The Use of ICT Tools in Teaching Mathematical Modeling to Students

Olha I. Matiash[1][a], Liubov F. Mykhailenko[1][b], Natalia Y. Dmitrenko[1][c], Halyna D. Kateryniuk[2][d] and Ihor V. Kalashnikov[1][e]

[1]*Vinnytsia Mykhailo Kotsiubynskyi State Pedagogical University, 32 Ostrozkoho Str., Vinnytsia, 21001, Ukraine*

[2]*Regional Scientific Lyceum of the Municipal Higher Education Institution "Vinnytsia Humanitarian Pedagogical College", 13 Nahirna Str., Vinnytsia, 21001, Ukraine*

Keywords: Mathematical Modeling, Information and Communication Technologies, Distance Learning.

Abstract: The study reveals that for the development of students' mathematical competency it is important to provide effective conditions for the formation of mathematical modeling skills. It is considered that methodologically balanced use of information and communication technologies (ICT) in the process of teaching mathematical models is a key means in the formation of students' mathematical modeling skills. In the article, the authors described the possibilities of using the electronic manual "Methodological Tool for Developing Students. Ability to Mathematical Modeling" and the mathematics teacher's website in today's conditions of forced distance learning. According to the results of the expert survey, it was elucidated that there was a request from the mathematics teachers for methodologically adapted and substantiated electronic manuals. For mathematics teachers, the most important aspects in such manuals were: methodologically successful selection of applied problems; modern, relevant content of tasks for students; methodological assistance to teachers in order to increase the effectiveness of teaching. The research on the effective use of ICT in the process of formation of students' mathematical modeling skills was based on the same principles as the development of computer-oriented methodical learning systems were based: the principle of interest in learning, the principle of adaptability to the individual characteristics of students, the principle of the student's search activity, the principle of self-evaluation and self-actualization, the principle of individualization of learning, and the principle of cooperation and mentoring. It is concluded that it is important to form skills of using ICT for creation and study of mathematical models to prospective teachers of mathematics.

1 INTRODUCTION

All recent Ukrainian educational documents on mathematics education emphasize that mathematical competency is the ability to see and apply mathematics in real life, understand the content and method of mathematical modeling, build a mathematical model and study it by using mathematical methods, and interpret the obtained results. We consider the ability to mathematical modeling as a direct feature of students' practical competency, which is formed in parallel with the formation and development of skills to use mathematical modeling in educational activities and practice.

[a] https://orcid.org/0000-0002-7149-9545
[b] https://orcid.org/0000-0001-5051-5561
[c] https://orcid.org/0000-0002-3556-0003
[d] https://orcid.org/0000-0002-5794-7743
[e] https://orcid.org/0000-0001-7961-8134

The analysis of scientific and methodological publications shows that research on the problems of teaching mathematical modeling to students in different countries around the world has evolved from simple quality research cases to large research projects. The analysis of the Ukrainian scholars' pedagogical research reveals that special attention should be paid to the problem of providing the effective conditions for the formation of students' mathematical modeling skills. We completely agree with opinion of Semerikov et al. (Semerikov et al., 2010), formulated more than ten years ago, that "today it is no longer necessary to prove the obvious fact that increasing the effectiveness of the study of all school subjects is based on the systematic use of ICT". The intensive development of computer technologies is leading to the emergence of new approaches to the educational process. The quarantine requirements for COVID-19 have accelerated this process. The program of education reform

Matiash, O., Mykhailenko, L., Dmitrenko, N., Kateryniuk, H. and Kalashnikov, I.
The Use of ICT Tools in Teaching Mathematical Modeling to Students.
DOI: 10.5220/0012066900003431
In *Proceedings of the 2nd Myroslav I. Zhaldak Symposium on Advances in Educational Technology (AET 2021)*, pages 675-687
ISBN: 978-989-758-662-0

activities developed by the Cabinet of Ministers of Ukraine provides the creation of modern electronic educational resources, in particular, electronic textbooks and manuals. In the context of this article, the particular interest is paid to the scientific heritage of Myroslav I. Zhaldak as to the methodological activities of teachers, in particular, "pedagogically balanced, theoretically and experimentally justified use of innovative ICT in the educational process harmoniously combining with the scientific-educational heritage of the past allow to form the knowledge that underlies many contemporary professions related to new information and production technologies" (Zhaldak, 2003).

The purpose of the article is an explanation of the methodologically balanced use of ICT by the mathematics teacher in the process of teaching mathematical modeling to students in the present-day forced conditions of distance learning.

2 LITERATURE REVIEW

The analysis of scientific publications has shown that the problem of formation and development of mathematical modeling skills is one of the most globally recognized problems in the study of mathematical education. The most important arguments for this conclusion are:

- Since 1983, the International Conference on the Teaching and Learning of Mathematical Modeling and Application (ICTMA) has been held every two years. The conference discusses the state and problems of teaching mathematical modeling to students in the form of an international discussion. The conference abstracts are regularly published in the series Springer's International Perspectives on the Teaching and Learning of Mathematical Modeling.
- At the CERME International Forum, organized by the European Society for the Study of Mathematical Education, one of the traditional sections is "Applications and Modeling".
- Springer search system issued 58,613 search results for "Mathematical modeling in secondary school". Moreover, these results are publications of recent decades, which address the problem of formation and development of mathematical modeling skills.

In order to obtain information about the areas of current studies and their results related to the use of ICT in teaching mathematical modeling, it is considered to observe some recent publications.

Tezer and Cumhur (Tezer and Cumhur, 2017) analyzed the results of research connected with the impact on mathematical achievement, problem-solving skills and students' views by means of the educational model 5E and the method of mathematical modeling in the process of studying the topic "Geometric Objects". The results of statistical analysis proved that learning with the 5E "Instructional Model" in experimental group 1 and the method of mathematical modeling in experimental group 2, improved students' academic achievements. However, the method of mathematical modeling was more effective for mathematical achievements and problem-solving skills.

Blum (Blum, 2011) and Kaiser (Kaiser, 2019), the most authoritative German researchers on the formation and development of mathematical modeling skills, founded the ISTRON group. Since 2014, the ISTRON group has published 20 volumes of publications to support teachers in solving real problems of teaching mathematical modeling to students at school. In all these studies, various aspects of the problem of modeling training were analyzed in complex. In particular, one of the learning environments, designed specifically for beginners in modeling, examined the KOMMA (computer learning environment). The KOMMA learning environment included four variants of heuristic activity samples. In these samples, two fiction characters solved simulation problems and explained their ideas, heuristic strategies, and tools. All samples were structured by means of a 3-step simulation cycle. The eight-grade students' competency of modeling (316 people) was tested before the experimental training, immediately after it, and four months after the experiment. The results of the research have shown a significant increase in the competency of modeling immediately after the implementation of the learning environment and a slightly lower long-term effect (Greefrath and Vorhölter, 2016; Blum, 2015).

In recent years, in Germany, the methodological activity of teachers in teaching mathematics (including modeling) has changed mainly due to the active development and implementation of digital technologies. Solving application problems, a computer or a well-equipped graphing calculator can be especially useful tools to support teachers and students. For example, Henn (Henn, 2007) suggested the use of digital tools, such as notebooks with algebra software, as it allows to incorporate complex programs and modeling into everyday learning.

One of the possibilities of using digital technologies is research and experiments (Hilscher, 2002). For instance, a real situation can be transferred to a geometric model, where one can experiment with dy-

namic geometry software or spreadsheet analysis that seems to be similar to experimental modeling. Modeling in the process of experiments is designed to provide an idea of the real system presented in the model (Greefrath and Weigand, 2012). Applied mathematical simulations performed by a computer can be perceived as part of a simulation cycle in which a numerical model, developed from a mathematical model, is tested and validated by comparing it with measurement results (Sonar, 2001). A common use of digital tools, especially computer algebraic systems, is to calculate or estimate numerical or algebraic solutions (Hilscher, 2002). Without the use of a computer, students would not be able to make these assessments, at least in a reasonable amount of time. In addition, digital tools can perform visualization of the studied object (Barzel and Hußmann, 2009; Hilscher, 2002; Weigand and Weth, 2002). Digital tools also play a useful role in controlling and testing the mathematical modeling skills (Barzel and Hußmann, 2009). If computers with Internet connections are provided for teaching mathematics, they can be used for research (Barzel and Hußmann, 2009), for example, in the context of programs.

German scientists state that there is currently little empirical knowledge about the possibilities of learning modeling and recommendations for working with digital tools in teaching mathematics. Only some case studies have been conducted, there are no large-scale experiments on the introduction of computer technology in the process of teaching mathematical modeling. Case studies (Greefrath et al., 2011; Geiger, 2011) note that digital tools can be useful for each step of the modeling process, especially for interpretation and validation. Open research questions can be found in the studies of Niss et al. (Blum et al., 2007; Niss and Højgaard Jensen, 2002): "How should digital tools be used in different classes to support modeling processes? What is the impact of digital tools on the range of modeling issues that need to be discussed? How does the use of digital technologies affect the learning culture? When do digital tools improve or hinder learning opportunities in the modeling process?"

Based on the mentioned analysis of teaching mathematical modeling in Germany, we indicate several current issues and tasks in formation of students' mathematical modeling skills for Ukrainian mathematics education:

- modeling activities can be significantly changed due to the development of digital technologies;
- the activity and role of mathematics teachers in the successful implementation of mathematical modeling in mathematics lessons is important;
- the focus should be on research of particular modeling lessons, as well as the entire modeling education environment.

In the article (Matiash and Mykhailenko, 2021) we analyzed the course of scientific discussions in terms of the International Forum CERME-2021 (we were lucky enough to be participants of it), which related to the use of information technology in the formation of mathematical competencies of students. Nowadays, the attention of researchers of mathematics education in the world is focused on problems: blended learning of mathematics students, blended learning in the process of professional training of mathematics teachers, and the developing tools, platforms and learning equipment for online mathematics education.

3 THEORETICAL BACKGROUND

Systematic application of the method of mathematical modeling at the process of teaching mathematics at school can be considered as a means of implementing the applied orientation of the school course of mathematics. In particular, it means: creating a bank of mathematical models that describe real phenomena and processes, have general cultural significance, and are studied in related courses; formation of students' knowledge and skills which are required for the study of these mathematical models; teaching students to build and study the simplest mathematical models of real phenomena and processes.

3.1 Mathematical Modeling

The essence of the concept of "mathematical modeling" is explained in different ways in many modern available sources:

1. Modeling is the study of knowledge objects on their models; construction of models of real objects and phenomena (living organisms, engineering structures, social systems, various processes, etc.). Mathematical modeling is the most modern comprehensive method of scientific research; it is the process of creating mathematical models. A mathematical model is a system of mathematical relations that describes the studied object, process, or phenomenon (Mathematical model, 2023).
2. Mathematical modeling or mathematical simulation is a research method of processes or phenomena by creating mathematical models and studying these models. Mathematical modeling allows

to replace a real object with its model and then study it (Mathematical model, 2023).

3. Mathematical modeling is one of the main modern methods of systems research. It usually involves the creation of a conceptual model of the studied object, its formalization and transformation into a mathematical or computer model, verification of adequacy and further study of the model by means of analytical or numerical methods and modern computer technology (Semenova, 2014).
4. Modeling is a process of real system research, which includes construction of a model, its research and transfer of the obtained results to the studied system. A model can be defined as an object that in some respects coincides with the prototype and is a means of describing, explaining and/or predicting its behavior. The mathematical model of the real system (process) means a set of relationships (formulas, equations, inequalities, logical conditions, operators, etc.) that determine the characteristics of the system depending on its parameters, external conditions (input signals, influences), initial conditions and time (Bakhrushin, 2004).
5. Modeling is the construction (or selection) and study of an object of any nature (model) that can replace the studied object (the original) and the study of model provides new information about the studied object. *Mathematical modeling* is the highest form of modeling. It contributed to the development of science and technology in industrial society, and the advent of electronic computing facilities led to the rapid development of contemporary – post-industrial society (Stanzhytskyi et al., 2006).

Thus, *mathematical modeling* is considered mostly as the study of the object properties in a mathematical model. Simultaneously, the mathematical model is an approximate description of a phenomenon or process of the external world, which is presented by mathematical symbolism. Mathematical modeling is one of the most up-to-date directions, which is closely related to the introduction of modern computer equipment and information technologies (Zahrai and Kotovenko, 2007).

From the pedagogical point of view, it does not matter where (mathematics, physics, computer science) students learn to model. It is important to understand that by means of modeling, the system-combinatorial thinking and the ability to solve real problems are formed (Teplytskyi, 2000). It is worth mentioning that modeling forms the world view and the scientific picture of the world not only for the student but also for the teacher. Modeling is the method, the most adequate to modern requirements for the education system, of including a computer in the learning process, which provides an active type of educational and cognitive activities (Morze et al., 2022). In particular, a computer model is a software environment for a computational experiment. Based on a mathematical model of a phenomenon or process, it combines tools of the experiment object analysis and information display. Using widely computer graphics, 3D modeling programs help to turn individual ideas into smart models and prototypes. 3D models are used in various fields: cinema, computer games, interior design, architecture, etc. Choosing software for modeling is a rather difficult process, as it is not easy to find a program that would have all the required functionality.

The use of computer modeling in the learning process (study of phenomena based on ready-made models, constructing models by students themselves) can increase the intensity of learning and the students' cognitive activity. The advantages of educational computer modeling are related to overcoming the formality of knowledge acquisition, the development of research and design skills, and the development of students' intellectual abilities (Morze et al., 2022).

Teplytskyi (Teplytskyi, 2000) proposed a methodological system for studying computer modeling and aimed to reveal the content of all school education through the introduction of the concepts of "model" and "modeling", which have developed both in specific sciences and in the methodology of science in general. The scholar considered the following definitions to be the most accessible to students: a model is a mentally imagined or materially realized system of reflecting or reproducing the studied object and the study of the model provides new knowledge about this object (Shtoff, 1963). Additionally, the model is considered as a system that does not differ from the studied object in some of its essential properties and differs in all other insignificant properties (Biriukov and Gutchin, 1982; Maliarchuk, 1997).

In our study, we reveal the content of the mathematical model as a set of mathematical relationships, equations, and inequalities that describes the basic laws in the studied process, object, or system (Kvietnyi et al., 2012). Modeling in the teaching of mathematics at school is understood as the process of building a model. Studying mathematics at school, students should realize that the process of any applied problem solving is divided into three stages: 1) formalization (transition from the situation described in the problem to a formal mathematical model of this

situation, and from it to a clearly formulated mathematical task); 2) solving the problem within the constructed model; 3) interpretation of the obtained solution of the problem and its application to the initial situation.

3.2 Key Aspects of the ICT Use in Teaching Mathematics to Students

In the publications of Zhaldak (Zhaldak, 2003; Zhaldak and Hrybiuk, 2014), ICT in education are considered as a set of computer-oriented educational and teaching materials, software, and hardware for educational purposes, as well as a system of scientific knowledge about the role and place of computer technology in the educational process, the methods and forms of their pedagogically balanced, methodologically motivated, and appropriate use in order to improve the educational process.

There is awareness among mathematics teachers that the introduction of ICT tools can significantly help to create more effective conditions for students' cognitive activity and contribute to the formation of their competencies. "The basis of informatization of the educational process is grounded on the creation and widespread introduction of new computer-based teaching methods into everyday pedagogical practice based on the gradual and non-antagonistic principles, omitting destructive reforms. Embedding information and communication technologies in existing didactic systems is considered as a harmonious combination of traditional and computer-oriented learning technologies, without denying and rejecting the achievements of pedagogical science of the past, but, on the contrary, their improvement and strengthening by means of including the use of advances in computer technology and communications as well" (Zhaldak and Hrybiuk, 2014). According to Zhaldak (Zhaldak, 2003), the specific components of a teacher's information culture are the ability to use ICT for training, support, analysis, adjustment, and management of the educational process; the ability to choose the most rational methods and tools of learning, and take into account the individual characteristics of students, their requests, inclinations, and capabilities; ability to combine effectively traditional teaching methods with new ICT. At the same time, the use of ICT in the educational process (Zhaldak, 2003; Zhaldak and Hrybiuk, 2014) should not promote only the study of certain learning material, but, first of all, the comprehensive and harmonious development of students' personalities and their creative capabilities. Thus, in the conditions of active use of ICT tools in teaching mathematics, the requirements for the methodological competency of the teacher increase significantly.

Nowadays, in the era of mobility and globalization, there is an urgent need to use the Internet, social networks, and personal sites. Looking at modern students' level of informatization, it becomes clear that the teacher needs "to keep up with the times", as traditional teaching methods, in some way, lose their effectiveness. Currently, a mathematics teacher should have to some extent universal, fundamental and modern knowledge in order to be able to use ICT in a methodically competitive and effective way, and create conditions to develop students' inclinations and capabilities, meet educational and cognitive needs.

The study and justification of the required directions of ICT use in the educational process should be considered as one of the most important pedagogical problems. Zhaldak (Zhaldak, 2003) note that the problems of teaching mathematics in secondary schools by means of ICT are studied insufficiently. Despite a significant amount of research on this topic, there is a lack of computer-based scientific and methodological support for teaching school subjects in the context of systematic pedagogically balanced and methodologically motivated use of ICT, particularly, in mathematics. The methods of studying the effectiveness of computer use in education need to be improved (Zhaldak, 2003; Zhaldak and Hrybiuk, 2014). Studying the effectiveness of teaching mathematics to students by means of ICT, it is necessary to identify its criteria and reasoning factors. The teacher can achieve the set goals only if these goals are accepted and achieved by the students. Evaluating the possibilities and expediency of using ICT in teaching mathematics to students, it should be mentioned that a computer is only teachers and students' tool for educational activities.

It was remarked principally by Razumovsky et al. (Razumovsky et al., 2013) that with the introduction of computers in the educational process, the possibilities of many methods of scientific knowledge are increased, especially the method of modeling, which can dramatically influence the intensity of learning. While modeling, the essence of phenomena is singled out and their commonality is cleared up, i.e. scientific and theoretical thinking is developed. However, the fascination with the use of ready-made models threatens to prematurely disconnect between the studied phenomenon and reality. It often happens when students are asked to work with ready-made models without disclosing the process of model creation. Since the objects of study must still be real phenomena, their replacement, by abstract concepts and symbols with an insufficient base of observations and experience, often leads to pernicious formalism, when

the pretended knowledge lacks the essence.

The issue of computerization of school education and the development of appropriate pedagogical software is the subject of constant attention of Ukrainian scholars (Zhaldak, 1989; Zhaldak et al., 2012, 2020; Zhaldak and Franchuk, 2020; Leshchuk et al., 2022; Kramarenko et al., 2019; Morze et al., 2022; Semerikov et al., 2021).

4 TEACHING MATHEMATICAL MODELING TO STUDENTS IN CONDITIONS OF ICT USE

4.1 Literature Review of Ukrainian Scientists' Research Results

The Ukrainian scientists study the problem of students' forming mathematical modeling skills in many ways. In particular, the content of teaching students the methods of mathematical modeling is determined; the main stages of constructing a mathematical model and their operational composition are highlighted; the functions of modeling in the educational process are described; some methodological recommendations for teaching mathematical modeling to students have been developed; the ways of using ICT in the process of teaching mathematical modeling to students are proposed. The main methodological fundamentals of teaching mathematical modeling to students are revealed in the studies of Vozniak and Vozniak (Vozniak and Vozniak, 2003), Gnedenko (Gnedenko, 2020), Slepkan (Slepkan, 1983), Shvets (Shvets, 2009).

Filimonova and Shvets (Filimonova and Shvets, 2010; Shvets and Filimonova, 2010) developed the main fundamentals of the methods of forming the middle-class students' knowledge, skills and abilities in mathematical modeling. Among them are the following aspects: the formation of mathematical modeling skills should be provided through successful use of organizational and methodological tools. In particular: a rational combination of traditional and innovative teaching methods. A special role should be given to interactive and project methods.

Panchenko and Shapovalova (Panchenko and Shapovalova, 2010), one of the goals of mathematical training of prospective mathematics teachers in higher education is teaching the basics of mathematical modeling and training students for the introduction of ideas and methods of mathematical modeling in the course of mathematics in secondary school. To achieve this goal, it is necessary to perform some tasks, namely: to teach students and pupils to use ICT in creating and studying mathematical models.

From the point of view of our research, the following position draws our attention: it is expedient to include elements of mathematical modeling in secondary school education to a somewhat greater extent and with greater penetration into the essence of models than it is done in many cases today. Obviously, it should be done in classes of an advanced study level of mathematics in secondary school. However, in classes with a general academic level of mathematics, physics or computer science, the appeal to mathematical modeling should be moderately dosed and balanced with the interests and abilities of students (Krasnytskyi and Shvets, 1997).

Since the method of mathematical modeling is a powerful tool for studying various processes and systems, according to many Ukrainian scholars, the concept of mathematical model and some general fundamentals related to it should be illustrated in one form or another at the learning process of the entire systematic course of mathematics in school. The samples of this method application for solving specific problems are presented in many well-known monographs and textbooks.

Ukrainian scientists' research emphasizes that for older adolescents it would be appropriate to organize the educational process with lessons-lectures, lessons-seminars, lessons-conferences, etc., involvement in writing research papers, and projects. Units of school curricula in various discipline courses related to solving problems for work, movement, interest, progression, application of derivatives and integrals can serve to develop the students' mathematical modeling skills. At the same time, Ukrainian researchers point out the current problems in the mathematical training of students, which often cause difficulties in the process of developing mathematical modeling skills. As for the method of mathematical modeling as a method of scientific research and learning cognition, in fact, it was not realized systematically and continuously in the school course of mathematics in the last three decades.

4.2 Methods of the Research

The Ukrainian and foreign researchers emphasize the need to use project technologies in the process of formation of students' mathematical modeling skills. We consider the organization of project activities of students at school as one of the priorities of modern education. Educational projects allow taking into account the individual characteristics of students that contribute to the formation of their active and inde-

pendent position in learning, and readiness for self-development. The project method, as a component of the education system, creates personal motivation to solve an interesting problem. The project method is associated with active practical activities and teamwork. We assign a significant role to ICT in the organization of students' project activities. In particular, ICT (mathematics teacher's website, social network group, and the electronic manual) were actively used in the process of teaching mathematics to students of the 9th grade in order to improve the conditions for the formation of mathematical modeling skills.

We tested the students' implementation of a research project on the topic "Geometry and Football". A group of nine-grade students from the Regional Sports and Humanitarian Lyceum-Boarding School of the Municipal Higher Education Institution "Vinnytsia Humanitarian Pedagogical College" took part in the project. The designed project was awarded the third place at the Ukrainian Conference-Olympiad of Geometric Creativity named after V. A. Yasinskyi. As the participants of the project were students of the sports and humanitarian profile, the sports direction was chosen for the project. Out of one hundred students of the lyceum – 70 students were engaged in various kinds of sport at that time. The most number of lyceum students played football – 22 students (31% of all lyceum students). Therefore, the play of football was stressed in the project. Students learned with great enthusiasm that mathematical models can be used to solve certain practical problems in football, helping the team and coaches to achieve the best results. Our experience has shown, that solving football problems has increased interest, motivation, and as a result, the effectiveness of the study of mathematics, geometry in particular. Cooperation with students was established by means of the teacher's personal website and social networks. All students in the class were asked to solve geometric tasks about football. Each student sent the solution personally, hidden from other project participants. If the student solved the problem correctly, he received the next task, in case he did not solve the task, he received instructions about mistakes and tips on how to correct them. The students were motivated by the fact that they did not know at what stage (what task) the rest of the classmates were currently solving.

The project aim was not only to increase the motivation of sports students to learn geometry with the help of new technologies but also to show the importance and practicality of geometric knowledge in everyday life and future activities. It was important to ensure that students learned to come up with their ideas, were not afraid to express their opinions, and were able to think logically and critically.

In order to do this, students were offered a research task: to create problems in geometry, which would be directly connected to football and things related to it (football field, football goal, football ball, etc.). Surprisingly, but not only the students with the high level of academic achievements worked hard. As samples, the students used the problem tasks that they solved remotely in the first phase of the project.

We managed to ensure that students learned to come up with their ideas, were not afraid to express their opinions, and tried to think logically, in particular, critically. In the process of implementing the designed project, it became clear that we managed to create conditions with good opportunities to convince students that mathematics is the way to improve a lot of circumstances of the surrounding reality. For further diversifying the methods of motivation to master the method of mathematical modeling, it is important to select and accumulate effective teaching tools and techniques by mathematics teachers. Among effective teaching tools and techniques are applied tasks, methods of implementing interdisciplinary links, preparation and conduct of special practical work with an applied focus.

As we have already mentioned in this article, one of the main problems in the process of forming students' skills of mathematical modeling in Methodology of mathematics is the lack of methodological materials, especially for high school students. Therefore, in order to help mathematics teachers, we have developed and experimentally tested the textbook "Methodological Tools for Developing Students' Ability to Mathematical Modeling" (Matiash and Kateryniuk, 2019). In the textbook, it is presented and substantiated the theoretical aspects of students' ability to mathematical modeling, offered a system of applied problems for the formation of mathematical modeling skills (tasks to find the smallest or largest values; geometric problem-tasks; physical problems; stochastic problems; production problems; everyday and professionally oriented tasks), explained the methodological aspects of solving problems based on mathematical modeling. The methodological recommendations on the organization of independent cognitive activity of students in order to form the skills of mathematical modeling, students' project activities, and diagnostic tools for the formation of students' mathematical modeling skills were given. As a separate unit of the textbook, we have provided teachers with a list of publications on the development of students' mathematical modeling skills. Thus, we have systematized relevant material for practical use in mathematics lessons at school, and

proposed authors' tasks. In the textbook, we have emphasized that at school, according to the learning outcomes identified in the mathematics curriculum, mathematics teachers must provide conditions for the development of students' mathematical modeling skills. The textbook is prepared for teachers of mathematics and prospective teachers of mathematics in order to provide methodological assistance in overcoming the problem of developing students' mathematical modeling skills.

Based on the printed textbook "Methodological Tools for Developing Students' Ability to Mathematical Modeling", we have developed an electronic manual. It is not just an electronic version of the printed textbook; it is also an electronic supplement to it. Simple and clear navigation allows turning pages quickly and easily. The use of an electronic appendix to the third section of the manual, which is devoted to the systematization of publications for further detailed study of materials on this topic, is of great practical value. It is much more convenient to use it as it contains all the publications we have mentioned, not just a list of them (as in the print version). They are available for viewing and downloading in *pdf* or *djvu* format. To download the e-manual, the following link can be used: https://drive.google.com/file/d/1K9B9VI6Yr12w0JHG4Y_aSOSJVsm3cuGd/view?usp=sharing.

According to the results of our research and experimental study aimed at finding ways to improve the effectiveness of forming the students' mathematical modeling skills, we can summarize that the personal website of the mathematics teacher is in great use. The personal website of a mathematics teacher should facilitate the exchange of experiences with colleagues, reduce the distance between teacher and parents, as well as between students and parents, and most importantly between students and teachers. The teacher's website allows implementing an individual approach, and focusing on the development of the personality of a particular student. With the help of the website, the teacher can distribute the required information in a short time, share important news, post educational and methodological hand-outs, creative work of students, their achievements, and much more. The personal website of the mathematics teacher has the opportunity to convey information to students in multimedia forms.

On the pages of the personal site, our teacher-experimenter (co-author of this article) added, edited and commented on materials, posted and analyzed test tasks, discussed various issues, and communicated with students. It is observed that modern students spend increasingly much time online and it is much easier for them to get the required materials online than to use libraries, books, reference books, and even writing home-assignment. According to most teachers' opinion, it is not very good. However, teachers should understand that they work with students of the new generation, and train them for life in a new society, which requires a modern personality, who is ready to live in a new information society. Out of 100 students (the survey was conducted at the Regional Sports and Humanitarian Lyceum-Boarding School of the Municipal Higher Education Institution "Vinnytsia Humanitarian Pedagogical College"), 95 have mobile phones (95% of respondents). 68% of students have mobile Internet access, and the rest students use Wi-Fi. After classes, 100% of respondents have access to the Internet. It confirms the ability of the mathematics teacher to communicate actively with students via the personal website.

During the experiment studying, the mathematics teacher's website provided an opportunity to draw students' attention by interesting learning materials, in particular, to the teacher's information competency. It expanded the possibilities of conveying interesting and important information to students and showing the wide application of the acquired mathematical knowledge in everyday life.

Such an environment allowed implementation of educational functions. The teacher, for example, placed the text of a problem task on the forum of the website and set the instruction: to construct a mathematical model. In the process of completing the task, students discussed, suggested their models, and when everyone expressed the opinion, the teacher corrected their steps, explained the incorrectness or inaccuracy of their reasoning, if a mathematical model did not correspond to the task in a result. On the contrary, the teacher could emphasize the right considerations, encourage further work, and summarize which model was still correct, or the most appropriate. The next step was the process of collective research of the mathematical model. Those students, who were not able to participate in the online discussion, had the opportunity to open the forum later and get acquainted with the discussion and the whole process of solving the tasks proposed by the teacher. Thus, we consider the function of the mathematics teacher's website "teacher-student interaction" to be quite effective. While experimental training, the question was put: "Why can't the teacher's personal website be replaced by communication with students on social networks?" According to the results of our research, we came to the conclusion that it is better to leave social networks for personal communication and entertainment, and everything that the mathematics teacher

wants to convey to his/her students is better to post on the pages of the teacher's personal website. In our opinion, the modern mathematics teacher can create a personal website by him/herself with the help of a site designer, using a simple and free service Google Sites, as well as due to information competency which was formed in the process of professional training in a higher educational institution.

We created the mathematics teacher's personal website (URL: https://sites.google.com/view/kateryniuk/). Its content and effectiveness in improving the conditions for the formation of students' mathematical modeling skills were essential for us.

Thus, modern content management systems (CMS), the availability of free hosting, and free website designers allow teachers to create and develop their personal websites. The personal pedagogical website is not just a matter of time, but a necessity for the fruitful work of any subject teacher who cares about providing conditions for improving the effectiveness of teaching. Distance learning and external studies are developing extremely, so the exchange of tasks between a student and a teacher is rapidly moving into the ICT medium. In our opinion, the main tasks that are effectively solved with the help of the teacher's personal website are searching for new ways to interact with students and the organization of distance learning and project activities on the Internet.

5 RESULTS AND DISCUSSION

Different research and experiment methods allowed to construct an up-to-date method for the formation of students' mathematical modeling skills and to explain the place and role of ICT use in this method. In order to determine the attitude of the mathematics teachers to some results of our research (expediency and quality of the electronic manual "Methodological Tools for Developing Students' Ability to Mathematical Modeling"), we used the method of expert evaluation (Delphi method). The main stages of implementation of the method of expert evaluations in our study were: expert selection, identifying factors for the survey, conducting the survey, analyzing the results of the survey, and processing the results.

32 experts were chosen from a group of mathematics teachers of Vinnytsia and Vinnytsia region, who took courses of continuing professional development for teachers in-service based on the Department of Algebra and Methods of Teaching Mathematics at Vinnytsia Mykhailo Kotsiubynskyi State Pedagogical University. Before the presentation of the electronic manual "Methodological Tool for Developing Students' Ability to Mathematical Modeling" to expert teachers, we found out the experts' preliminary vision of the importance of certain characteristics of the use of electronic manuals in teaching mathematics to students.

The survey suggested 10 factors:

(1) quality of the manual design;

(2) the presence of clear scientifically grounded recommendations for the formation of students' mathematical modeling skills;

(3) increasing the number of school mathematics lessons;

(4) modern, relevant for students the content of tasks;

(5) the conviction of the mathematics teacher in the need for special conditions for the formation of students' mathematical modeling skills;

(6) high teacher salaries;

(7) successful selection of applied tasks;

(8) the teacher's methodological competency;

(9) students' interest in learning material;

(10) teacher's knowledge and understanding of the criteria and indicators of the formation of students' ability to mathematical modeling.

The used methods of mathematical statistics were based on ranking. In our case, each factor was assigned a rank from 10 to 1 by each expert (in descending order to determine their relative importance). The required condition for the reliability of the collective assessment was sufficient consistency of opinions of the interviewed experts. To determine the consistency of group assessments, we used the concordance coefficient – a common rank correlation coefficient for a group of experts. Statistical processing of the results of the teacher survey allowed us to state that the most significant ranks were:

5. Conviction of the mathematics teacher in the need for special conditions for the formation of students' mathematical modeling skills;

7. Successful selection of applied tasks;

4. Modern, relevant for students the content of tasks;

9. Students' interest in learning material;

8. Teacher's methodological competency.

The consistency of experts' opinions was assessed by the concordance coefficient:

$$W = \frac{12}{m^2(n^3-n)} \cdot \sum_{j=1}^{n} \left[\sum_{i=1}^{m} x_{ij} - \frac{m(n+1)}{2} \right]^2,$$

where m – the number of experts, n – the number of factors.

In our case, $m = 32$, $n = 10$. The obtained value is $W = 0.64$, according to the scale for rank correlation coefficients, which falls into the interval $(0.6; 0.8)$. It demonstrates a good consistency of experts.

According to the results of the expert survey, it was found that the designed electronic manual contained the important characteristics of the use of electronic manuals in the teaching of mathematics to students that were relevant for mathematics teachers, and therefore would be useful for the formation of students' mathematical modeling skills. In order to identify the needs of mathematics teachers in printed or electronic manuals for the formation of students' mathematical modeling skills, the survey was proposed. The seventh question of the questionnaire was: "Would you like to have a manual in printed or electronic version?".

The response results of 500 surveyed teachers to the proposed question are presented in the following diagram (figure 1).

Analysis of respondents' answers to other questions of the proposed survey allowed to assert that 78% of mathematics teachers (out of 500 participants) need educational and methodological literature on the formation of students' mathematical modeling skills; 97.2% of teachers find it useful to develop a system of applied tasks; 66.4% of teachers need to master the method of solving problem tasks based on the mathematical modeling; 58.4% of teachers state that it is appropriate to develop lesson outlines for teachers oriented on the topic of mathematical modeling, and 30.8% of teachers admit the usefulness of such outlines. 90.4% of teachers are interested in diagnostic tools for the formation of students' mathematical modeling skills. 95.6% of respondents express a desire to get acquainted with the scientific and methodological publications on the formation of students' mathematical modeling skills. 58.6% of teachers want to have a textbook on teaching students mathematical modeling in the printed version and 68.3% of teachers – in the electronic version.

In the process of pedagogical experiment, and active communication with mathematics teachers, it was confirmed that "the use of ICT can significantly improve the efficiency of learning messages and data circulating in the educational process, due to their timeliness, usefulness, appropriate dosage, availability (intelligibility), noise minimization, and operational relationship between the source of educational information and the student, adapting the pace of presentation of educational material to the speed of its assimilation, taking into account individual characteristics of students, effective combination of individual and collective activities, teaching methods and tools, organizational forms that to some extent contributes to solving the problems of educational process humanization. Students and teachers remain the main participants in the educational process. Computers, software, and communication tools, are the only means of their activities. The effectiveness and efficiency of educational and cognitive activities of students depend on the teacher's awareness and skillfulness" (Zhaldak, 2003).

6 CONCLUSIONS

ICT and communication tools play an important role in the forming technology of students' mathematical modeling skills. Nowadays, it is important to teach prospective mathematics teachers to use ICT in the creation and study of mathematical models, and in the process of teaching mathematics to students. Due to forced conditions, distance learning development is accelerating, so communication between teachers and students is rapidly moving into the ICT medium. The results of teachers' surveys and the pedagogical experiment lead to the conclusion that the task of methodologically balanced use of ICT is complicated by the necessity to change the personal attitude of both teachers and students to self-education, self-development, and cooperation. Simultaneously, the Ukrainian scientists' ideas are confirmed that the readiness of mathematics teachers to use ICT depends significantly on:

- understanding the effectiveness of the use of ICT and communication tools in the educational process;
- ability to use modern computer equipment and tools of communication in professional activities;
- ability to competently assess the benefits, opportunities, and limitations of the use of ICT in the educational process;
- understanding the place and role of autonomous work of students using ICT;
- understanding the changes in the functions of the teacher in the organization of the educational process with using ICT (Zhaldak and Hrybiuk, 2014).

Our research on the effective use of and other methodological tools and techniques for forming students' mathematical modeling skills was based on the same principles as the principles of the development of computer-based teaching methodological educational systems were grounded by Zhaldak and Hrybiuk (Zhaldak and Hrybiuk, 2014): the principle of interest in learning; the principle of adaptability to

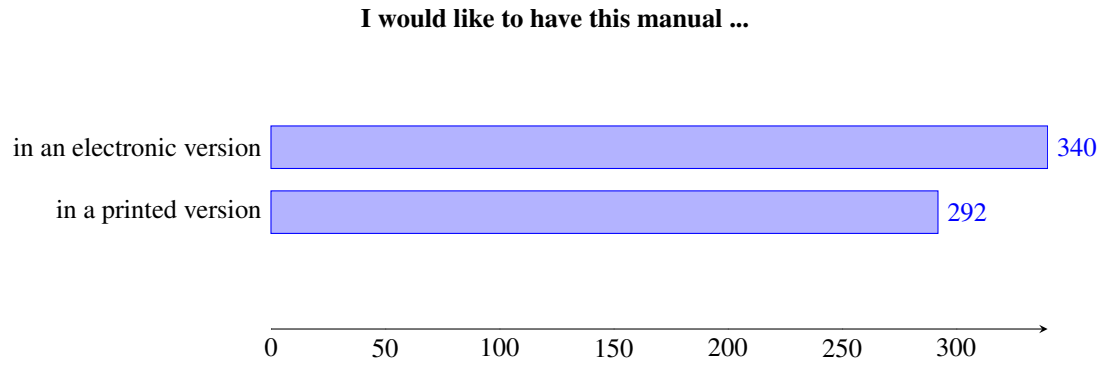

Figure 1: The results of the teacher survey.

the individual characteristics of students; the principle of the student's search activity; the principle of self-evaluation and self-actualization; the principle of individualization of learning; the principle of cooperation and mentoring. Thus, adherence to these principles in the process of formation of students' mathematical modeling skills is an important component of the methodological activities of mathematics teachers at school.

REFERENCES

Bakhrushin, V. E. (2004). *Matematychne modeliuvannia [Mathematical modeling]*. ZIDMU, Zaporozhye.

Barzel, B. and Hußmann, S. (2009). *Computer, Internet & Co. im Mathematik-Unterricht*. Cornelsen Vlg Scriptor.

Biriukov, B. V. and Gutchin, I. B. (1982). *Mashina i tvorchestvo [Machine and creativity]*. Radio i sviaz, Moscow.

Blum, W. (2011). Can Modelling Be Taught and Learnt? Some Answers from Empirical Research. In Kaiser, G., Blum, W., Borromeo Ferri, R., and Stillman, G., editors, *Trends in Teaching and Learning of Mathematical Modelling*, volume 1 of *International Perspectives on the Teaching and Learning of Mathematical Modelling*, pages 15–30, Dordrecht. Springer Netherlands. https://doi.org/10.1007/978-94-007-0910-2_3.

Blum, W. (2015). Quality Teaching of Mathematical Modelling: What Do We Know, What Can We Do? In Cho, S. J., editor, *The Proceedings of the 12th International Congress on Mathematical Education*, pages 73–96, Cham. Springer International Publishing. https://doi.org/10.1007/978-3-319-12688-3_9.

Blum, W., Galbraith, P. L., Henn, H.-W., and Niss, M., editors (2007). *Modelling and Applications in Mathematics Education: The 14th ICMI Study*, volume 10 of *New ICMI Study Series*. Springer, New York, NY. https://doi.org/10.1007/978-0-387-29822-1.

Filimonova, M. and Shvets, V. (2010). Matematychne modeliuvannia v kursi osnovnoi shkoly: zmist i vymohy do pidhotovky uchniv [Mathematical modelling in the course of mathematics at secondary school: contents of the course and requirents for training schoolchildren]. *Didactics of mathematics: Problems and Investigations*, 34:72–76. http://www.dm.inf.ua/_34/%D1%81.72-76.pdf.

Geiger, V. (2011). Factors Affecting Teachers' Adoption of Innovative Practices with Technology and Mathematical Modelling. In Kaiser, G., Blum, W., Borromeo Ferri, R., and Stillman, G., editors, *Trends in Teaching and Learning of Mathematical Modelling*, volume 1 of *International Perspectives on the Teaching and Learning of Mathematical Modelling*, pages 305–314, Dordrecht. Springer Netherlands. https://doi.org/10.1007/978-94-007-0910-2_31.

Gnedenko, B. V. (2020). *Matematika i matematicheskoe obrazovanie v sovremennom mire [Mathematics and mathematical education in the modern world]*. URSS.

Greefrath, G., Siller, H.-S., and Weitendorf, J. (2011). Modelling Considering the Influence of Technology. In Kaiser, G., Blum, W., Borromeo Ferri, R., and Stillman, G., editors, *Trends in Teaching and Learning of Mathematical Modelling*, pages 315–329, Dordrecht. Springer Netherlands. 10.1007/978-94-007-0910-2_32.

Greefrath, G. and Vorhölter, K. (2016). Teaching and learning mathematical modelling: Approaches and developments from german speaking countries. In *Teaching and Learning Mathematical Modelling: Approaches and Developments from German Speaking Countries*, ICME-13 Topical Surveys, pages 1–42. Springer International Publishing, Cham. https://doi.org/10.1007/978-3-319-45004-9_1.

Greefrath, G. and Weigand, H.-G. (2012). *Simulieren: Mit Modellen experimentieren*, volume 174 of *mathematik lehren*. Friedrich Verlag, Hannover.

Henn, H.-W. (2007). Modelling Pedagogy — Overview. In Blum, W., Galbraith, P. L., Henn, H.-W., and Niss, M., editors, *Modelling and Applications in Mathematics Education: The 14th ICMI Study*, volume 10 of *New ICMI Study Series*, pages 321–324. Springer US, Boston, MA. https://doi.org/10.1007/978-0-387-29822-1_33.

Hilscher, H. (2002). *Mathematikunterricht und Neue Me-*

dien Hintergründe und Begründungen in fachdidaktischer und fachübergreifender Sicht. Franzbecker.

Kaiser, G. (2019). ZDM Mathematics Education—Its Development and Characteristics. In Kaiser, G. and Presmeg, N., editors, *Compendium for Early Career Researchers in Mathematics Education*, ICME-13 Monographs, pages 481–492. Springer International Publishing, Cham. https://doi.org/10.1007/978-3-030-15636-7_25.

Kramarenko, T. H., Pylypenko, O. S., and Zaselskiy, V. I. (2019). Prospects of using the augmented reality application in STEM-based Mathematics teaching. *Educational Dimension*, 1:199–218. https://doi.org/10.31812/educdim.v53i1.3843.

Krasnytskyi, M. P. and Shvets, V. O. (1997). Peredumovy zdiisnennia dyferentsiatsii pry pohlyblenomu vyvchenni matematyky [Prerequisites for the implementation of differentiation in the in-depth study of mathematics]. In *Modern information technologies in the educational process*, pages 156–164.

Kvietnyi, R. N., Bohach, I. V., Boiko, O. R., Sofyna, O. Y., and Shushura, O. M. (2012). *Kompiuterne modeliuvannia system ta protsesiv. Metody obchyslen. [Computer modeling of systems and processes. Calculation methods]*. Vinnytsia National Technical University.

Leshchuk, S. O., Ramskyi, Y. S., Kotyk, A. V., and Kutsiy, S. V. (2022). Design a progressive web application to support student learning. *CEUR Workshop Proceedings*, 3077:83–96. https://ceur-ws.org/Vol-3077/paper16.pdf.

Maliarchuk, S. M. (1997). *Formuvannia osnov informatsiinoi kultury uchniv 6–7 klasiv za dopomohoiu seredovyshcha Loho [Formation of bases of information culture of pupils of 6–7 classes by means of the Logo environment]*. PhD thesis, National Pedagogical Dragomanov University. The thesis for the degree of candidate of pedagogical sciences on speciality 13.00.02 – theory and methods of teaching informatics.

Mathematical model (2023). Mathematical model. https://en.wikipedia.org/w/index.php?title=Mathematical_model&oldid=1138362942.

Matiash, O. and Mykhailenko, L. (2021). Aktualni aspekty mizhnarodnykh doslidzhen vykorystannia informatsiinykh tekhnolohii navchannia v haluzi matematychnoi osvity [Current aspects of international research on the use of information technologies of learning in the field of mathematical education]. *Modern information technologies and innovation methodologies of education in professional training: methodology, theory, experience, problems*, 60:81–90. https://doi.org/10.31652/2412-1142-2021-60-81-90.

Matiash, O. I. and Kateryniuk, H. D. (2019). *Metodychnyi instrumentarii formuvannia zdatnosti uchniv do matematychnoho modeliuvannia [Methodological Tools for Developing Students' Ability to Mathematical Modeling]*. Works. https://drive.google.com/file/d/1K9B9VI6Yr12w0JHG4Y_aSOSJVsm3cuGd/view?usp=sharing.

Morze, N., Buinytska, O., Varchenko-Trotsenko, L., Vasylenko, S., Nastas, D., Tiutiunnyk, A., and Lytvynova, S. (2022). System for digital professional development of university teachers. *Educational Technology Quarterly*, 2022(2):152–168. https://doi.org/10.55056/etq.6.

Niss, M. and Højgaard Jensen, T., editors (2002). Number 18 in Uddannelsesstyrelsens temahæfteserie. Undervisningsministeriets forlag. http://static.uvm.dk/publikationer/2002/kom/hel.pdf.

Panchenko, L. L. and Shapovalova, N. V. (2010). Tsilovi aspekty navchannia studentiv pedahohichnoho universytetu matematychnoho modeliuvannia [Target aspects of teaching students of pedagogical university of mathematical modeling]. *Scientific Journal of NPU Named after M.P. Drahomanova*, 6:101–108. Series 3. Physics and mathematics in high school and high school.

Razumovsky, V. G., Saurov, Y. A., and Sinenko, V. Y. (2013). *Siberian Teacher*, (2(87)):5–16. http://sibuch.ru/sites/default/files/%D0%90%D0%B1%D1%80%D0%B8%D1%81%D0%A0%D0%B0%D0%B7%D1%83%D0%BC%D0%BE%D0%B2%D1%81%D0%BA%D0%B8%D0%B9_%D0%B8_%D0%B4%D1%80.pdf.

Semenova, I. Y. (2014). *Matematychni modeli MSS]*. Kyiv. http://www.mechmat.univ.kiev.ua/wp-content/uploads/2018/03/MatModelMSSlast.pdf.

Semerikov, S., Osadchyi, V., and Kuzminska, O. (2021). 1st Symposium on Advances in Educational Technology: Outlook. *Educational Technology Quarterly*, 2021(4):429–604. https://doi.org/10.55056/etq.53.

Semerikov, S. O., Mintii, I. S., Slovak, K. I., Teplytskyi, I. O., and Teplytskyi, O. I. (2010). Mobilne prohramne zabezpechennia navchannia informatychnykh dystsyplin u vyshchii shkoli [Mobile software for teaching computer science disciplines in high school]. *Scientific Journal of NPU Named after M.P. Drahomanova. Series 2: Computer-Based Learning Systems*, 8(15):18–28. https://doi.org/10.31812/0564/939.

Shtoff, V. A. (1963). *O roli modelej v poznanii [On the role of models in cognition]*. Publishing House of Leningrad University.

Shvets, V. O. (2009). Matematychne modeliuvannia yak zmistova liniia shkilnoho kursu matematyky [mathematical modeling as profound line of the school math course]. *Didactics of mathematics: Problems and Investigations*, 32:16–23. http://www.dm.inf.ua/_32/16-23_32_2009.pdf.

Shvets, V. O. and Filimonova, M. O. (2010). Evoliutsiia matematychnoho modeliuvannia yak metodu piznannia i navchannia [Evolution of mathematical modeling as a method of cognition and learning]. *Mathematics at school*, 4:22–25.

Slepkan, Z. I. (1983). *Psihologo-pedagogicheskie osnovy obucheniya matematiki [Psychological and pedagogical foundations for teaching mathematics]*. Radianska shkola, Kiev.

Sonar, T. (2001). *Angewandte Mathematik, Modellbildung und Informatik: Eine Einführung für Lehramtsstudenten, Lehrer und Schüler. Mit Java-Übungen im Internet von Thorsten*

Grahs. Vieweg+Teubner Verlag, Wiesbaden. https://doi.org/10.1007/978-3-322-80225-5.

Stanzhytskyi, O. M., Taran, Y. Y., and Hordynskyi, L. D. (2006). *Osnovy matematychnoho modeliuvannia [Fundamentals of mathematical modeling]*. Kyiv University.

Teplytskyi, I. O. (2000). *Rozvytok tvorchykh zdibnostei shkoliariv zasobamy kompiuternoho modeliuvannia [Development of pupils' creative capacities by means of computer simulation]*. The thesis for the degree of candidate of pedagogical sciences on speciality 13.00.02 – theory and methods of teaching informatics, Kryvyi Rih State Pedagogical University. http://elibrary.kdpu.edu.ua/handle/0564/1599.

Tezer, M. and Cumhur, M. (2017). Mathematics through the 5E Instructional Model and Mathematical Modelling: The Geometrical Objects. *Eurasia Journal of Mathematics, Science and Technology Education*, 3(8):4789–4804. https://doi.org/10.12973/eurasia.2017.00965a.

Vozniak, H. M. and Vozniak, O. H. (2003). *Matematyka. Prykladni zadachi: vid teorii do praktyky [Mathematics. Applied tasks: from theory to practice]*. Mandrivets, Ternopil.

Weigand, H. and Weth, T. (2002). *Computer im Mathematikunterricht: Neue Wege zu alten Zielen*. Mathematik Primarstufe und Sekundarstufe I + II. Spektrum Akademischer Verlag, Heidelberg.

Zahrai, Y. M. and Kotovenko, O. A. (2007). *Modeliuvannia i prohnozuvannia stanu dovkillia*. KNUBA, Kyiv. https://tinyurl.com/42k55m5k.

Zhaldak, M. I. (1989). *Sistema podgotovki uchitelya k ispol'zovaniyu informatsionnoy tekhnologii v uchebnom protsesse [The system of preparing a teacher for the use of information technology in the educational process]*. A dissertation in the form of a scientific report for the degree of Doctor of Pedagogical Sciences on specialty 13.00.02 – theory and methods of teaching informatics, Kyiv State Pedagogical Institute named after A. M. Gorky. http://elibrary.kdpu.edu.ua/handle/0564/1365.

Zhaldak, M. I. (2003). Pedahohichnyi potentsial kompiuterno-oriientovanykh system navchannia matematyky [Pedagogical potential of computer-based mathematics teaching systems]. *Scientific Journal of NPU Named after M. P. Drahomanova. Series 2: Computer-Based Learning Systems*, 7:3–16. https://sj.npu.edu.ua/index.php/kosn/article/view/584.

Zhaldak, M. I. and Franchuk, V. M. (2020). Some applications of cloud technology in mathematical calculations. *Scientific journal of National Pedagogical Dragomanov University. Series 2 Computer-based learning systems*, (22(29)):3–17. https://doi.org/10.31392/NPU-nc.series2.2020.22(29).01.

Zhaldak, M. I., Goroshko, Y. V., Vinnychenko, E. F., and Tsybko, G. Y. (2012). *Mathematics with a computer: The teacher's guide*. National Dragomanov Pedagogical University, Kyiv, 3 edition. http://erpub.chnpu.edu.ua:8080/jspui/handle/123456789/1523.

Zhaldak, M. I. and Hrybiuk, O. O. (2014). Psykholoho-pedahohichni vymohy do kompiuterno-oriientovanykh system navchannia matematyky [psychological and pedagogical requirements for computer-based mathematics education systems]. *Scientific Journal of NPU Named after M. P. Drahomanova. Series 2: Computer-Based Learning Systems*, 14(21):3–19. https://sj.npu.edu.ua/index.php/kosn/article/view/486.

Zhaldak, M. I., Kuzmina, N. M., and Mykhalin, H. O. (2020). *Teoriia ymovirnostei i matematychna statystyka : Pidruchnyk dlia studentiv fizyko-matematychnykh ta informatychnykh spetsialnostei pedahohichnykh universytetiv [Probability Theory and Mathematical Statistics. Textbook for students of physics, mathematics, and computer science specialties of pedagogical universities]*. National Dragomanov Pedagogical University, Kyiv, 4 edition. http://enpuir.npu.edu.ua/handle/123456789/35207.