 Performance Comparison of the Two Decentralized System

When the supplier decides the buy-back price of each component, the optimal output Q of the system solves (26). When the assembler decides the buy-back price of each component, the optimal output Q of the system satisfies

$$p-C-(p-V)F(Q)-(W-C)\varphi(Q)=0. \tag{29}$$

As described in the previous theorem, in both mechanisms, the parameters of the system will affect the final performance of the system. It can be seen that (26) and (29) are very similar, except that only one coefficient is different, i.e., $n(p-W)$ and $(W-C)$.

According to Theorems 1 and 2, the output Q and the performance $\pi(Q)$ are the same in the two decentralized systems only if $n(p-V)=W-C$ holds. This equality is equivalent to $(n+1)(p-W)=p-C$.

Let $\delta=\frac{p-W}{p-C}$. If and only if $\delta(n+1)=1$, the system performances in the two cases are the same. If $\delta(n+1)>1$ holds, the system performance is better in the case that the assembler decides the buy-back prices. On the contrary, if $\delta(n+1)<1$ holds, the system performance is better in the case that the suppliers decide the buy-back prices. δ is a threshold value of the assembly system, to determine which mechanism is better for the decentralized system.

In the case that the assembler decides the buy-back prices, the output Q and expected profit $\pi(Q)$ of the system are increasing in δ, and not affected by the number of suppliers. In the case that suppliers decide the buy-back prices, the output Q and expected profit $\pi(Q)$ of the system are decreasing in l, and the number of suppliers.

The parameter δ can be interpreted as the proportion of the unit sales profit owned by the assembler. The above results can be intuitively understood as follows: When the assembler has a strong market position (strong ability to obtain profits), the whole system will benefit from the assembler's dominant role in the negotiation of buy-back prices. On the contrary, when the suppliers are strong, the whole system will benefit from the suppliers' dominant role in the negotiation of buy-back prices.

4.3 Numerical Example

Here is a simple numerical example to illustrate the results in the previous subsections. Let $f(x)=2x$ and $F(x)=x^2$, $x\in[0,1]$. The other parameters are as follow: $p-C=100$, $p-W=50$, $p-V=150$. Then $W-C=50, \delta=\frac{1}{2}$.

According to $F(Q)=\frac{p-C}{p-V}$, the optimal output of the centralized system is $Q=0.816$. In the decentralized system where the buy-back prices are determined by the assembler, the output of the system satisfies

$$\frac{d\pi_0(Q)}{dQ} = \frac{250}{3} - 300Q = 0. \tag{30}$$

The solution is $Q = 0.278$. It is obvious that the system output and system profit are lower than the centralized system. In the decentralized system where the supplier sets the buy-back prices, the output of the system satisfies

$$100 - \frac{50}{3}n - 300Q = 0. \tag{31}$$

Take $n = 2$. The solution is $Q = 0.222$, which is lower than 0.278. In fact, $\delta(n+1) > 1$ holds in the above example. Thus, the system profit is higher when the assembler decides the buy-back prices. In addition, (31) can be written as follows:

$$Q = \frac{1}{3} - \frac{1}{18}n. \tag{32}$$

Obviously, Q is a decreasing function of n, and so is the profit of the system. This is consistent with the theoretical result in the previous section.

5 CONCLUSIONS

The purpose of this paper is to explore the principle of designing the buy-back contract for the assembly system. Between the supply chain members, cooperation and confrontation coexist. The key feature of the assembly system is that the components are complementary. In this context, two different buy-back pricing mechanisms are studied, and the influence of various parameters on the system performance is analyzed. By comparison, a critical condition about the proportion of profit is provided to identify which mechanism is more beneficial to the whole system. It is shown that the two mechanisms will lead to the same system performance only when the proportion of profit owned to the assembler is equal to the number of members in the system.

The model in this paper is not without limitation. In order to facilitate the analysis, only two extreme cases are considered: The buy-back prices of all components are determined by either the assembler or the suppliers. In reality, the buy-back prices may be set partly by the assembler and partly by the suppliers. This is a very complicated case which may be worth further exploration.

REFERENCES

Das D. (2017). An approach to improve supply chain profit through buy-back contract with reference to a book publishing firm. J. *International Journal of Logistics Systems and Management*. 28(3), 338-354.

Donohue K L. (2000). Efficient supply contracts for fashion goods with forecast updating and two production modes. J. *Management Science*. 46(11), 1397-1411.

Emmos H, Gilbert S M. (1998). The role of returns policies in pricing and inventory decisions for catalogue goods. J. *Management Science*. 44(2), 276-283.

Giri B C, Bardhan S. (2014). Coordinating a supply chain with backup supplier through buyback contract under supply disruption and uncertain demand. J. *International Journal of Systems Science: Operations & Logistics*. 1(4), 193-204.

Gong Q. (2008). Optimal buy-back contracts with asymmetric information. J. *International Journal of Management and Marketing Research*. 1(1), 23-47.

Hou Y, Sun X, Wang L. (2009). Buyback contract of supply chain with supplier's sales effort. J. *Science-Technology and Management*. 11(6), 21-23.

Lau H S, Lau A H L. (1999). Manufacturer's pricing strategy and return policy for a single-period commodity. J. *European Journal of Operational Research*. 116(2). 291-304.

Padmanabhan V, Png I P L. (1995). Returns policies: Make money by making good. J. Sloan Management Review. 37, 65-65.

Pasternack B A. (1985). Optimal pricing and return policies for perishable commodities. J. *Marketing Science*. 4(2), 166-176.

Wang J, Wang Q, Sang S. (2008). Buyback policy in supply chain under stochastic demand. J. *Operations Research and Management Science*. 17(3), 164-167.

Wang Y, Gerchak Y. (2008). Capacity games in assembly systems with uncertain demand. J. *Manufacturing & Service Operations Management*. 5(3), 252-267.

Webster S, Weng Z K. (2000). A risk-free perishable item returns policy. J. *Manufacturing & Service Operations Management*. 2(1), 100-106.

Xiao Y, Wang X. (2008). Analysis on cordination and risk sharing of supply chain based on buyback contract. J. *Control and Decision*. 23(8), 905-909.

Xu T, Zhao Z, Nie Q. (2000). Study on profit incentive of supplier and demander under buy-back contract. J. *Science & Technology Progress and Policy*. 25(6), 48-51.

Xu Z, Zhu D, Zhu W. (2008). Supply chain coordination under buy-back contract with sales effort effects. J. *Systems Engineering - Theory & Practice*. 2008(4), 1-11.

Zhang Y, Donohue K, Cui T H. (2016). Contract preferences and performance for the loss-averse supplier: Buyback vs. revenue sharing. J. *Management Science*. 62(6), 1734-1754.