How Concept Learning Changes Strategic Choice in Guessing Games?

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Abstract: This paper deals with the k-beauty contest game. The research question formulated in this work is how players (mainly children and undergraduates) learn in complex strategic situations that they have never faced before and how we can model this learning process. We examine data from different games played during popular lectures about game theory and present findings of players' progress in strategic learning while competing with other players. The raw data gathered is available in an open repository for replication and analysis and an R file with data manipulations, metrics calculations, and plots. Based on the conclusions from experimental data, we create the agent-based model and launch ten thousand simulations for different setups. Then we apply analysis from experimental data and discuss findings and similarities between agents and humans.

1 INTRODUCTION

Game theory is a field of science investigating rational players' decision-making under uncertainty. The source of the uncertainty can be strategic structure, e.g., probability of certain events, lack of information about future possibilities, or decisions of other agents that can generate it. In the last case, we can talk about the interdependence of strategic actions, when some players' decisions affect others' payoffs. Such situations arise around us daily, and we, consciously or unconsciously, participate in them. The success heavily relies on our perception of the actions of other players.

The problem is how we can know the future actions of other players. We cannot, but we can start with some assumptions that will help create a framework, model, or theory of "mind", which will predict future (reasonable) actions. Game theory proposed approach, which is now under questioning (especially from the side of the experimental or behavioral economy). Nevertheless, we will start from standard notions and proceed to experimental data.

One can expect that other players will play "reasonably", and this game theory means they will try to achieve a better result in some agent's sense. This idea is grasped by the term **rationality**. Every rational player must calculate the best possible result, taking into account the rules of the game and the interests of other participants. In other words, think strategically. It is well known from theory that rational players will play Nash equilibrium (NE) if there is any, which is very useful in games where only one unique NE exists. The notion of rationality was indeed fundamental for the development of game theory. However, the problems with this notion are also quite famous.

First, it is very demanding because it presupposes that the agent has complete, transitive preferences and is capable of computing equilibrium in a given strategic situation. However, this is not feasible in many natural situations (for example, we know about NE in chess, but still no computer can compute it). Secondly, probably more important, there are many games where NE is the poor prediction of actual human behavior. This paper investigates some of the data from such a game and discusses the difference.

All this makes decision making exciting problem to investigate. This is a rich area of research, where theoretical constructions of the game theory seem to fail to work, and experimental data shows unusual patterns. These patterns are persistent and usually do not depend on age, education, country, and other things. During the last 25 years, behavioral game theory in numerous studies has examined bounded rationality (best close concept to the rationality of game theory), cognitive distortions, and heuristics people use to reason in strategic situations. For example we can note surveys of Crawford et al. (Crawford

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et al., 2013) and Mauersberger and Nagel (Mauersberger and Nagel, 2018). Also, there is a comprehensive description of the field of behavioral game theory by Camerer (Camerer, 2003).

We will concentrate on the guessing games, which are a significant part of research because of their simplicity for players and easy analysis of rules from a game-theoretic perspective. In this paper, we present the results of games played during the 2018-2021 years as part of popular lectures about game theory (Ignatenko, 2020). The audience of these lectures was quite heterogeneous, but we can distinguish the following main groups:

- children at schools (strong mathematical schools, ordinary schools, alternative education schools);
- students (bachelor and master levels);
- mixed adults with almost any background.
- adults with a business background
- participants of Data Science School
- participants of summer STEM camps for children

We propose a framework of four types of games, each presenting one idea or concept of game theory. These games were introduced to players with no prior knowledge (at least in the vast majority) about the theory. On the other hand, games have simple formulation and clear winning rules, making them intuitively understandable even for kids. This makes these games the perfect choice to test the ability of strategic thinking and investigate the process of understanding complex concepts during the play, with immediate application to the practice. This dual learning, as we can name it, shows how players try and learn in natural conditions and react to interaction challenges with other strategic players.

In this paper, we will concentrate on the first game – the famous p-beauty contest game. For this game, we analyze data and try to formulate simple rules which are plausible for an explanation of players' behavior. In the next section, we created the agent-based model using the Netlogo environment and discussed the model's main features. To investigate the model and its properties, we performed simulations using the BehaviorSpace tool of Netlogo; as a result, about 10000 games were simulated. Such a volume of data is impossible to get using human-based experiments. We recreated plots and metrics developed for human data and analyzed its similarity and differences. In the end, we formulate conclusions and future work directions.

First, let us start with some definitions.

1.1 Game Theory Definitions and Assumptions

Consider games in strategic or normal form in a non-cooperative setup. A non-cooperativeness here does not imply that the players do not cooperate, but it means that any cooperation must be self-enforcing without any coordination among the players. The strict definition is as follows.

A non-cooperative game in strategic (or normal) form is a triplet $G = \{\mathcal{N}, \{S_i\}_{i=\in\mathcal{N}}, \{u_i\}_{i\in\mathcal{N}}\}$, where:

- $\mathcal{N}$ is a finite set of players, $\mathcal{N} = \{1,\ldots,N\}$;
- S_i is the set of admissible strategies for player i;
- $u_i : S \to \mathcal{R}$ is the utility (payoff) function for player i, with $S = \{S_1 \times \cdots \times S_N\}$ (Cartesian product of the strategy sets).

A game is said to be static if the players take their actions only once, independently of each other. In some sense, a static game is a game without any notion of time, where no player has any knowledge of the decisions taken by the other players. Even though, in practice, the players may have made their strategic choices at different points in time, a game is still static if no player has any information on the decisions of others. In contrast, a dynamic game is one where the players have some (full or imperfect) information about each others' choices and can act more than once. In this work, we deal with repeated static games, which means that the same game is played twice (sometimes three times) with the same players.

Agents' rationality is a significant issue; sometimes, it is called full rationality (to differentiate it from bounded rationality – a less restricting notion). When a fully rational agent tries to find the best action, it usually depends on the action of other self-interest agents. So the first agent must form beliefs about the second agent's beliefs about the beliefs of the first agent, and so on. Such constructions seem too complicated, but they are based on the predictions of classical game theory, which assumes all agents to be fully rational.

One quite famous result by Aumann (Aumann, 1995) is that for an arbitrary perfect-information extensive-form game, the only behavior that is compatible with (1) common knowledge of rationality, and in particular by (2) each agent best responding to their knowledge is for each agent to play according to the strategy, obtained by the backward induction. Aumann and Brandenburger (Aumann and Brandenburger, 1995) showed that common knowledge of rationality, the game payoffs, and the other agent's beliefs are sufficient conditions for Nash equilibrium in an arbitrary game.

In this regard, the most accepted solution concept for a non-cooperative game is that of a Nash equilibrium, introduced by John F. Nash (Kuhn et al., 1996). Loosely speaking, a Nash equilibrium is a state of a non-cooperative game where no player can improve its utility by changing its strategy if the other players maintain their current strategies. Of course, players also use information and beliefs about other players, so we can say that (in Nash equilibrium) beliefs and incentives are important to understand why players choose strategies in real situations.

The NE is the core concept of game theory, but it differs from experiments and sometimes reality. In some games, humans demonstrate convergence to equilibrium, but in others do not. This gap between similarly-looking games is slim and not easy to catch. We will consider guessing games as a playground to work with players' behavior.

2 GUESSING GAMES HISTORY

In early 90xx, Rosemary Nagel started a series of experiments of guessing games, summarized in (Nagel, 1995). She was not the first to invent the games; they were used during the lectures by different game theory researchers (for example, Moulin (Moulin, 1986)). In recent work (Nagel et al., 2017) authors provide extensive research of the origins of the guessing game with unexpected links to the editor of one french newspaper Alain Ledoux, who, as far as it is known today, was the first who used the rules and then publish an article about unusual patterns observed (Ledoux, 1981). The work of Nagel (Nagel, 1995) was the first experimental try to investigate the hidden patterns in the guessing game, and in this working framework of k-level models was proposed.

Later, Ho et al. (Ho et al., 1998) gave the name "p-beauty contest" inspired by Keynes's comparison of stock market instruments and newspaper beauty contests.

The beauty contest game (BCG) has become an important tool to measure the "depth of reasoning" of a group of people using simple abstract rules. To begin with, we should note that behavioral game theory aims to develop models, which explain human behavior in a game-theoretic setup more accurately, based both on experiments and theory (Camerer, 2003). There are two main approaches how to dealing with the problem of replacing full rationality with bounded rationality. The first view is to consider boundedness as an error. For example quantal response notion (Camerer et al., 2002) or ε-equilibrium (Leyton-Brown and Shoham, 2008) assume that agents make an error by choosing not optimal strategy profile. They play near-optimal response because they do not have the capacity to calculate the exact best action.

The second approach is to treat bounded rationality as a structural property of an agent's reasoning process. One of the most prominent classes of models of this type is the iterative model's scheme. They include the k-level reasoning (Nagel, 1995; Costa-Gomes et al., 2001), cognitive hierarchy (Camerer et al., 2004) and quantal cognitive hierarchy models (Wright and Leyton-Brown, 2017). All these models consider boundedness as an immanent part of the reasoning. Each agent has a non-negative integer level representing the degree of strategic reasoning (i.e., modeling of recursive beliefs) of which the agent is capable. Level-0 agents are nonstrategic – they do not model other agents' beliefs or actions at all; level-1 agents model level-0 agents' actions; level-2 agents model the beliefs and actions of level-1 agents; and so forth (Wright and Leyton-Brown, 2016). In this work, we support the latter idea, analyzing experimental data to estimate changes in numbers of different levels in the learning and teaching process.

2.1 Learning Models

Recently, game theorists began to research the process of reasoning toward equilibrium actively. Two prominent simple learning models are reinforcement and belief learning (e.g., fictitious play). In reinforcement, strategies have numerical attraction levels, which are reinforced (increased) when a strategy is chosen, and the result is good. Reinforcement is a good model of animal learning but does not gracefully accommodate the fact that people often choose strategies that have not been directly reinforced.

In the fictitious play, players form beliefs based on a weighted average of what others have done in the past, and best respond given their beliefs. Remarkably, weighted fictitious play is equivalent to a generalized reinforcement model in which unchosen strategies are reinforced by the forgone payoffs they would have yielded.

There are a lot of other approaches; we will mention the approach which enriches 0-level reasoning (Wright and Leyton-Brown, 2017). Specifically, they investigate general rules that can be used to induce a level-0 specification from the normal-form description of an arbitrary game.

Also, we can note work (Gill and Prowse, 2016), where participants were tested on cognitive abilities and character skills before the experiments. Then authors perform statistical analysis to understand the impact of such characteristics on the quality of making strategic decisions (using a p-beauty contest game with multiple rounds). In more recent work (Fe et al., 2019) even more elaborate experiments are presented. It is interesting that in the mentioned paper, experiments are very strict and rigorous (as close to laboratory purity as possible) in contrast to games played in our research. However, at the end of the day, the results do not differ very much.

As we know, there are not many works about game theory experiments for children. In our previous work (Ignatenko, 2020) we presented data from games with participants 15-18 years old. There is a master thesis (Povea and Citak, 2019), with the study of the behavior of children aged 8-11 in a beauty contest game with ten repetitions. The author found evidence that children are able to play a beauty contest game using not only cognitive skills but also empathy.

To deal with these problems, computer simulation, mainly agent-based modeling ABM can be used. Agent-based models are essentially a tool to discover patterns in behaviors that emerge from simple rules – micro behavior. Agent-based modeling for guessing games is not a very developed area of research. For example see paper (Nichols and Radzicki, 2007).

3 EXPERIMENTS SETUP

We claim that our setup is closer to reality than the laboratory, and this is the point of this research: how people learn under real-world situations. All games were played under the following conditions:

1. Game was played during the lecture about the game theory. Participants were asked not to comment or discuss their choice until they submitted it. However, this rule was not enforced, so usually, they have this possibility if they wanted;
2. Participants were not rewarded for a win. The winner was announced (so get some "good feelings"), but no more;
3. During some early games, we used pieces of paper, and we got some percentage of joking or trash submission, usually very small. Later we switched to google forms, which is a better tool to control submission (for example, only natural numbers are allowed).
4. Google forms give a possibility to make multiple submissions (with different names) since we did not have time for verification, but a total number of submissions allows us to control that to some extent.

The aim of this setup was to free participants to explore the rules and give them the flexibility to make a decision in an uncertain environment. We think it is closer to real-life learning without immediate rewards than laboratory experiments. Naturally, this setup has strong and weak sides. Let us summarize both.

The strong sides are:

1. This setup allows to measure how people make decisions in "almost real" circumstances and understand the (possible) difference with laboratory experiments;
2. These games are part of an integrated approach to active learning, when games are mixed with explanations about concepts of game theory (rationality, expected payoff, Nash equilibrium), and they allow participants to combine experience with theory;
3. Freedom and responsibility. The rules do not regulate manipulations with conditions. So this setup allows (indirectly) to measure the preferences of players: do they prefer to cheat with rules, choose random decisions without thinking or put effort into solving the task;
4. During the 2020-2021 years, lectures were mainly online. That fact brings new challenges for our experiments, but since we initially rely on google spreadsheets for gathering the answers, all routines remain mainly the same.

Weak sides are:

1. Some percentage of players made "garbage" decisions. For example, choose the obviously worse choice to spoil efforts for others;
2. Kids has (and often use) the possibility to talk out decision with the neighbors;
3. Sometimes participants (especially kids) lost concentration and did not think about the game but made a random choice or did not make decisions at all;
4. Even for the simplest rules, sometimes participants failed to understand the game the first time. We suppose it is due to conditions of lecture with (usually) 30-40 persons around;
5. Still, we should note that online lectures are less involving. It is demotivating for a lecturer not to see listeners and also demotivating for students to listen to lectures online.

3.1 Rules

In this paper, we concentrate on the p-beauty contest. The winning number is the closest to p of average. As usual, $p = 2/3$, but we have used other setups as well (for clarity, we omit data from that games in this analysis). Participants are asked to choose an integer number in the range 1 – 100, margins included. Note that some setups investigated in references use a range starting with 0. However, the difference is negligible. To provide quick choice calculation, we have used a QR code with a link to google form, where participants input their number. All answers were anonymous (players indicated nicknames to announce the winners, but then all records were anatomized).

4 RESULTS AND DATA ANALYSIS

In this section, we present a summary of data gathered during the games. A summary of the results of the game is given in table 1. Columns descriptions are:

- id is the id of the experiment;
- type is the type of group. Alternative H and M are for alternative schools (not in the governmental system) with humanitarian and mathematical directions, respectively. Math lyceum also goes for summer camps with participants from different lyceums;
- age is the approximate age of participants, only indicated for children, to distinguish possible borderline between stages of strategic reasoning;
- round is the round of the game;
- average is the average of choices;
- winning number is the average * 0.66;
- zlevel is the percent of players, choosing numbers bigger than 50. It is an estimation of 0-level players in this round. As one can expect, it is declining with round;
- median is the median of choices (sometimes it is more informative than average);
- count is the number of choices;
- irrationality is the percent of choices wider than 90.

First, we observe statistics of choices for different types of participants.

As we can see from the plot (figure 1) some setups have one round, some two and two cases have three rounds. It was limited by format (popular scientific lecture), so we can only safely compare two rounds.

Almost all winning numbers fall (roughly) in the experimental margins, obtained in (Nagel, 1995). With winning number no more significant than 36 and not smaller than 18 in the first round. Two exceptions in our experiments were Facebook online game (15.3) when players could read information about the game in, for example, Wikipedia. Moreover, Another is the alternative humanitarian school (40.1), where participants seem did not get the rules the first time.

4.1 Metrics and Analysis

The first metric to observe is the percent of "irrational choices" – choices that cannot win in (almost) any case. Let us explain, imagine that all players will choose 100. It is impossible from practice but not forbidden. In this case, everybody wins, but if only one player deviates to a smaller number – he/her will win, and others will lose. So playing numbers bigger than 66 is not rational unless one does not want to win. Furthermore, here we come to an important point, in all previous experiments, this metric drops in the second round and usually is very low (like less than 5%) (Ho et al., 1998). However, in our case, there are experiments where this metric becomes higher or changes very slightly. Moreover, initially, values are much higher than expected. So here, we should include the factor of unique behavior; we can call it "let us show this lecturer how we can cheat his test!" what is more interesting – is that this behavior is more apparent in the case of an adult than in kids.

It is also interesting to see a distribution of choices for different groups. We can summarize choices on the histograms (figure 2). Using models of strategic thinking, we will adopt the theory of k-levels. According to this idea, 0-level reasoning means that players make random choices (drawn from a uniform distribution), and k-level reasoning means that these players use the best response for the reasoning of the previous level. So 1-level reasoning is to play 33, which is the best response to the belief that the average will be 50, 2-level is the best response to the belief that players will play 33, and so on.

As we can see from the diagram (figure 2), some spikes in choices are predicted very well, but it depends on the background of the players. The best prediction is for the Data Science conference attendees, which presume a high level of cognitive skill and computer science background.

On figure 3, we can see boxplots defined by several players with different levels of perception for different types of players. We compare here ordinary schools and mathematical lyceums. Levels are defined in the following subsection, but we can see a pattern of behavior. The number of "irrational" (choices with big numbers) is decreasing, so as "next-to-win-but-bigger" numbers. Several 2-level reasoning, especially after explaining the equilibrium concept, is growing substantially, while some "too smart" choices from [1, 5] are more or less the same.

Table 1: Summary of first game for id of experiment and type of players. Explanation of columns is in the text

Id	Type	Age	Round	Average	Winning	Zlevel	Median	Count	Irrationality
1	Alternative H	12-14	1	66.7	44.5	69.23	78	13	46.15
1	Alternative H	12-14	2	3.91	2.61	0	3.5	12	0
2	Alternative M	12-14	2	42.82	28.54	23.52	45.0	17	0
2	Alternative M	12-14	2	24.37	16.24	0	26.5	16	0
3	Adults		1	40.57	27.05	31.57	40.0	19	5.26
4	Alternative H	12-14	1	52.54	35.03	63.63	55	11	9.09
4	Alternative H	12-14	2	15.41	10.27	8.33	6	12	8.33
5	Adults		1	22.98	15.32	11.76	17.0	102	0
6	TechSchool	16-18	1	43.41	28.94	35.29	45.0	51	3.92
6	TechSchool	16-18	2	46.5	30.99	35.48	29.0	62	32.25
7	Math lyceum	16-18	1	30.58	20.38	16	27.5	50	2.0
7	Math lyceum	16-18	2	14.26	9.5	5.26	7	57	5.26
8	Math lyceum	15-16	1	37.06	24.71	20.68	33.0	29	3.44
8	Math lyceum	15-16	2	26.20	17.47	10.34	17.0	29	6.89
9	Math lyceum	14-16	1	42.0	27.99	44.44	42.5	18	11.11
9	Math lyceum	14-16	2	23.1	15.39	5.0	19.0	20	0
10	Ordinary school	14-16	1	48.69	32.46	46.15	46.5	26	0
10	Ordinary school	14-16	2	19.78	13.18	0	22.0	23	0
11	DS conference		1	37.25	24.83	28.33	33.0	60	8.33
11	DS conference		2	21.44	14.29	15.78	9.0	57	12.28
12	Students		1	42.40	28.27	33.33	40.0	27	3.7
13	Students		1	27.37	18.24	12.5	25.5	8	0
13	Students		2	8.62	5.74	0	8.5	8	0
14	Math lyceum	14-16	1	41.05	27.37	22.22	35.0	18	11.11
14	Math lyceum	14-16	2	17.23	11.49	5.88	13.0	17	0
15	Adults		1	34.32	22.88	20.73	30.0	82	1.21
15	Adults		2	12.48	8.32	2.19	8.0	91	2.19
16	Adults		1	43.05	28.70	33.96	40.0	53	1.88
16	Adults		2	14.69	9.79	1.88	11.0	53	1.88
17	Adults		1	50.33	33.55	41.66	50.0	12	8.33
17	Adults		2	13.50	8.99	0	12.0	46	0
18	Math lyceum	14-16	1	41.72	27.81	36.36	37.0	11	9.09
18	Math lyceum	14-16	2	26.36	17.57	0	30.0	11	0
19	Math lyceum	14-16	1	29.43	19.62	13.63	25.0	44	0
19	Math lyceum	14-16	2	27.25	18.16	20.45	9.5	44	20.45
20	Students		1	30	19.9	5.2	27	19	0
20	Students		2	24.9	16.6	20	11.5	20	15
21	Ordinary school	14-16	1	43	28.7	33.9	40	53	1.88
21	Ordinary school	14-16	2	14.7	9.7	1.88	11	53	1.88

Interesting hypotheses that need to be tested in detail can be formulated: **Higher number of choices from [50, 100] in the first round leads to the higher number of choices from [1, 5] in the second round and vice verse.** We can support this hypothesis with the following plot (figure 5).

Another metric (Güth et al., 2002) is how much winning choice in the second round is smaller than in the first. Due to multi-level reasoning, every player in this game is trying their best to win but can't do all the steps to winning. So there are players who have 0-level reasoning. They choose random numbers. First-level players choose 33, which is the best response for players of 0-level and so on. Based on the result of the first round and, in fact, an explanation of the Nash equilibrium, players must know that it is better to choose much lower numbers. However, the graph shows that the decrease is quite moderate. Only students show good performance in this matter. Moreover, the tech school shows a (small) increase in winning number in the second round!

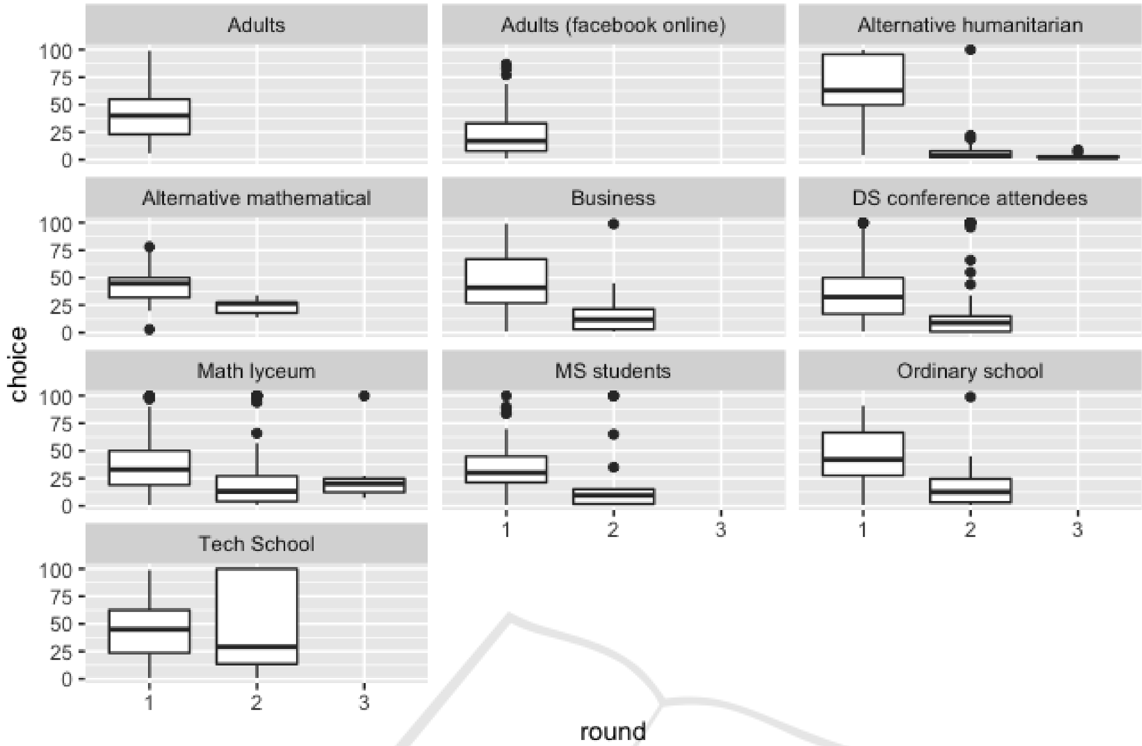

Figure 1: Histogram of choices for each round.

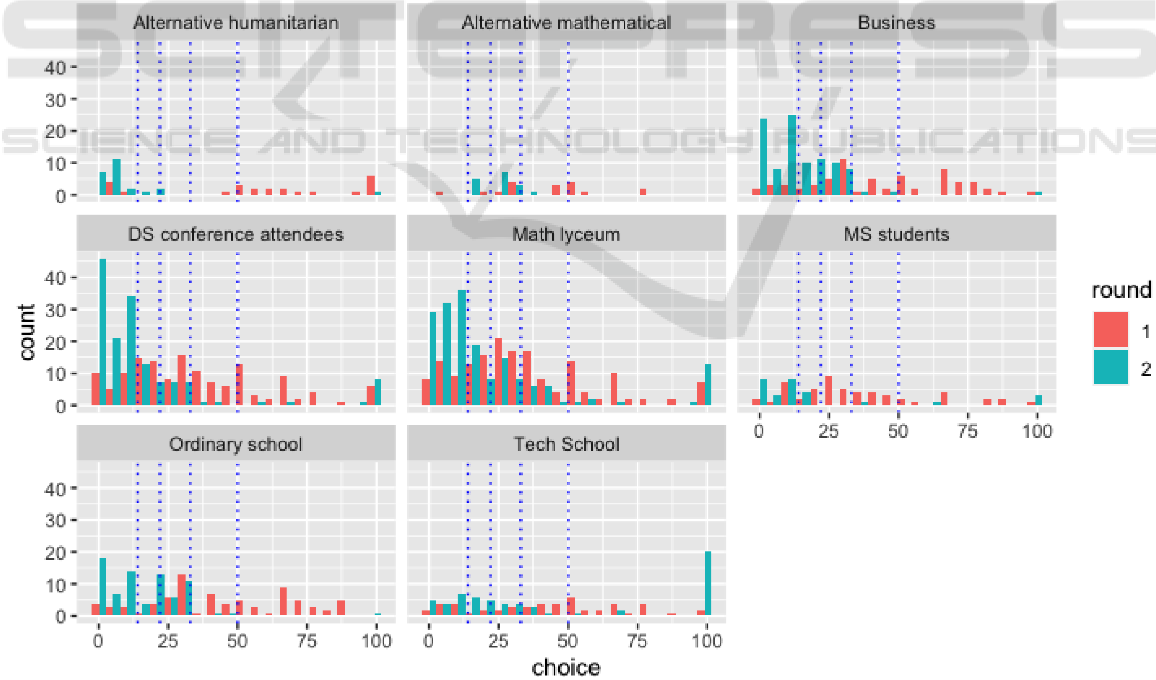

Figure 2: Histogram of choices for each round.

4.2 Levels of Reasoning Snalysis

Another point about the learning process in this game is how players' decisions are distributed over the space of strategies. We claim that there is a distinct difference in changes between the first and second rounds for different groups. To perform this analysis, we apply the idea of k-level thinking.

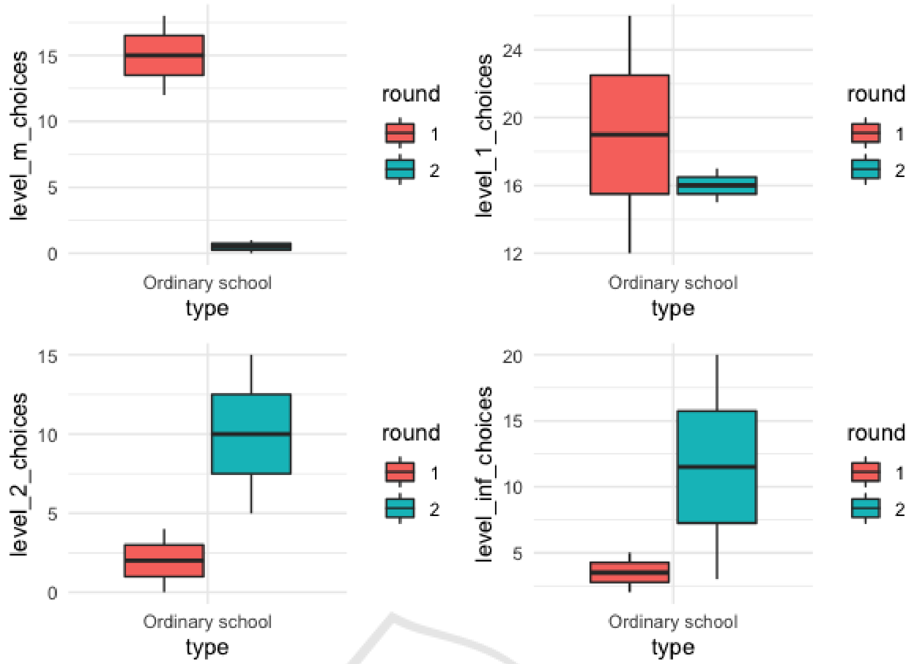

Figure 3: Change in winning number for number of ordinary schools participants.

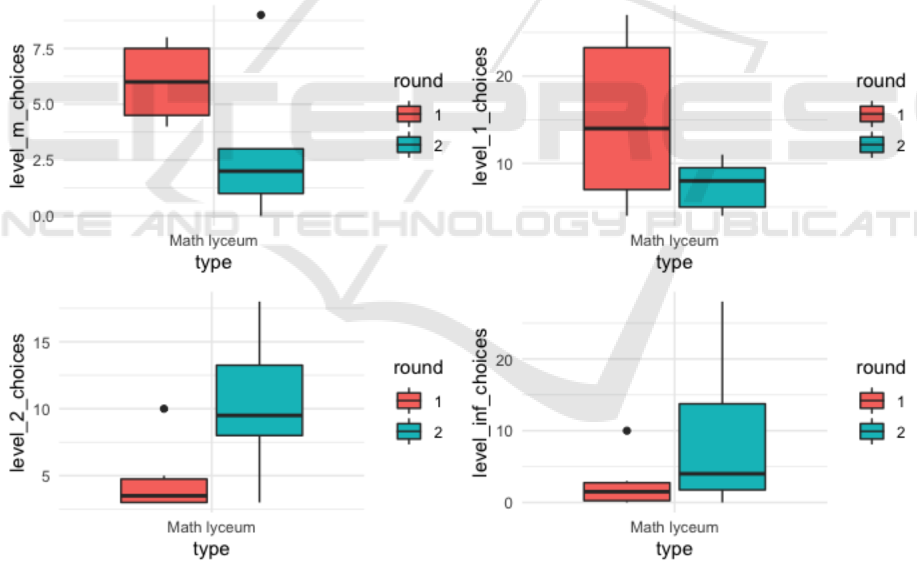

Figure 4: Change in winning number for number of mathematical lyceums schools participants.

To find differences, we need to simplify this approach. First, we define **b-level** players players who choose numbers from the range [50, 100]. Beginner players who do not understand rules (play randomly) or do not expect to win or want to lose intentionally (for reasons discussed above). The substantiation for such a range is that numbers above 50 did not win in any game. Second level we call **m-level**, it is for range [18, 50]. It is for players with middle levels of reasoning. The first-round winning number is usually in this range (and in part of the second rounds). Third level is **h-level**, it is for range [5, 18]. It is for high level reasoning and finally **inf-level** ([1, 5] range) is for "almost common knowledge" level of thinking.

By calculating the number of levels for each game, we can estimate change (in the percentage of the number of players) in adopting different strategy levels.

What conclusions can we draw from this data? There are no clear differences in changes, but at least we can summarise a few points:

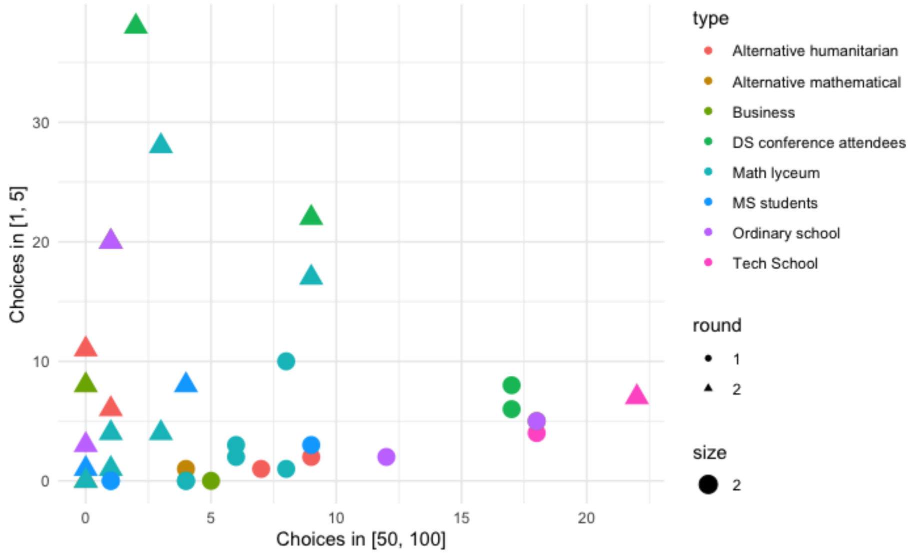

Figure 5: Choices number in [1, 5] range and in [50, 100] range by rounds and type.

Table 2: Summary of change in strategy levels

Type	b-difference	m-difference	h-difference	inf-difference
Alternative humanitarian	-72	-8	0	72
Alternative mathematical	-24	-6	30	-6
Alternative humanitarian	-52	0	17	43
Math lyceum	-9	-36	24	34
Math lyceum	-10	-24	28	7
Ordinary school	-49	12	20	4
DS conference attendees	-14	-32	14	27
MS students	-12	-50	50	12
Alternative mathematical	-17	-34	23	23
DS conference attendees	-17	-30	23	35
Business	-32	-17	21	28

- Usually after first round and equilibrium concept explanation there is decrease in **b-level** and **m-level**;
- Symmetrically, there is an increase in two other levels, but sometimes it is more distributed, sometimes it is (almost) all for **inf-level**;
- the Last situation is more likely to happen in schools, where kids are less critical of new knowledge;
- Usually second round winning choice in the realm of **h-level**, so groups with the biggest increase in this parameter are the ones with better understanding.

Another possible approach to the measurement of irrationality is to calculate the percent of choices wider than 50 (quite low chances for the win) and the percent of choices wider than 90 (no chances to win). We can see an interesting picture when we plot these metrics for different types.

As we can see here, there is a nice direct line of green dots of unknown nature. And all red dots are gathered below. This is interesting dependence that needs to be investigated in detail.

4.3 Size and Winning Choice

This game is indeed rich for investigation. Let us formulate the last (in this paper) finding of this game. Can we in some way establish the connection between the number of players and the winning number (actually with strategies players choose during the game)? To clarify our idea, see at figure 7. It is a scattered plot of a two-dimensional variable; the x-axis is for the number of participants in the game, and the y-axis is for the winning choice per round. Different colors are for different types of groups where games were played.

As we can observe, the first and second rounds form two separate clusters. This situation is expected

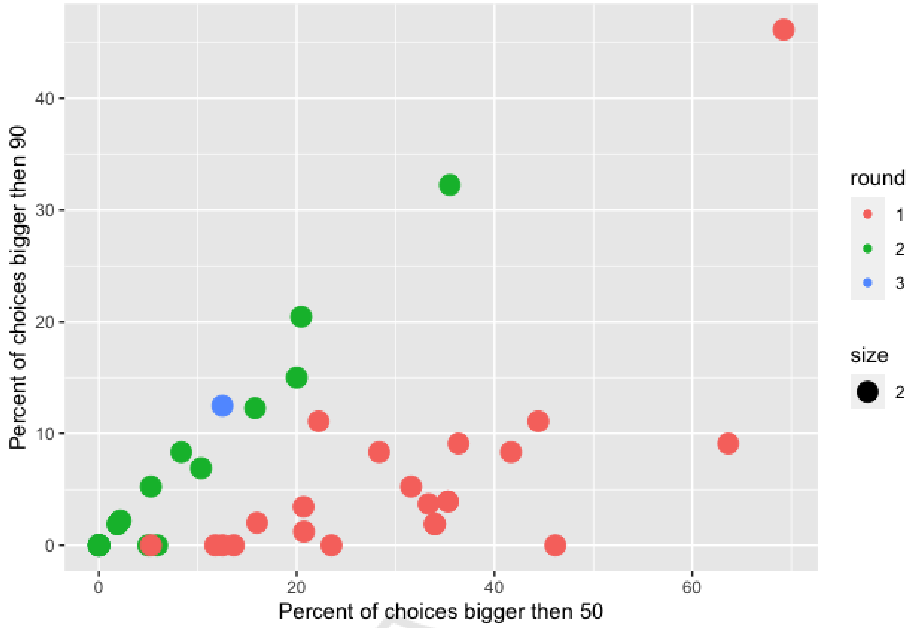

Figure 6: Irrationality of participants.

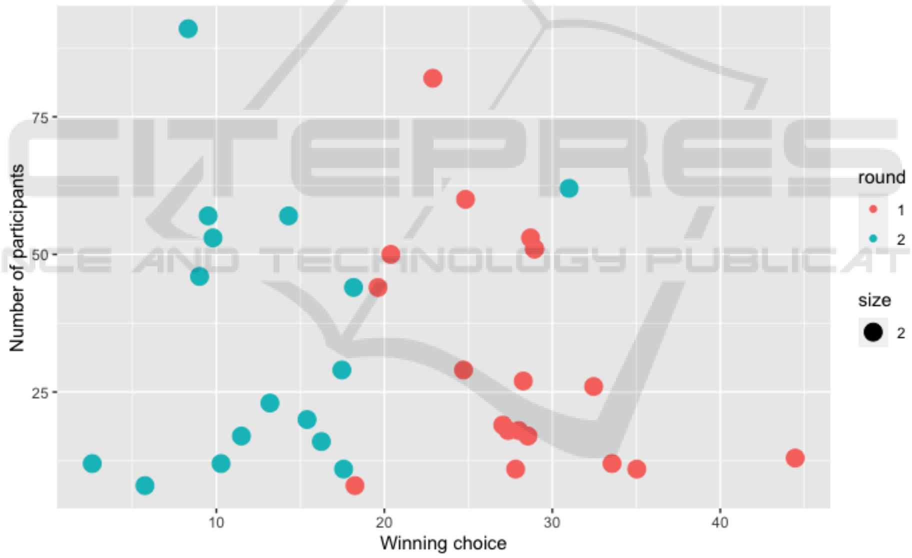

Figure 7: Change in winning number for number of participants.

and informs us that players learn about the equilibrium concept between rounds and apply it to practice. Also, there is a mild tendency for smaller groups to have bigger winning numbers. At least the variation is more significant.

This is yet too bold to formulate a connection between the size of the group and the winning number, but probably the reason is that when the size of the group is bigger, the number of “irrational” players increases. It can be due to some stable percentage of such persons in any group or other reasons, but it is an exciting connection to investigate.

4.4 Intentionally Irrational?

Another interesting finding is that after the first round finished, observing the result and listening explanation about the NE number of players who choose over 90 (it is an non-winning choice) increases. It is not accidental; data from (Povea and Citak, 2019) also show an increase in 2-5 rounds. We believe that this is quite an important part of the play. This phenomenon is evident in high school children with solid math backgrounds (usually, they have more freedom and self-confidence in choosing non-standard strategies).

All experimental data and R files for graphs can be accessed in the open repository (Ignatenko, 2021).

5 AGENT-BASED MODEL

The existence of irrational behavior challenges the basic game-theoretic assumption about self-interest and the capability to calculate the best option. In other words, real people do not think like machines or algorithms. They form hypotheses or expectations using simple rules. These rules are influenced by emotions and social norms and can be changed depending on feedback (reinforced). This use of inductive reasoning leads to two issues. First, what rules that people follow? Second, suppose we know these rules. How do we model the behavior of many interacting, heterogeneous agents in that situation? We start with the definition of agents and the formulation of rules of their behavior.

So we have agents of one type – players. Each agent has three variables: level of thinking (current level of reasoning), choice (a current number he chooses), and Boolean variable irrational, which is true or false.

Also, we define interface parameters, which define the setup of players. First of all, it is the number of players. We can also define the percentage of level 0 players, p of the game, and irrational setup, which will be explained later.

The setup of the game is following:

1) creates several players;
2) defines the level of reasoning for each player using the formula: level-0 percent from the interface, 100 – level-0 is divided into three parts. Two parts are level-1 reasoning, and one part is level-2 reasoning;

One round of the game proceeds in the following way:

1) each player chooses a number using their level of reasoning and some randomization. Basically, player with k level generates normal variable with mean $50 * p^k$ if $k > 0$ and $k \leq 4$. If $k = 0$, the choice is uniformly random from 1 to 100. When $k = 5$, the choice is 1 (this is an infinity level of reasoning);
2) if the player is irrational, he chooses 100. It is to model irrational behavior that can be observed from experiments;
3) the winner is calculated using the game formula;
4) players, with a choice wider than the winning number, increase their understanding level by 1. If the level is already equal to 5, it remains the same;
5) if the player is not winner, he becomes irrational with some small probability if boolean variable irrational is on.

The model is available in COMses library of Netlogo models. To measure experiments data we use BehaviorSpace tool with following parameters:

```
["percent-level-0" [50 5 100]]
["num-players" [10 10 100]]
["irrational1" false true]
["p" 0.66]
```

Here [10 10 100] means we launch simulation for 10, 20, and so on several players. On each step, we wrote to file the choices of players. 10 steps limited each particular games. In total, we had 11000 runs.

Epstein (Epstein, 1999) defines following characteristics of agent-based model:

1) heterogeneity; agents are different in some ways;
2) autonomy; each agent make own decisions;
3) explicit space; agents interact in a given environment;
4) local Interaction; agents generally interact with their neighbors and immediate environment.
5) bounded Rationality; agents have limited information and computing power. Agent behavior is generated by simple rules that may adapt over time;
6) non-equilibrium dynamics.

In this model, we consider only five levels, where level 5 means common knowledge when a player chooses 1. When no irrationality is in the model, we can observe typical convergence to equilibrium (figure 8) left, and this is a stable pattern.

But as we already know from the experiments, it is not what we can observe in real life. So irrational behavior was included to meet the pattern from the data. Irrationality in our model is implemented as 'anger' when a player who is currently a loser sometimes goes to irrational mode and chooses 100 in one next round. This leads to an exciting pattern (figure 8) right, when sometimes the winning number increases in the second round, but convergence to equilibrium is inevitable.

5.1 Data Analysis

In this section, we analyze data from simulations and compare them to previous results. In the following plot (figure 9) we can observe convergence towards

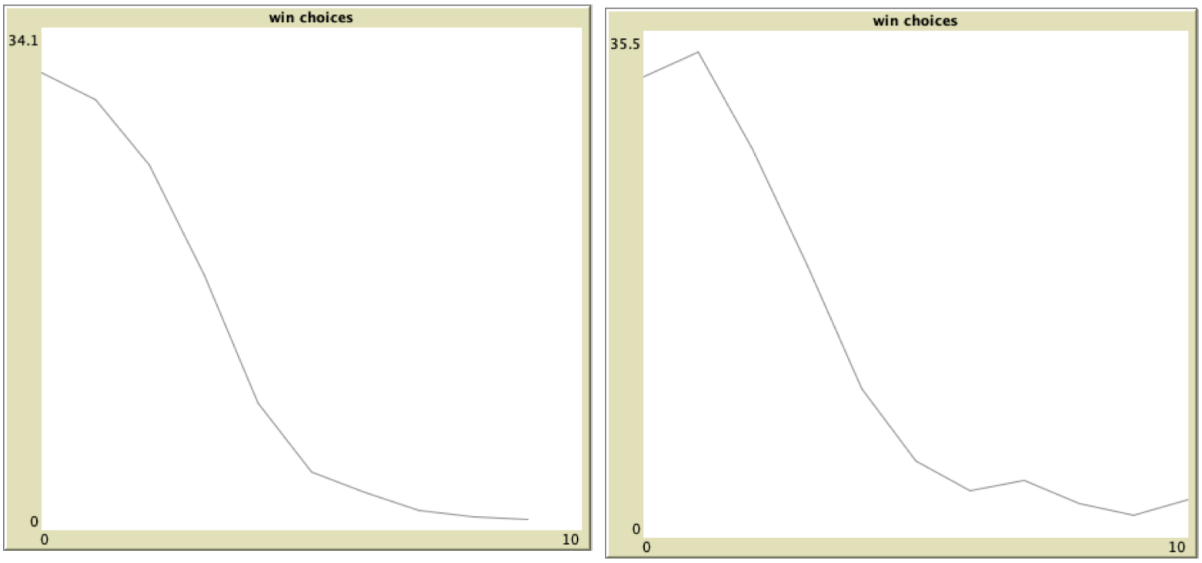

Figure 8: Plots of players win choice over round.

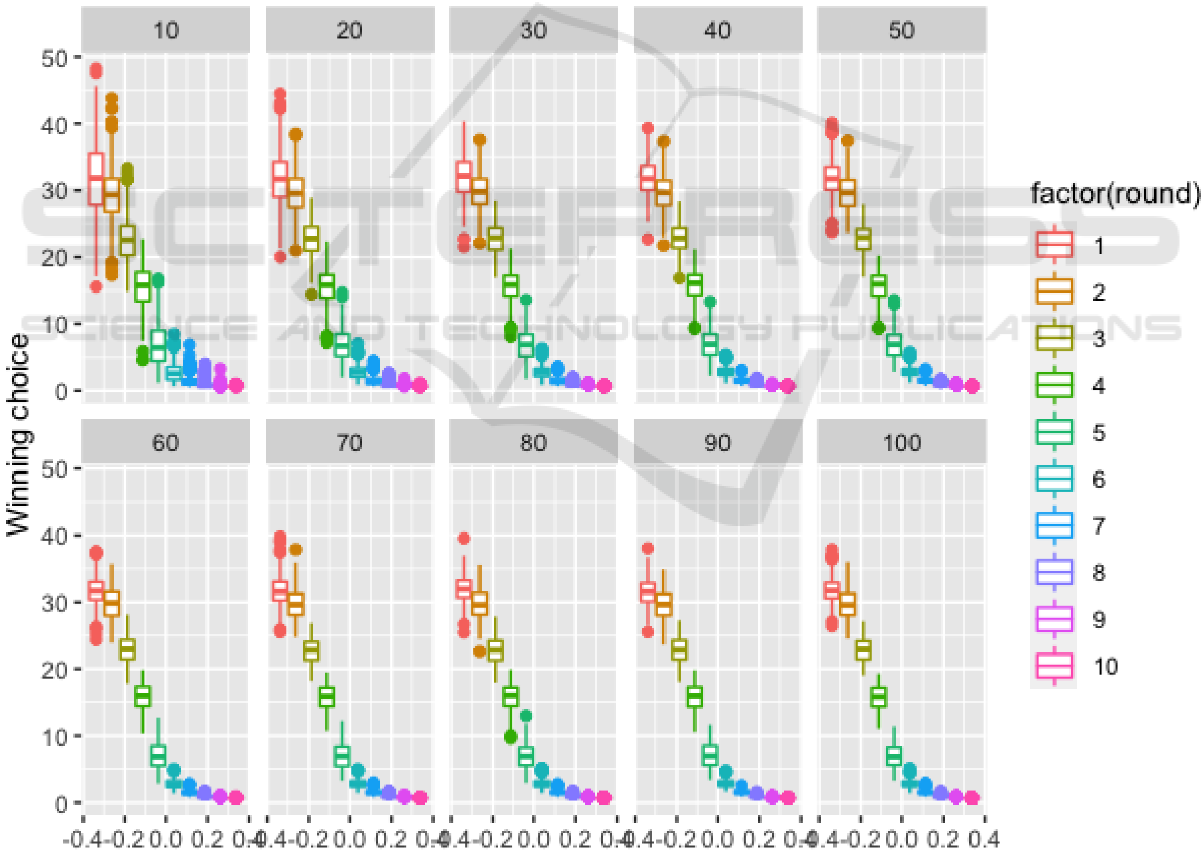

Figure 9: Plots of players win choice over round.

equilibrium. As expected volatility of choices is more significant in smaller groups.

Now let us plot (figure 10) winning choice in the first round compared to the winning choice in the second round. As we can see, there is a big cluster around (33,31). The color here is the initial percentage of low-level players. High level means that almost all players initially are randomizers, and then they learn

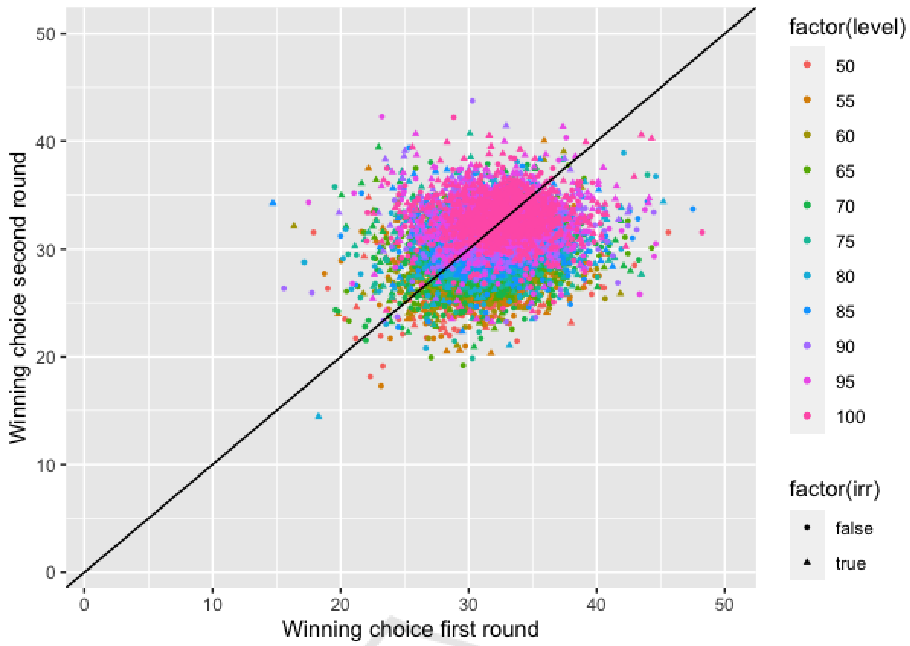

Figure 10: Plots of players win choice for first and second rounds.

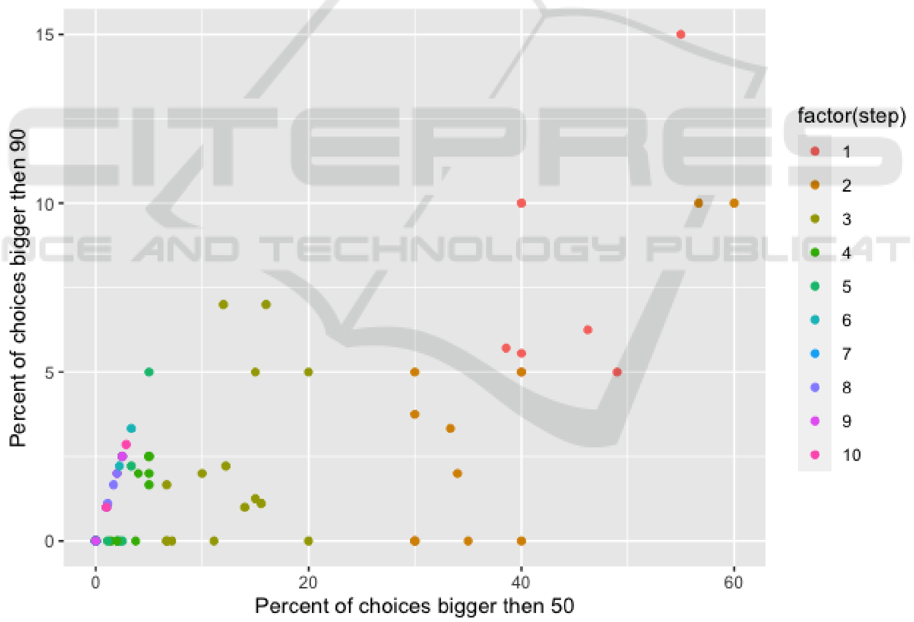

Figure 11: Irrationality of players.

from round to round. A trim level allows for more efficiency when points are like (30,20). This is quite similar to human experiments data.

The next plot is about the irrationality of players. It seems that there is a connection between the percent of players who chooses numbers bigger than 90 and the percent of players who chooses numbers bigger than 50. It is very similar to figure 8 from experimental data. The reason, as we can understand it, is in the nature of irrational behavior in games like the k-beauty contest. It was a common situation when players at some point (we have data for round 2) lost hopes of winning and just chose 100 for fun. At the same time, the majority of players are still thinking strategically and choose numbers around winning numbers (20 – 30 range). This gives a characteristic line of dots because, at some point, all players who chose numbers bigger than 50 are the players who chose 100.

6 CONCLUSIONS

In this paper, we have presented data from experiments on the k-beauty contest game. We provide access to raw data files and files with data manipulation, metrics calculation, and plot building. This will hopefully support reproducibility in this area of research. Then we discuss the results of the experiments and provide an analysis and explanation of patterns of behavior. It seems possible to confirm the existence of a pattern in decision-making – every group behaves almost the same way when dealing with an unknown strategic situation. We can formulate findings in a few short notes. First, participants have chosen not winning moves (> 66) partly because of a new situation and trouble understanding the rules. However, a high percentage of such choices was present in the second round, when players knew what was going on. This effect was especially notable in the cases of high school and adults and almost zero in the case of special math schools and kids below 9th grade. We can hypothesizeesis that high school is the age of experimentation when children discover new things ared do not afraid to do so. Second, the winning number as the decision of a group is decreasing in all cases, so we can see that group is learning fast and steady. Even if some outliers choose 100, the mean still declines with every round. There seems to be an unspoken competition between players that leads to improvement in the aggregated decision even if no prize is at stake. It is a plausible scenario when all participants choose higher numbers. However, this did not happen in any experiment. Third, a a stable percent of people choose about 100, and it is not about learning how to play the game. We think this is something like a -1 level of reasoning when the player intentionally plays a “bad move”, and this is an essential part of the model. If we neglect such persons and their motivation, our model will not be correct.

In the second part of the paper, we presented an agent-based model using conclusions about human behavior in the game. Simplification is the key to building good ABM, so in the model, agents have only a few parameters: level and irrationality. Based on these two parameters, agents choose a number on each step. Depending on other players’ choices model determine winners and losers: this state influences agents’ future level and irrationality. We perform about ten thousand games (each ten rounds long) and apply the same analysis as before. This approach shows that in some aspects, agents’ behavior is close to humans. Future investigation of this model will concentrate on the following possible modifications:

- deterministic choices of agents;
- different numbers of reasoning levels;
- rewards and punishments as elements of learning;
- implementation learning models from current research;
- build testing environment to automatically compare different learning strategies.

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