Digital Learning Space to Improve the Conceptual Understanding of Mathematics of non-Mathematical Specialties Students

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Abstract: The article considers the problem of improving the conceptual understanding of mathematics of non-mathematical specialties students by means of digital learning space. The results of a survey of students on the effectiveness of their mathematical training are presented. It is established that most often students use procedural rather than conceptual knowledge, which complicates the application of mathematical methods on practice. As a result of the analysis of the most common strategies for teaching mathematics, the author's vision of teaching mathematics using digital tools is presented. Such teaching is aimed at forming students' conceptual mathematical knowledge, which is harmoniously combined with procedural knowledge. A learning space has been developed to study the topic "Derivative", where you can explore mathematical concepts and relationships between them, formulate hypotheses, experiment, ask questions, draw conclusions and discuss the results. One of the developed educational spaces is described, namely for studying the topic "Derivative". In it, you can explore mathematical concepts and relationships between them, formulate hypotheses, experiment, ask questions, draw conclusions, and discuss the results obtained. Digital tools are built into the space, which allow to implement different types of educational activities of students (individual, group and frontal) and thus ensure the effectiveness of the formation of conceptual knowledge about the derivative. On the example of studying Lagrange's theorem on finite increments and solving various applied mathematical problems, methods of forming a conceptual understanding of students' mathematics are given.

1 INTRODUCTION

In modern high-tech and digitalized society, the main trend in the development of mathematics is the process of its penetration into various sciences – the mathematization of science, which, in turn, leads to mathematization and computerization of human practice. In these conditions, a competent specialist must know the mathematical methods and models used in his profession, be able to use them, be able to use appropriate computer programs to develop new models for his/her professional needs. That is, mathematical training is a necessary and integral part of the training of highly qualified specialists at all levels of higher education.

Mathematical knowledge is dual: it is knowledge of "what" and "why" (conceptual knowledge) and knowledge of "how" (procedural knowledge). Therefore, learning mathematics will be successful if this process is aimed at understanding concepts and mastering procedures.

Many works are devoted to the definition of the content of the concepts "conceptual knowledge" and "procedural knowledge", their relationships, theoretical and methodological principles of formation (Hiebert and Lefevre, 1986; Cobb, 1988; Byrnes and Wasik, 1991). Based on them, it can be argued that conceptual knowledge involves, in addition to knowledge of concepts, facts, methods, understanding of the relationships and interdependencies between them; the ability to see the key idea of a method, to assess in what contexts it may be useful; find different solutions to one problem; analyze and evaluate the obtained result.

Procedural knowledge involves a number of steps that must be performed to solve the problem. Proce-

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dural knowledge includes knowledge of algorithms, techniques and methods.

There are different views on the causal relationship between conceptual and procedural knowledge (Byrnes and Wasik, 1991; Rittle-Johnson et al., 2001; Forrester and Chinnappan, 2010). Most researchers believe that the effectiveness of learning is higher if conceptual knowledge precedes procedural and its knowledge is based on conceptual (Balka et al., 2012; Nahdi and Jatisunda, 2020; Forrester and Chinnappan, 2010). For example, Forrester and Chinnappan (Forrester and Chinnappan, 2010) found in testing students the dominance of their procedural skills over conceptual knowledge. All common errors were related to incorrect application of procedures due to misunderstanding of their conceptual basis.

It should be noted that the purpose of mathematical training of students of non-mathematical specialties is not to master the knowledge of higher mathematics as a value that is important in itself. The goal (and at the same time the motivation to study mathematical disciplines) is the opportunity and ability to apply mathematics in his/her field. Therefore, questions arise: 1) how do different types of knowledge (conceptual and procedural) affect this ability? 2) what should be the educational strategy aimed at forming in students of non-mathematical specialties conceptual mathematical knowledge that is harmoniously combined with procedural knowledge?

There are many methodological techniques that allow you to implement this pedagogical task. But the most effective, in our opinion, is research-oriented teaching of mathematics using digital technologies (Leung, 2006; Smetana and Bell, 2012; Astafieva et al., 2020). It involves active interaction between teacher and student, which includes identifying problems, joint search for solutions, research, discussion, consideration of alternatives, rethinking and evaluating the result. On the other hand, the use of ICT provides active communication between teachers and students, their independent research work (both individual and group), competent and effective organization of educational space and its management.

The aim of research – development and theoretical substantiation of some methodical methods of using digital learning space to improve the conceptual understanding of mathematics of non-mathematical specialties students.

Research methods:

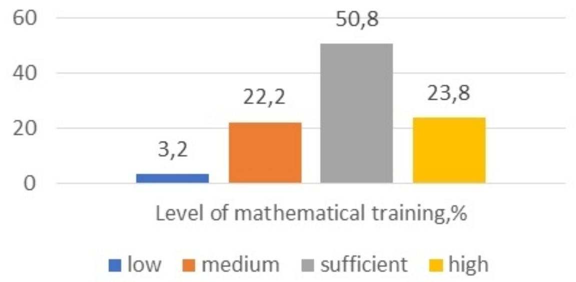
- theoretical – analysis, synthesis, systematization and generalization of scientific, methodological literature, to determine the impact of conceptual and procedural mathematical knowledge on the ability to apply mathematics and study educational strategies of mathematical training of non-mathematical specialties, definition of conceptual and categorical research apparatus (“conceptual knowledge”, “procedural knowledge”, “educational and research spaces”); generalization of progressive ideas and existing shortcomings in modern higher education to justify ways to improve the mathematical training of students by means of digital technologies that allow active learning, in particular, to conduct experiments, research and modeling;
- empirical: surveys to determine the effectiveness of mathematical training of non-mathematical specialties students.

2 RESULTS AND DISCUSSION

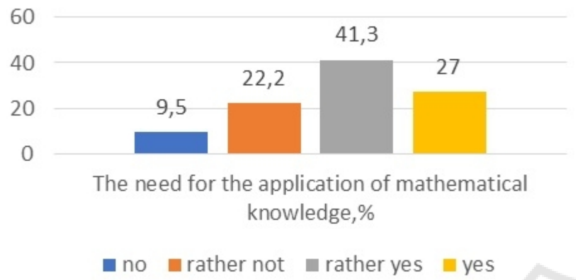
Borys Grinchenko Kyiv University trains, among others, specialists in “Computer Science”, “Management”, “Economics”, “Finance, Banking and Insurance”. These educational programs include the study of higher mathematics. In June 2022, we conducted a survey of students of these specialties on the effectiveness of their mathematical training. A total of 63 first-fourth year students took part in the survey. All students took a course in higher mathematics and had the opportunity to use the acquired mathematical knowledge in practice in the process of studying professional disciplines.

Firstly, we asked students to assess the general level of their mathematical training after studying the discipline “Higher Mathematics” (on a 4-point scale: 1 – low, 2 – medium, 3 – sufficient, 4 – high). It turned out that the vast majority of students (74.6% of respondents) believe that they have a sufficient and high level of mathematical training, 22.2% – medium, 3.2% – low. Interestingly, the answers of junior students (I – II courses) were not fundamentally different from the answers of senior students (III – IV courses). Next, we found out if they feel the need to apply mathematical knowledge in the study of professional disciplines. The majority of students, 68.3%, answered in the affirmative – “yes” and “rather yes”, 22.2% of respondents – “rather not”, 9.5% of students do not feel the need to apply mathematical knowledge in the study of professional disciplines.

We also tried to determine whether students had to apply mathematical knowledge in the study of professional disciplines (on a 4-point scale). 68.3% of respondents answered that they use mathematical knowledge in the study of professional disciplines, 23.8% – rather no, 7.9% – no. Figure 1 shows the results of students’ survey.



(a)



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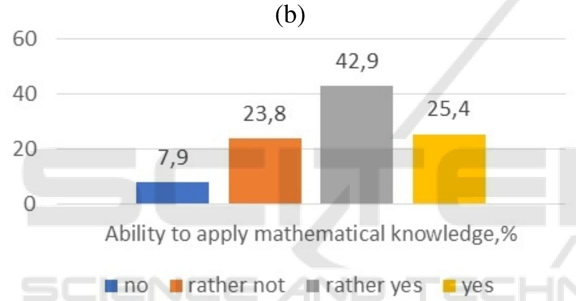

(c)

Figure 1: The results of students' survey.

We compared the available empirical distributions using Pearson's test χ^2. It turned out that $\chi^2_{emp} = 1.631$ at $\chi^2_{cr} = 12.562$ (at $p = 0.05$). Thus, we can say that there is a certain relationship between the level of mathematical training of students (figure 1a), a sense of their need (figure 1b) and the real opportunity to apply mathematical knowledge on practice (figure 1c). For example, 25.4% of students do not appreciate the level of their own mathematical training, while 31.7% of respondents do not see a special need and opportunity to apply mathematical knowledge.

One of the key mathematical concepts is the concept of "derivative of a function". We asked students to comment on the essence of this concept. As a result of self-assessment it was found that only 12.7% of students believe that they remember the definition well, understand the essence of this concept and use it in the study of professional disciplines, 34.9% – remember the definition well, understand the essence of the concept, but do not use it. study of professional disciplines. The majority of students (52.4%) stated that they had some information about the derivative function or did not remember anything about it.

We decided to check the obtained results and asked students to choose the correct definition of the derivative function from the proposed options. The majority of students (34 people or 54.0%) chose the wrong answers. It should be noted that among them 14 people (22.2%), answering the previous question, thought that they well understood the essence of the concept of "derivative function".

To test the conceptual understanding of the concept of a derivative, we also asked students to explain in one word what a derivative is. Options were proposed: speed, productivity, tangent, consequence, area, work, mass. The correct options were chosen: "speed" – 46%; "productivity" – 17.5% of students. The majority of respondents (57.1%) chose the "tangent" option, which indicates a lack of conceptual knowledge, first of all, an understanding of the essence of the concept of a derivative (a derivative is not a tangent, but an angular coefficient of a tangent). Another number of students chose other wrong options: consequence (28.6% of respondents), area (11.1%), work (4.8%), mass (1.6%). It can be assumed that the reason for the incorrect answer was mainly the formal following of a certain algorithm for using the derivative in solving applied problems. In our opinion, students generally remember the rules of differentiation, can use the table of derivatives, but do not feel the main thing – the essence of the concept of "derivative function". Thus, most of them are not able to use this concept in non-mathematical contexts. Interestingly, among the 29 students who recognized the correct definition of the derivative function from the proposed ones, there were only two in which the derivative was associated with both speed and productivity, and they did not choose the wrong options. And this indicates the formality of knowledge and lack of conceptual understanding of the concept of derivative in the remaining 27 students.

The results of the survey, as well as the real experience of teaching mathematical disciplines to the authors of the study showed that students of all specialties to some extent feel the need to apply mathematical knowledge in the study of professional disciplines, as well as use this knowledge in practice. At the same time, most students are more likely to use procedural rather than conceptual knowledge. In particular, students work mechanically according to known algorithms, schemes or rules, but have a poor understanding of the essence of mathematical concepts, facts, methods and tools.

The results of our survey are close to the results of the study of Aydın and Özgeldi (Aydın and Özgeldi, 2019). The authors of this study, analyzing the success of prospective elementary mathematics teachers in solving PISA-2012 problems, found that few students were able to give "mathematical explanations for conceptual knowledge items, and that their contextual knowledge was fragmented" (in this article the authors understand contextual knowledge those that allow you to connect the real world – the context in which the problem arises, with mathematics (Sáenz, 2009)). In addition, students had particular difficulty with tasks that required a combination of conceptual and procedural knowledge.

Thus, we approached the main question: what teaching of mathematics to non-mathematical specialties students will be effective, how to teach that the acquired mathematical knowledge became useful in the future professional activity?

There are two extremes in teaching mathematics to non-mathematical specialties students. The first is the desire for formal rigor of proofs of theoretical facts, formulations of definitions. Teachers are supporters of this point of view, pay too much attention to strict deductive reasoning. They, striving for formal rigor (which is still not achieved due to the limited time allocated in the educational programs of non-mathematicians to study mathematical disciplines), often have little concern about how this material will "work" in the future to solve problems that lie outside mathematics.

The second extreme is a minimum of attention to theoretical justification, learning algorithms, solving typical problems according to the instructions, the use of certain formulas, and etc. Teachers are apologists for this point of view, believing that they teach applied mathematics, do not attach due importance to the justification of certain actions, clarifying the limits of the method, and etc. Such training critically limits the ability of the future specialist to use mathematics as a tool, because the mechanical implementation of certain procedures can lead to unreasonable and incorrect decisions.

In both cases, students do not develop the ability to use mathematics to solve applied problems, which is the purpose of teaching mathematics to non-mathematical specialties students.

In our opinion, the course of higher mathematics (or a block of mathematical disciplines) for non-mathematical specialties students should have an applied orientation. But it should not be narrowly utilitarian and prescription, because applied mathematics is not a simplified version of "pure" mathematics. Courses of mathematical disciplines should be based on the necessary theoretical concepts: key mathematical concepts, facts, a set of ideas and methods that have a wide range of applications. For mathematics to be applied, deep knowledge of the essence of mathematical concepts, facts, methods is required, i.e. conceptual knowledge is required. It is also necessary to have not only (and not so much!) deductive, but also empirical thinking, which would be convincing, although not necessarily strict from the standpoint of "pure" mathematics. Rigor should be in the exact (correct) presentation of the idea, not in the fetishization of the form of presentation.

Teaching mathematics to "applicators" should, in our opinion, pursue the following goals:

1) formation of conceptual understanding of key mathematical concepts, facts, methods with illustration of their applied application, study of the corresponding mathematical apparatus;
2) developing basic skills of mathematical research and mathematical modeling.

In view of this, we consider it effective to use (as tools) digital technologies for the formation of conceptual and procedural knowledge, which make it possible to implement active learning, in particular, to conduct experiments, research and modeling.

One of the platforms that has these tools is Go-Lab. The Go-Lab project (Global Online Science Labs) is a research innovation project co-financed by the European Commission. Its goal is to bring science closer to pupils and students by providing open access to online science laboratories created by scientists and teachers from different countries (Next-Lab, 2022).

On the portal (https://www.golabz.eu) for the teacher there is an opportunity to work with the base of ready laboratories and training spaces (ILS). And also create your own learning spaces in the Graasp environment (https://graasp.eu/), using Go-Lab tools and resources.

Go-Lab Inquiry Learning Spaces (ILSs or Inquiry Spaces) are research-oriented activities structured through Go-Lab Inquiry Cycles, which can include laboratories, training resources, programs, online services and other digital tools to provide and support research learning (de Jong, 2015).

The functionality of such a space allows you to organize a special environment. Here, under the guidance of the teacher, students explore mathematical concepts and the relationships between them, formulate hypotheses, experiment, ask questions, draw conclusions and discuss the results.

In fact, with the help of ILS, the teacher can encourage students to actively learn, using methods and techniques that require students to conscious learning activities, to involve them in the process of constructing new knowledge, research skills. The active participation of the student in the educational process is the key to the formation of conceptual understanding and achievement of high results during training, as well as his ability to apply the acquired knowledge and skills in further professional activities.

An example of such a space can be created by educational and research space to study the topic "Derivative".

The learning space contains tasks that involve working with a variety of digital tools (figure 2 – 6). These tools are selected so that you can implement different types of student learning activities (individual, group and frontal) and ensure maximum efficiency in the formation of conceptual knowledge in solving the problem.

There are things that integrated into the space:

- a ready – made Go-Lab laboratory (Labs), which allows students to conduct research using computer simulations;
- built – in GeoGebra environment for research and geometric modeling;
- built – in online spreadsheets with the ability to co-edit;
- TEXT Input forms for answers (the teacher has the opportunity to see the answers of all students from his profile, and students see only their answers);
- Apps Sticky Notes, in which participants in the process can write down their ideas on stickers;
- Apps Concept Mapper for compiling problem-solving algorithms;
- Padlet board for joint work of students and teacher;
- Apps Quest 2.0 – tests to check the mastery of the topic.

Let's illustrate the use of these tools on some tasks of our educational space "Derivative".

Task 2. The motorcyclist moves in a straight line according to the law $x(t)$. When changing some parameters of the motorcyclist's movement, observe how others change. Record the results of your observations in the form below. Can you independently build graphs of speed and acceleration according to a known graph of the law of motion? And the schedule of the passed way according to the schedule of speed?

The task is performed in Go-Lab. When performing this task, students have the opportunity to conduct an experiment by changing certain parameters of the motorcyclist's movement (figure 2).

One of the most important theorems of differential calculus is Lagrange's theorem on finite increments.

Theorem. If the function $f(x)$ is continuous on the interval $[a;b]$ and differentiable on the interval $(a;b)$, then on this interval there exists a point c such that the equality (Lagrange formula):

$$\frac{f(b)-f(a)}{b-a} = f'(c).$$

The theorem has a simple (even obvious!) geometric meaning: on a solid smooth curve connecting two points, there is a point where the tangent is parallel to the chord. Students are given the task to experimentally "prove" the validity of the theorem. To do this, they should draw a line parallel to the chord AB, and then move this line parallel to itself and make sure, thus, the validity of the theorem formulated in geometric terms. The task is performed using the built – in environment GeoGebra (figure 3).

Such a geometric illustration (and argumentation!) is quite convincing, it is intuitively clear. Therefore, it is not necessary to "frighten" students, for whom mathematics will not be the sphere of their professional activity in the future, with a strict mathematical formulation of the theorem, especially – by proving it. Students will easily come to the analytical formulation when they have before their eyes the appropriate graphic image. The teacher can (if necessary) provide assistance in the form of a question: "What is the condition of parallel lines?"

With the pedagogical mediation of the teacher (or without him), students, changing the curve, just as easily establish graphically that the point c, which is discussed in the theorem, may not be one (figure 4).

To improve the conceptual understanding of mathematics and the formation of conceptual knowledge, it is important to clarify the implication links between statements. Therefore, the chance will be lost if, when studying Lagrange's theorem, we do not stimulate students to answer the question: "Are the conditions of the theorem necessary, essential, is each of them individually, sufficient?" (**Task 5**). In this particular situation, we organize the collective work of students using the built – in online Google Sheet. Students fill in the so-called. conceptual table on the topic "Lagrange's theorem" (figure 5).

What is a "conceptual spreadsheet"? A conceptual table is a summary, organized, and structured information about the content of a particular topic, ways to solve a particular problem, the results of a study, and etc. Students fill in the spreadsheet collectively, most often – working in small groups. At the same time, they must demonstrate an understanding of the essence of concepts, facts of this topic, their connection with previously studied, physical, economic,

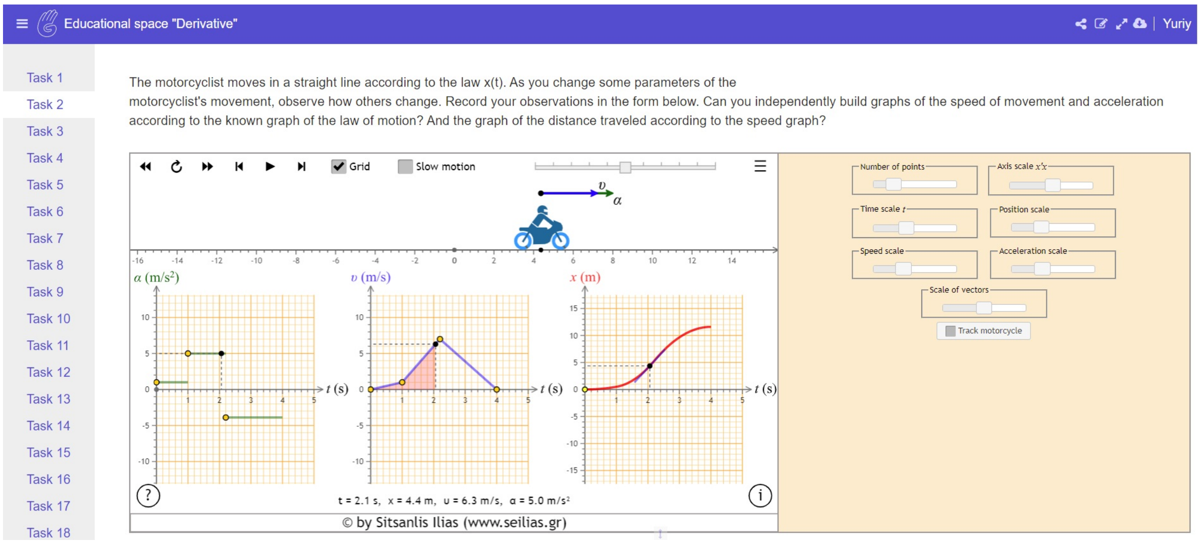

Figure 2: Computer simulation of the mechanical content of the derivative.

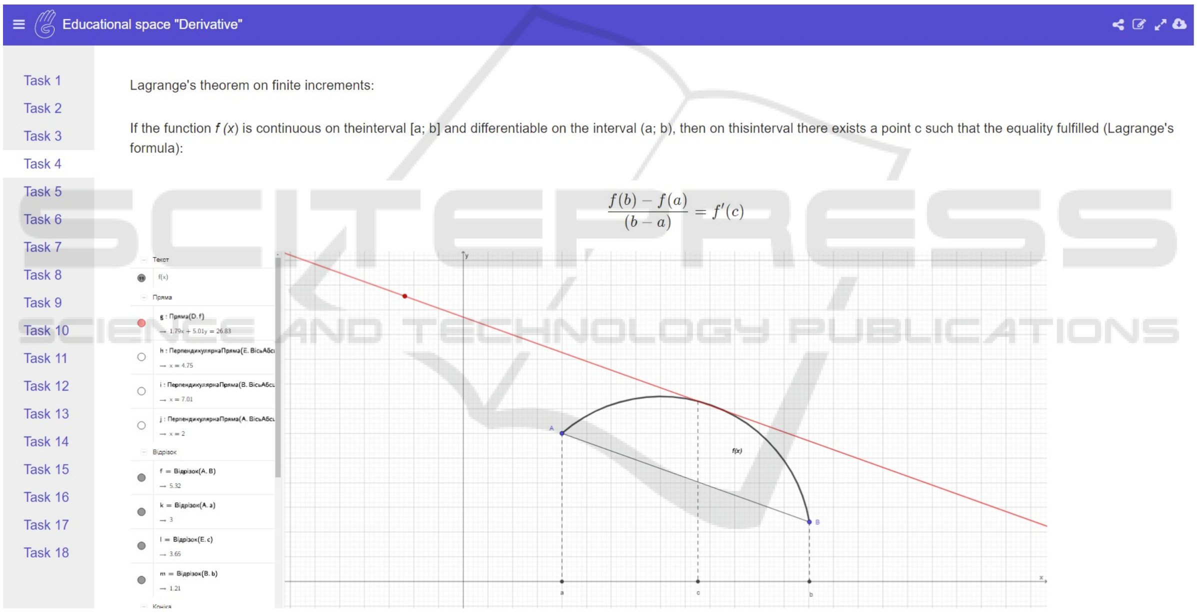

Figure 3: Graphic illustration of Lagrange's theorem (point c is one).

etc. content, the ability to correlate different forms of presentation of a mathematical content (verbal, symbolic, graphic).

The work with the conceptual spreadsheet, which summarizes the results of the study of the conditions of Lagrange's theorem (whether each of them is necessary, essential, sufficient) can be organized in different ways. For example, one group of students formulates conclusions, and another one provides a graphical argumentation of the correctness (or incorrectness) of the verbal conclusion. Or vice versa – one group gives a graphic image, and the other one makes a verbal conclusion that can be made on the basis of this image.

It will be useful to find out the mechanical meaning of Lagrange's formula (it indicates that when a body moves according to the law $s(t)$, then at some point in time the velocity of the body will be equal to its average velocity over a period of time).

Instead of a rigorous analytical proof of the theorem, which will not add any more convincing (for geometric simulation) arguments to students, it is worth considering various applications of the theorem in physics, economics, computational mathematics, and so on. It will be appropriate to draw students' attention to the fact that the number c in the Lagrange for-

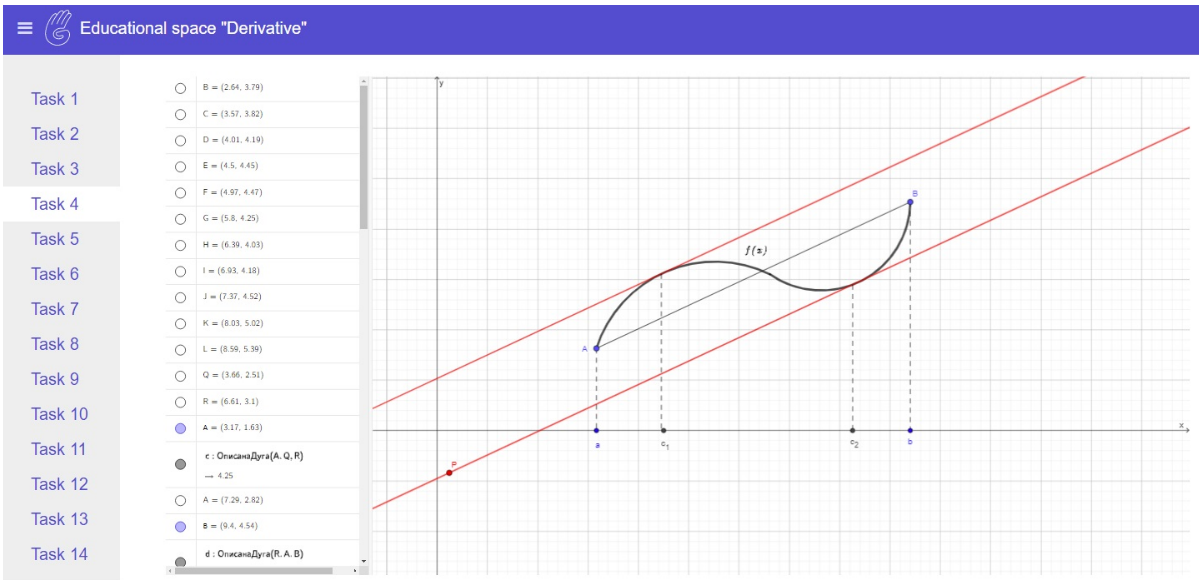

Figure 4: Graphic illustration of Lagrange's theorem (points c are two).

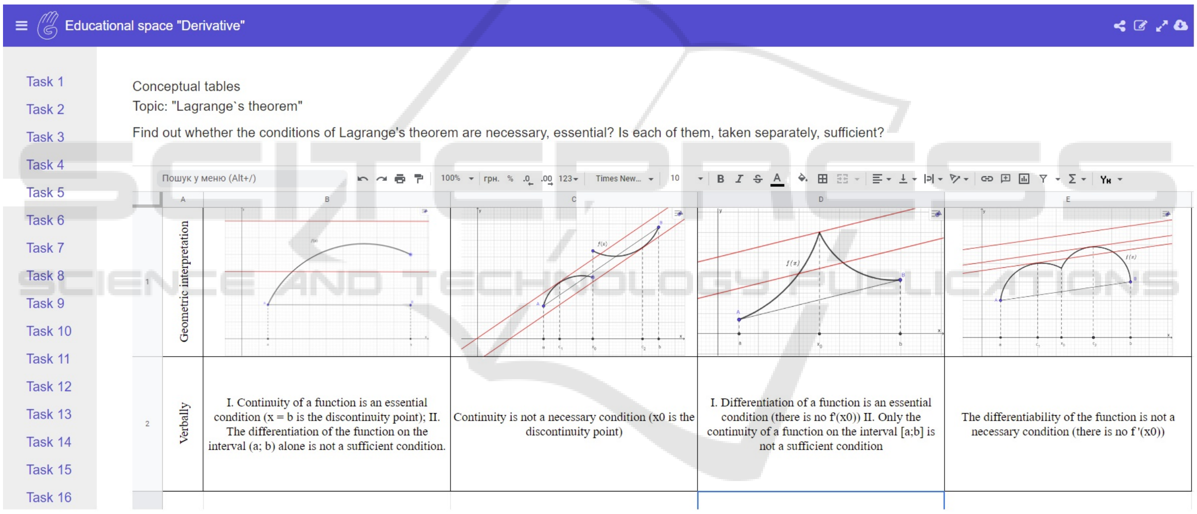

Figure 5: Conceptual spreadsheet on the topic "Lagrange's theorem".

mula is unknown, but the formula is useful in problems to estimate the difference between the values of the function, and hence – profit, work, distance traveled and other quantities.

In the context of what has been said, it will be useful, for example, to estimate the difference (**Task 6**):

$$\arctan 0.55 - \arctan 0.45$$

Solution. The function $\arctan x$ is continuous and differentiable for all real x, so Lagrange's theorem can be applied. According to this theorem, we can say that between the numbers 0.45 and 0.55 there is a number c that the equality

$$\frac{\arctan 0.55 - \arctan 0.45}{0.55 - 0.45} = (\arctan x)'|_{x=c}$$

that is

$$\arctan 0.55 - \arctan 0.45 = \frac{0.1}{1+c^2}$$

Hence, given that function $\frac{1}{1+x^2}$ decreases for positive x, we have

$$\frac{0.1}{1+0.55^2} < \frac{0.1}{1+c^2} < \frac{0.1}{1+0.45^2}$$

or:

$$0.0767 < \arctan 0.55 - \arctan 0.45 < 0.0832$$

Students were asked (**Task 8**) to formulate questions on the topic "Derivative" to this figure, which shows a graph of the function $f(x)$ (figure 6).

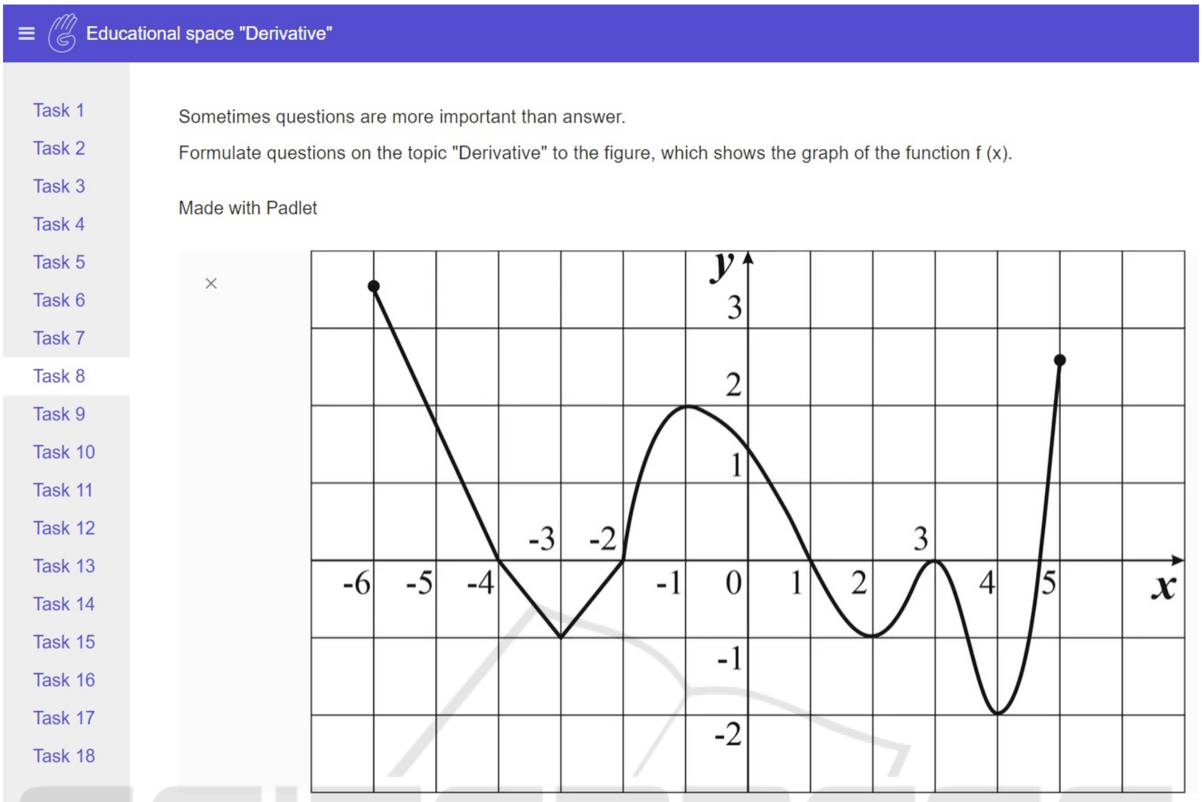

Figure 6: Task 8.

The ability to ask questions (form tasks) is one of the signs of conceptual knowledge. Therefore, the purpose of such tasks is to form a conceptual understanding of the concept of a derivative function, in particular, its geometric content, and the ability to graphically illustrate this understanding.

Students are divided into two subgroups. One formulates questions or tasks, and the other provides answers. In our space for this purpose the use of the Padlet board where each student attaches the questions – answers is provided. At the same time, an "editor" is appointed, who groups the same type of questions, "forbids" questions that are not related to the topic, and etc. Students attach the answers to the questions next to the questions themselves. Then, under the guidance of the teacher, questions and answers are discussed with a vote for the best question.

Here are the most interesting, in our opinion, questions (tasks) of freshmen:

1. Does it have the roots of an equation $f'(x) = 0$. If so, is it possible to find out: a) how much? b) which ones?
2. Solve the inequality: $f'(x) > 0$ ($f'(x) < 0$)..
3. Does the equation $f''(x) = 0$ have roots? If so, how much?
4. Does the inequality $f''(x) < 0$ have integer solutions? If so, how much? What are these solutions?
5. Does the function $f'(x)$ have a breakpoint? If so, what is the nature of these gaps?
6. Is it possible to determine the approximate value of its derivative at the point: a) -1; b) 1. If possible, explain how to find this value and find it. If not, then argue.

Task 10. Write an algorithm for approximating the value of the function $f(x)$ at the point $x_0 + \Delta x$ using a differential. Such tasks form a conscious understanding of the algorithm, i.e. the construction of procedural knowledge on the basis of conceptual.

It is suggested to perform this task in the Concept Mapper application, which contains tools for plotting.

Task 11. The plant received an order from the cannery to produce a batch of cylindrical cans of a certain volume for canned olives. Design the shape of the tin cans so that as little material as possible is used to make them. Check out the various canned cylindrical cans at your nearest supermarket. Is their packaging economical in terms of the cost of materials for their manufacture?

This is an optimization problem "from life", which requires the use of a differential calculus to solve it. We plan to use the Sticky Notes application to solve this problem. Students describe their research on stickers, which are placed in a common field. In this way, each student sees the results of others. Then you can have a general discussion.

Conceptual understanding of mathematics involves the ability to translate a real problem into the language of mathematics and vice versa, to interpret a mathematical result in the language of a real non-mathematical context, in particular, to predict the course of the process or the end result. To develop such an ability (and, at the same time, to check its formation) we offer the following task.

Task 17. The volume of products manufactured at the enterprise is determined by the function

$$f(t) = -2t^3 + 6t^2 + 9,$$

where t is the time (in years). Give reasonable answers to questions (1 – 6), without using the graph of the function $f(t)$:

1. During what period will production increase (decrease)?
2. Indicate the periods when the growth rate of production accelerates (slows down), if any?
3. Is there a time when the nature of the rate of growth (decline) of production changes to the opposite? If so, indicate this point.
4. When will the volume of production be the largest?
5. Predict whether the volume of output may fall to zero? If so, when will this happen. If not, why not?
6. Imagine that you became the head (manager) of this company after a year of its work. Would you consider it necessary to change the production strategy (function $f(t)$)? If not, justify (for example, all is well, because production is growing, i.e. the company operates efficiently). If your answer is yes, then explaining what exactly would alarm you?
7. On the basis of conducted in pp. 1 – 6 research and analysis provide a graphic illustration of the results.

Since all questions are formulated by the teacher in advance, and the answers to them must be individual and explained, we consider it convenient to perform this task form TEXT Input. For graphic illustration (p. 7 of the task), we suggest using the GeoGebra tools.

At the end of the study of the topic it is necessary to assess the level and quality of knowledge. In the digital space, these can be tests. To test the conceptual understanding of mathematical concepts, facts, methods, the most appropriate are open tests. Because in test tasks with the choice of the correct answer among the proposed ones there is a temptation to just guess it. And, even if we exclude guessing, there is a big difference between choosing the right answer among the several given (i.e. recognizing) and formulating it yourself. Therefore, we believe that closed-type tests should be avoided or minimized.

An example of an open – ended test task with **Task 18** is: “A body moves rectilinearly according to the law $s(t) = -4 + 2t + t^2$ (s is measured in meters, t is measured in seconds). Find the speed of this body at time ($t = 3s$). In response, write a number, for example, 7”.

3 CONCLUSIONS AND PROSPECTS FOR FURTHER RESEARCH

1. As a result of a survey of non-mathematical specialties students, it was found that students to some extent feel the need to apply mathematical knowledge in the study of professional disciplines, as well as to use this knowledge in practice. However, students do not sufficiently understand the essence of mathematical concepts and facts, the relationships and interdependencies between them. Therefore, they cannot always recognize mathematical structures in non-mathematical contexts and, therefore, apply mathematics effectively.
2. The purpose of studying mathematics by non-mathematical specialties students is to provide them with the ability to apply mathematics to solve professional problems. An analysis of the various practices of teaching mathematics has revealed that in many cases this goal is not achieved due to the lack of a reasonable balance between the formal rigor of the presentation of theoretical principles and the teaching of procedures. Therefore, the teaching of mathematics should pursue the following goals: the formation of a conceptual understanding of key mathematical concepts, facts, methods with an illustration of their application, the study of the relevant mathematical apparatus; developing basic skills of mathematical research and mathematical modeling.
3. The described methods of using the digital educational space are designed to improve the conceptual understanding of mathematics among students of non-mathematical specialties through the

study of mathematical concepts and their connections, experimentation, formulation of questions, hypotheses, conclusions, and discussion of the obtained results.

4. Prospects for further research are to develop criteria and indicators of conceptual understanding of mathematics by non-mathematical specialties students, as well as quantitative analysis of the effectiveness of the use of digital learning space for the formation of conceptual mathematical knowledge of students.

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