

# Complex Systems and Complex Thinking Within the Framework of Education 4.0

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**Abstract:** The presented paper raises the question of how the principles of Education 4.0 and the theory of self-organization (synergetics) can help in the reformation of the higher education system, and how interdisciplinary research can be useful for both teachers and students. In this paper, we give a brief review of different studies devoted to Education 4.0 and synergetics concepts. Next, we demonstrate the most important characteristics of complex systems and conceptually simplest methods for complex systems modelling. As part of the complex systems modeling course, which will first be presented to students of physics and mathematics, and then, possibly, to students of other specialties, we present signals of seismic activity, gravitational waves, magnetic activity, and stress-strain signal for a typical metal in the process of destruction. Our study demonstrates that complex systems theory and its toolkit can help to study phenomena of various nature and identify (forecast) their catastrophic states. This kind of analysis can serve as a good basis for the formation of professional skills and universal competencies.


## 1 INTRODUCTION


In 2021, Syukuro Manabe, Klaus Hasselmann, and Giorgio Parisi were awarded the Nobel Prize in Physics “for groundbreaking contributions to our understanding of complex physical systems” (Nobel Foundation, 2021). That is a sign that the study of complex systems is of paramount importance. Nevertheless, we need to deal with the problems of their implementation in the educational process.


The education system in the world today is in a state of crisis. This is evidenced by the following trends: a further increase in the number of illiterate people in the world; the widespread decline in the quality of education; the growing gap between education and culture, education and science; alienation of the student from the educational process.

This situation in the world at the present stage makes the problem of finding a new paradigm of education urgent, since the possibility of sustainable development of society, successful overcoming of global problems, regional and national conflicts characteristic of the present time of the development of civilization is closely related to the achieved level of education of all members of society (Karlov, 1998). But the education system is always based on a certain scientific understanding of the world and man, which determines the goals and objectives of education, its content, principles and methods.

Education 4.0 is such paradigm of education in which complex thinking, reasoning, teaching methods, and techniques become central to support educational processes for the formation of citizens committed to society and its complexity (Ramírez-Montoya et al., 2022). Modern generation of students meet business tasks which nowadays demand a wide range of knowledge, skills, and abilities: integrative, critical, systemic, scientific, innovative thinking; enabling

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analysis, synthesis, continues learning, problem solving. Without an extensive range of different fields of science, it is problematic to be an active transformer of the society. The complexity paradigm proposes new point of view in which contradictory parts of a system compose into interrelated. For solving complex task, the encounter and the exchange between all researchers and academicians in disjunctive domains are necessary.

Figure 1 represents core components that enable to design innovative pedagogical environments in terms of Education 4.0 with correct technologies and infrastructures which will carry out best practices.

Modern technological environment embraces advances of humanity that provide high capacities and performance capabilities in many systems and platforms. Such technologies provide high level of digitalization, virtualization, and datafication. Due to corresponding spectrum of possibilities and student-central environment, we are able to seek, prepare, and graduate new highly competitive professionals capable to propose innovative solutions for current world. Searching for real-world challenges and combining educational experience with ICTs, students are able to transfer from theory to practice very quickly.

Open education, innovations, science, and technologies are the cornerstones of Education 4.0. It relies on personalized learning pathways, innovative digital and management tools complemented with such trending computer science topics as artificial intelligence (AI), blockchain, robotics, virtual reality, etc. Especially should be emphasized AI which provides a framework for understanding complex systems behavior: how multi-agent, interconnected, and intelligent environments interact with each other, mostly producing non-linear and non-predictable dynamics.

The heyday of education in the XVII-XVIII centuries, which happened through the development and spread of classical mechanics of the New Time, led to the determination of the picture of the world, where the studied elements are unchangeable, and the laws of classical mechanics are universal and apply to all types of motion of matter.

Such real-world systems as a pandemic, storm, transport systems, the world-wide web, stock and crypto indices are presented to be complex, irreversible, and sensitive to initial perturbations (Hipkins, 2021). Following deterministic paradigm, where each phenomenon has a cause and at the same time there is a cause of other phenomena, i.e., all the processes taking place in the world are predetermined and predictable, we would encounter that real-world systems neither precisely random nor deterministic.

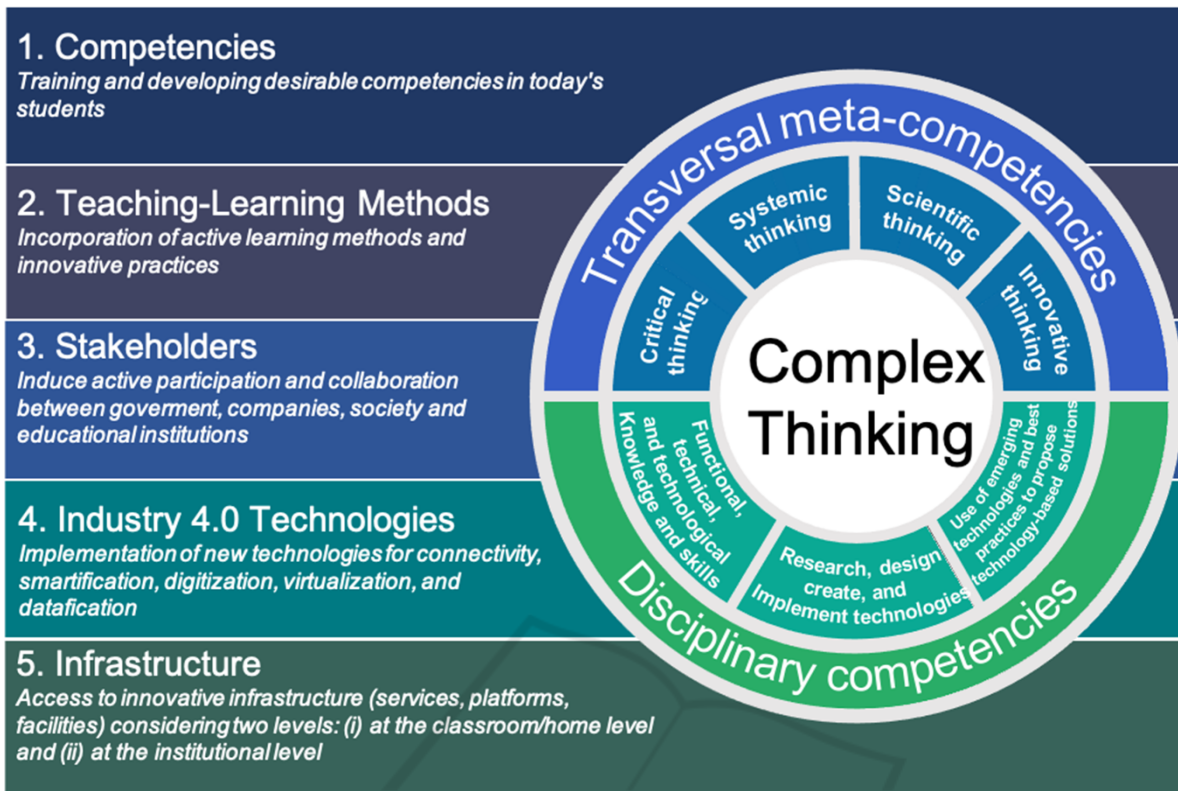
Complex systems tend to display ordered features and unpredictable dynamics simultaneously (Ovens et al., 2013).

Therefore, such ideological and methodological principles as rationalism, determinism, mechanism and reductionism began to dominate in scientific knowledge, which also had a decisive influence on the education system: on the forms of knowledge acquisition, presentation of material, organizational principles of education.

The discovery by synergetics of the processes of self-organization in inanimate nature clearly shows that the transition from disorder to order, accompanied by the emergence of self-organization and stable structures, the replacement of old structures with new ones occurs according to specific internal laws inherent in certain forms of the movement of matter. Ultimately, it is the qualitative and quantitative criteria of self-organization that characterize the level of complexity and perfection of the corresponding forms of movement (Haken, 1977, 1982). Based on these ideas, it is possible to develop a classification of types, forms, properties of matter according to their degree of complexity, perfection of organization, and thereby the degree of development. In this regard, development itself appears as a very complex, self-organizing process of movement from simple to complex, from less organized and perfect to more organized and perfect. In other words, development, in contrast to the movement that characterizes any changes in general, acts as a directed change associated with the emergence of a new one.

The post-non-classical stage of the development of science shows that rigid determinism and reductionism, which serve as the basis of the mechanistic view of the world, cannot be considered as universal principles of scientific knowledge, since an extensive class of phenomena and processes does not fit into the framework of linear, equilibrium and reversible schemes. In the world around us, a very real irreversibility plays an essential role, which is the basis of the majority of self-organization processes. Reversibility and rigid determinism in the world are applicable only in simple limiting cases, and irreversibility and randomness should be considered not as an exception, but as a general rule.

To integrate the synergetics approach into the educational process, it is important to instill in students ways of setting and solving problems of being and developing complex systems in various spheres: economic, social, natural, etc. It is equally important, at the beginning of studying the methods of studying complex systems, to instill in students at first or repeat with them the concepts of self-organization,



### Contexts and Challenging Real-Life Scenarios

Figure 1: Core components for effective learning in Education 4.0 (Ramírez-Montoya et al., 2022).

chaos, destructive phenomena, to voice the difference between complex and complicated systems, etc.

Complex systems are a field of research that is now acquiring the characteristic features of a well-formed area of science with its own object, conceptual apparatus, and methods of analysis (Thurner, 2017). The concept of a complex system is gradually becoming one of the fundamental concepts of modern science, or, more broadly, it is increasingly appearing in a general cultural context. The expansion of the scope of application of this concept, as well as the identification and awareness of an increasing number of phenomena where it is applicable, causes difficulties in its exact definition. Although the science of complex systems covers a broad interdisciplinary field of research, the methods and concepts of physics (dynamical systems theory, quantum mechanics, statistical physics) are central to it.

So, the processes of self-organization in non-equilibrium conditions correspond to the dialectical interaction between chance and necessity, fluctuations and deterministic laws. Near bifurcations, the main role is played by chaos, randomness, while deterministic connections dominate in the intervals be-

tween bifurcations. The ways of development of self-organizing systems are not predetermined. Probability appears not as a product of our ignorance, but as an inevitable expression of chaos at the points of bifurcations. This means the end of the classical ideal of omniscience and creates the need to revise the principle of mechanical rationalism as the dominant scientific explanation of reality. The traditional education system, based on the principles of classical science, cannot effectively fulfill the role of a means of mastering the world by a person.

Hence, there is a need to provide new principles and ideas of the complex systems paradigm in the sphere of Education 4.0.

## 2 ANALYSIS OF PREVIOUS STUDIES

For building a new way of learning and education, we must be aware that linear thinking and methods are very dangerous in non-linear world (Jörg et al., 2007). Consequently, we should tend to a new way of thinking beyond dualism, reductionism, and the idea

of controllable and perfectly predictable events.

Analysis of scientific sources and publications shows that today there is an opinion that synergetics could provide significant assistance in the search for a new paradigm of education. A synergistic approach to understanding patterns operating in nature is associated with the names of Haken (Haken, 1977, 1982, 1984, 2004; Haken and Schiepek, 2006), Prigogine (Prigogine, 1989, 1980; Prigogine and Stengers, 1984, 1997; Nicolis and Prigogine, 1989). Some scientists believe that synergetics, as a theory of self-organization of complex systems, describes the general (common) that is in their development, education is a complex system, and therefore synergetics, which today is developed by various branches of scientific knowledge, necessarily becomes its new philosophy. However, despite the existence of a sufficient number of works devoted to the application of synergetics in various spheres of human activity, the methodological and practical context of synergetics in the philosophy of education remains insufficiently developed. This is especially true for applying a synergistic approach to understanding the higher education system.

In contrast to the traditional interdisciplinary approach in education, the goal is not only to provide knowledge, but also to teach to hear and understand colleagues working in different specialties, to develop skills of dialogue between specialists in different branches of scientific knowledge. Thus, complexity theory is *transdisciplinary* rather than *interdisciplinary*: members of research team from different fields of science such as physics and economics are able to work together if they are sufficiently informed about one another's perspectives and motives (Bentham, 2002). The need for such a dialogue is becoming more and more palpable. Since the theoretical physicist Haken (Haken, 2004) introduced this concept into scientific use, the world has been accumulating some experience in the use of synergetics and in the study of social and educational systems.

Research conducted in schools and universities shows that interactive chaotic environments are very productive for developing creative thinking. The results of work in this area were presented by Davis-Seaver et al. (Davis-Seaver et al., 2000), who analyzed the learning process at three levels – from a single point of balance, statement of fact, statement of a single point of view to learning on the verge of chaos, when there are many points of view, when reasoning develops in different directions, when students listen to the opinions of others and on this basis develop their own judgments. The role of the teacher is not to spread knowledge and evaluate the correctness

of judgments, but to monitor the progress of reasoning and transfer the learning process from one level to another. As a result, the understanding becomes deeper, more versatile, and the incentives for learning are largely created by the energy of the group, and not by the diligence of the teacher. In the context of revealing a person's creative abilities, a synergistic approach to education seeks not to eradicate chaos, but to find the relationship between order and disorder that would be most fruitful (Kremen, 2013).

The above-mentioned concept of chaos from the point of view of synergetics loses its negative connotation. As Prigogine and Stengers (Prigogine and Stengers, 1997) notes, instability can be a condition for stable and dynamic development. Only systems that are far from equilibrium are able to organize and evolve spontaneously. Thus, there is no development without instability. And if the system is strict against the implementation of new units, new units ('innovators') die". In higher education, self-organizing systems are the Student, Teacher, their interrelation, etc. (Taranenko, 2014).

Jacobson and Wilensky (Jacobson and Wilensky, 2006; Wilensky and Jacobson, 2014) emphasize different research issues that need to be explored. They present such principles in studying complex phenomena as

- experiencing complex systems phenomena;
- making the complex systems conceptual framework explicit (Council, 2000);
- encouraging collaboration, discussion, and reflection; the design of environments for learning about complex systems needs to take advantage of lessons learned from the extensive research on pedagogy that foster collaboration, discussion, and reflection (National Research Council, 2000);
- constructing theories, models, and experiments;
- learning trajectories for deep understandings and explorations.

With a given appropriate conceptual and representational scaffolding in the learning environment, students should be able to tap into their everyday experiences and channel and enhance these experiences to construct understandings of complex systems that are cognitively robust. Nowadays, students should have more possibilities to explore world through computational modeling which progressive scientists use almost everyday.

Jackson (Jackson, 1995) and other, such as Pagels (Pagels, 1988), have observed how the use of computational tools in science allows dramatically enhanced capabilities to investigate complex and dynamical systems that otherwise could not be systemat-

ically investigated by scientists. These computational modeling approaches include cellular automata, network and agent-based modeling, neural networks, genetic algorithms, Monte Carlo simulations, and so on that are generally used in conjunction with scientific visualization techniques. Examples of complex systems that have been investigated with advanced computational modeling techniques include climate change (West and Dowlatabadi, 1998), urban transportation models (Balmer et al., 2004; Helbing and Nagel, 2004; Noth et al., 2003), and economics (Anderson et al., 1988; Arthur et al., 1997; Axelrod, 1997; Epstein and Axtell, 1996b). New communities of scientific practice have also emerged in which computational modeling techniques, in particular agent-based models and genetic algorithms, are being used to create synthetic worlds such as artificial life (Langton, 1989, 1995) and societies (Epstein and Axtell, 1996a) that allow tremendous flexibility to explore theoretical and research questions in the physical, biological, and social sciences that would be difficult or impossible in “real” or nonsynthetic settings.

Jörg et al. (Jörg et al., 2007) addressed their study to the theory of complexity, arguing that the present paradigms in the field of education neglect the inherent complexity of educational reality and therefore are not able to give an adequate understanding of reality. They discussed the importance of studying complex systems paradigm and its integration into educational process. In their opinion, complexity paradigm should help to uncover some of the myths we live by, but it is not necessary an unlimited source of truth. It is rather a better alternative for our rapidly evolving world in which we already encounter ‘deprivation of our culture’ (Midgley, 2001) and ‘perversion in system of education’ (Baistrocchi, 2018)

Costan et al. (Costan et al., 2021) investigated the existing barriers to Education 4.0 implementation. They collected a systematic review of the 30 journal articles on Engineering, Social Sciences, Computer Science, Business, Management, and Accounting generated from the initial search on Scopus, which were in turn related to Education 4.0. Their analysis provided 12 existing barriers for Education 4.0 implementation: cybersecurity threat, costly, skills gap of human capital, apprehensive stakeholders, lack of training resources, lack of collaboration, knowledge gap for the customization of curriculum design, insufficient available technologies, health issues, time constraint for material preparation, complexity of learning platforms, and insufficient foundation of basic education. Furthermore, a theoretical predictive model was constructed to present the causal relationships in modeling the problems associated with implementing

Education 4.0.

Sigahi and Sznelwar (Sigahi and Sznelwar, 2022) studied following questions: (1) how complexity thinking could be applied to engineering education; (2) how that could contribute to current engineering challenges; (3) what were different complexity approaches in engineering and how to integrate them. They conducted a review from fifty eight journal articles and five book chapters. They discussed: engineering axiology; epistemological and ontological perspectives; complex thinking and competences; systematic transformations of engineering education, etc. Were identified main gaps of such education and discussed different thoughts on topic complexity.

Complexity captures even physical education (Bielinskyi et al., 2022). Swedish National Agency for Education presented new curriculum which included such term as *complex movement*. Researchers (Janemalm et al., 2019) provided insights into the meanings of *complex movements* in the context of physical education in Sweden. Using a discourse analytic methodology, six policy texts were examined. The study suggests that there is needed greater consensus as complex movement can have a wide range of meaning, have a context-dependent meaning, and for different audiences will be understood in individual ways.

New paradigm of thinking and teaching concerns even sustainable development. It aims to equip learners with necessary knowledge about complex sustainability problems and develop in students creative thinking to acquire innovative sustainable solutions. Green et al. (Green et al., 2022) formed a randomized controlled trial to understand whether an innovative sustainability learning tools help to increase the understanding of a specific sustainability problem. Their learning toolkit incorporates two factors – system thinking and system dynamic simulation. They also tested whether those factors help to transfer knowledge to a second problem with a similar system structure. They used different statistical techniques to analyse the effect of the factors on sustainability understanding. Their research presented that the effectiveness of education for sustainable development increased significantly. Participants gave qualitative feedback on usefulness of systems thinking and simulation.

Network science (graph theory) is the key data analysis instrument for solving problems through their *graph representations*. For Education 4.0 it is one of the main fields of science which must be included into learning process. Many real-world complex systems exhibit common organizing principles, non-trivial patterns that were derived with graph the-

ory. Therefore, network science can be considered as highly interdisciplinary research field (Börner et al., 2008). Weber et al. (Weber et al., 2021) addressed their study to sustainability problems through the tool of network science and presented schematically how complex, real-world sustainability problems can be considered through the prism of graph theory (figure 2).

As the environmental, economic, and political problems of humanity have become global, complex and nonlinear, traditional ideas about individual responsibility are becoming questionable. We need to study and teach new models of collective behavior that take into account the different degrees of our individual abilities and understanding of what is happening.

We believe that the study of the apparatus of physics, graph theory, and computer science is now of paramount importance for the further development of both our society and the entire universe.

In further we need to understand how to grow an interest of students in constructing and revising computational models with multi-agent or qualitative modeling software, and how model building activities may enhance student conduct of real world experiments related to the phenomena under consideration (Abrahamson and Wilensky, 2005a,b; Jackson et al., 2000).

### 3 THE MOST IMPORTANT PROPERTIES OF COMPLEX SYSTEMS TO BE STUDIED

Based on the previously described characteristics and the direction in which we should move, it becomes clear that synergetics (the theory of complex systems) is the foundation of almost any system. Including pedagogical. Although the initial direction of research within this paradigm was physical systems, the latest objects of research on various manifestations of complexity also appear in the context of business organization and economics. For example, Wheatley (Wheatley, 2006) suggests that we view organizations as being more like living organisms than machines. As such, we need to modify traditional views on controlling organizations. Wheatley (Wheatley, 2006) argues that organizations are dynamic, nonlinear networks of relationships and cannot be separated into parts while maintaining their essential identity.

Complexity thinkers have been seeking for common characteristics in a tremendous range of simple and complex systems: dependence on initial per-

turbations, long-term correlations, multi-layered and multi-scale, mutual and reciprocal, etc.

In general, they are

- dynamic;
- non-equilibrium and have the potential to change suddenly and may take one path out of an infinite number of others (bifurcate);
- open systems, that is interchange energy (and information) with their surroundings;
- depended. What happens next depends on what happened previously;
- systems where the whole is more than the sum of its parts;
- causal and yet indeterminate;
- irreversible, since the interaction of parts together is transforming;
- multi-agent. They composed of a diversity of agents that interact with each other, mutually affect each other, and in so doing generate novel, emergent behavior for the system as a whole. The system is constantly adapting to the conditions around it and over time it evolves;
- co-evolving and move spontaneously towards the edge of chaos.

#### 3.1 Time Series Data

In order to maintain students' interest in studying complex systems and their corresponding data analysis tools, programming languages, etc. (Shumway and Stoffer, 2016; Fulcher et al., 2013), it is important to select truly interesting and complex systems (series). It is equally important that the studied systems are within the framework of the specialty that students are guided by. However, since we strive for an interdisciplinary approach, the study, for example, by biologists of the corresponding nonlinear methods on the example of the same socio-economic series or physical ones can also be beneficial for general development.

Complexity theory is subdivided into hard and soft complexity. Hard complexity theory stands for analytical analysis that concern with the nature of reality, while soft complexity aims to describe social and living systems. Davis and Sumara (Davis and Sumara, 2006) proposes such term as "complexity thinking" which lies somewhere in between hard and soft skills. We support such idea and would like to promote it among ordinary citizens who are not specialists and, particularly, among universities and their student. Focusing on interdisciplinarity, both hard and soft skills,

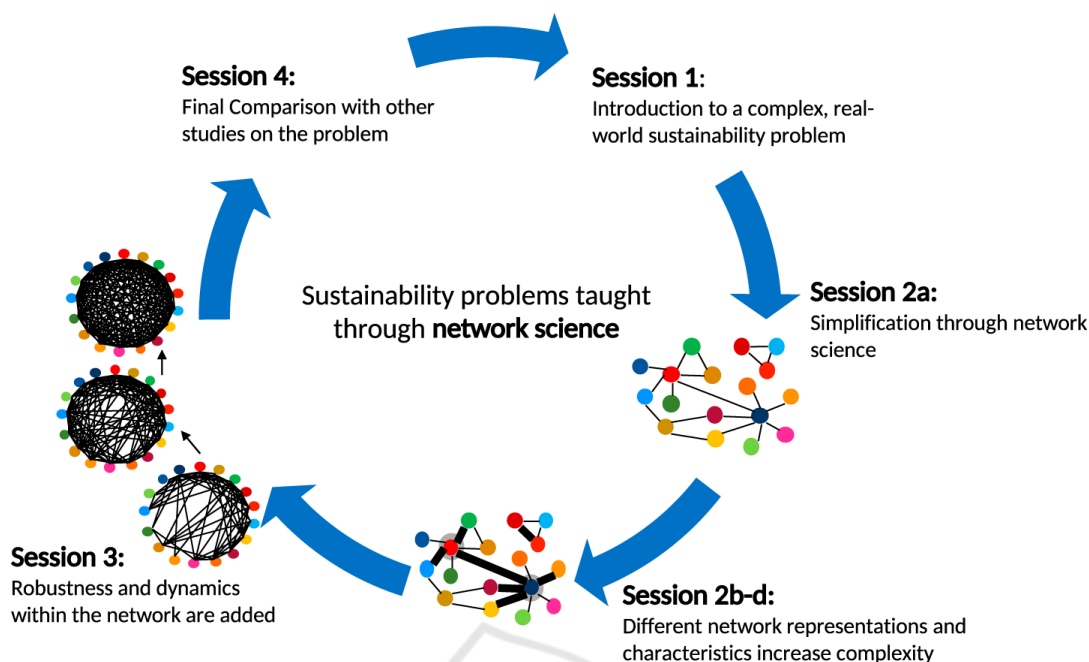


Figure 2: Sustainability problems that are solved through network science.

teachers and students will be more creative and productive in their further research. Knowing about interconnections across different disciplines, there are much more possibilities for collaborative research between different faculties and there is larger probability that people will be able to find common topics for communication and will be engaged to cooperate.

The goal of this work is to present the basic characteristics of complex systems, which should be introduced to students during the course of studying complex systems, and the basic sets of methods that allow analyzing the varying randomness (complexity) of the system during the development of the studied signals.

In this paper, we present some of the most fundamental, applied, robust, and powerful methods on the example of four physical signals: seismic (SEI), gravitational wave (GW), the distribution storm time (Dst) index, and stress-strain ( $\sigma(\epsilon)$ ) signal for a typical metal in the process of destruction.

SEI dataset constructed by Bladford (Bladford, 1993). Each event has 2048 points fixed at a seismic recording station in Scandinavia.

We used GW data GW150914 from Events of LIGO Open Science Center and select strain data (H1 and L1) after noise subtraction (The LIGO Scientific Collaboration and the Virgo Collaboration, 2016) (<https://www.ligo.org/detections/GW150914.php>).

The Dst index is an index of magnetic activity derived from a network of nearequatorial geomagnetic observatories that measures the intensity of the globally symmetrical equatorial electrojet ("ring cur-

rent"). Dst is maintained at National Centers for Environmental Information (National Centers for environmental information, 2021) from 1957 to the present. Dst equivalent equatorial magnetic disturbance indices are derived from hourly scalings of low-latitude horizontal magnetic variation. They show the effect of the globally symmetrical westward flowing high altitude equatorial ring current, which causes the "main phase" depression worldwide in the H-component field during large magnetic storms. In this paper, the time series of hourly values of the storm on March 13, 1989 is investigated. It is the strongest storm in the space age in several ways; the power system of the province of Quebec was out of order. The peak of the storm falls in the middle of the time series (point 1000).

The stress-strain signal  $\sigma(\epsilon)$  contains integrated information about the structural transformations of the spectrum of defects in the material under study (point, dislocations, pores, cracks) depending on the applied stress.

In order to study changes of complexity dynamically, i.e., to get not only one value that will characterize the whole system, but an array of values, where each value will reflect the complexity of a signal in a specific period, we use sliding window approach (Soloviev and Belinskyi, 2018a; Bielinskyi et al., 2021b,c).

In figures 3a and 3b is presented the dynamics of all physical signals that could be studied during physics classes. However, students of other faculties

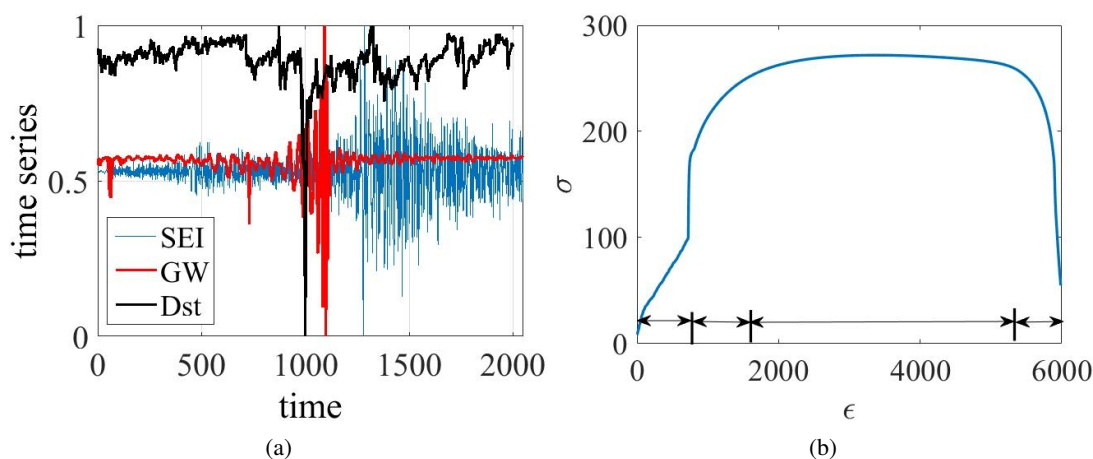


Figure 3: The signals of SEI, GW, and Dst in normalized scale (a). Stress-strain curve in absolute scale (b).

can also be interested.

Figure 3b shows a typical dependence  $\sigma(\epsilon)$  with 4 highlighted characteristic areas. In the first of them, elastic (reversible), point defects dominate. In the second region of plastic flow and hardening, dislocations multiply and move. It is the most informative. The third region is characterized by a quasi-stationary process of accumulation of pores and microcracks, as well as the nucleation of a neck. Finally, the last region is the phase of the formation of a global crack, ending with the destruction (rupture) of the material.

### 3.2 Fat-Tailed Distribution

When studying complex systems, we inevitably encounter power distributions characterized by thick tails. A classic example is the power-law of dividing words by their frequency of use in a text, known as Zipf's law (Zipf, 1950).

In economics, this is the law of wealth distribution among individuals (Pareto, 1896); in demography, the distribution of cities by their size (Auerbach, 1913); in biology, the distribution of the size of forest patches (Saravia et al., 2018); in scientometry, the distribution of citations (Brzeziński, 2014). In general, a wide class of phenomena is described in the framework of distributions with a degree dependence, but the researcher (student) will have to find out the nature of such a dependence, which can be caused by many factors: critical phenomena, processes with preference, self-organized criticality, multiplicative processes with connections, optimization and path-dependent nonergodic processes, the phase space of which decreases with evolution (Domp, 1996; Sornette, 2006; Bak et al., 1987; Mandelbrot, 1953; Corominas-Murtra et al., 2015).

First of all, it will be important to build an empirical distribution for our data (figure 4). Having vi-

sualized the series we study in this paper, we can already be convinced of the non-Gaussian dynamics of the presented systems.

In the course of our research, we have determined that the Lévy  $\alpha$ -stable distribution most successfully covers the key statistical characteristics of both the economic (Bielinskyi et al., 2019, 2021a,c) and those systems that are presented in this paper. Figures 5a to 5d show the window dynamics of the  $\alpha$  index derived from the Lévy distribution that characterizes the "heaviness" of tails.

From the figures above we can observe that the dynamics of all signals is beyond normal. Index of stability  $\alpha$  decreases during regions of instability, indicating an increase in the tails of the distribution.

### 3.3 Multifractality

When studying various types of systems, we often encounter both fractal (self-similar) structures and sets of different fractal dimensions (Stanley and Meakin, 1988). In such problems, it is necessary to take into account the entire range of critical indicators that characterize different moments in the distribution of observed quantities. Such properties usually relate to the term "multifractality" (Sreenivasan and Meneveau, 1986).

There are several different algorithms that allow the obtention of multifractal spectra from time series. The most famous is the MF-DFA (Kantelhardt et al., 2002; de Freitas et al., 2019; Eghdami et al., 2018).

Based on the MF-DFA procedure, we select the maximum value of such a quantitative characteristic of multifractality as the singularity strength (Ashkenazy et al., 2003), although in the corresponding section of fractal (multifractal) analysis, it would be necessary to characterize and demonstrate the dynamics of all multifractality indicators. The following figure



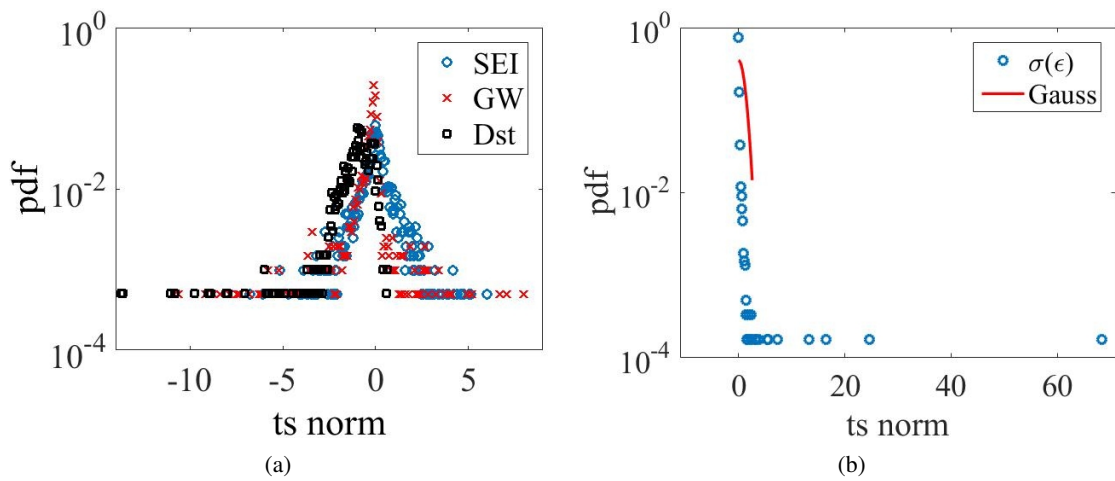


Figure 4: Probability density functions (PDF's) of Dst, Sei, and GW (normalized time series – ts norm) (a). PDF of  $\sigma(\epsilon)$  signal comparing to along with the Gaussian curve (b).

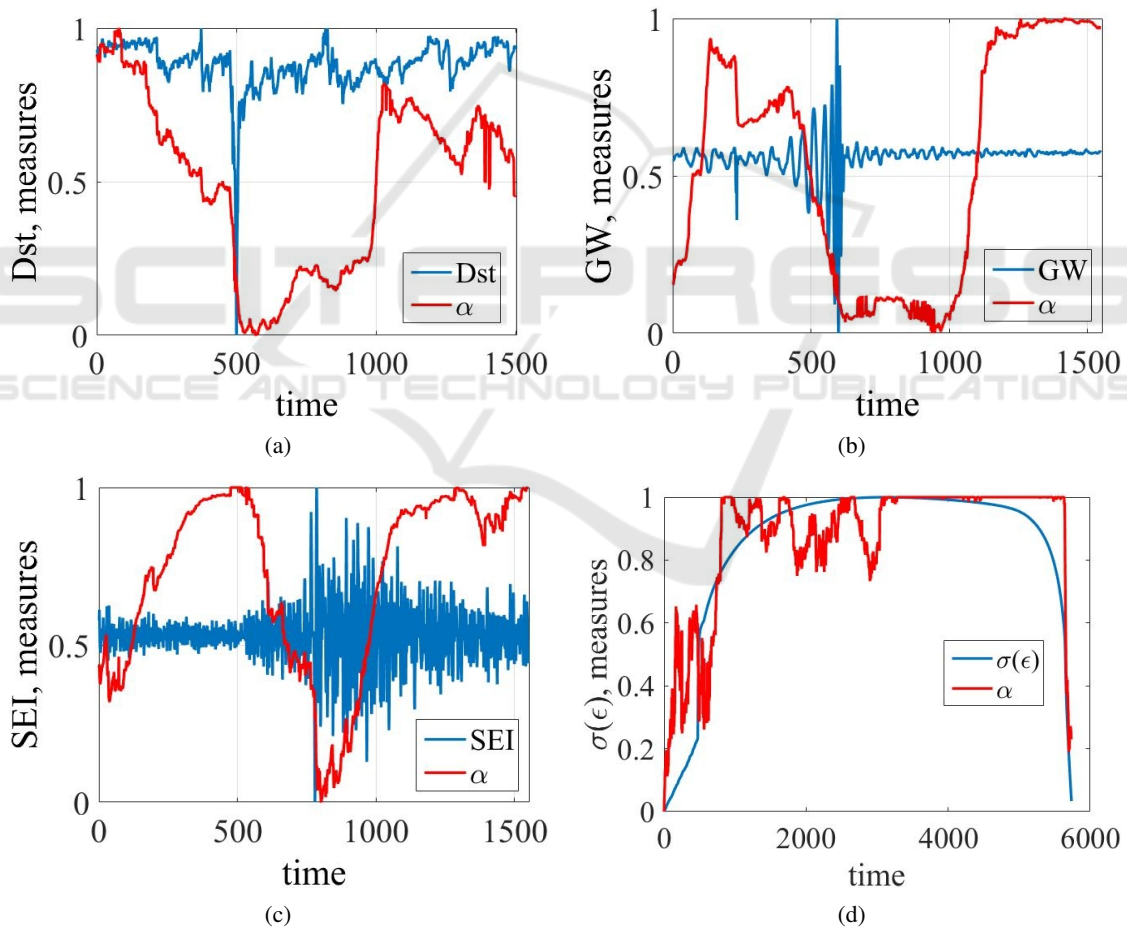


Figure 5: The dynamics of four signals and their  $\alpha$  index of stability.

shows the window dynamics of the maximum value of the singularity strength.

Figure 6 demonstrates the increase of multifractality during period of collapse. For Dst, SEI, and  $\sigma(\epsilon)$

critical periods become more multifractal, whereas for GW we have the opposite relation.

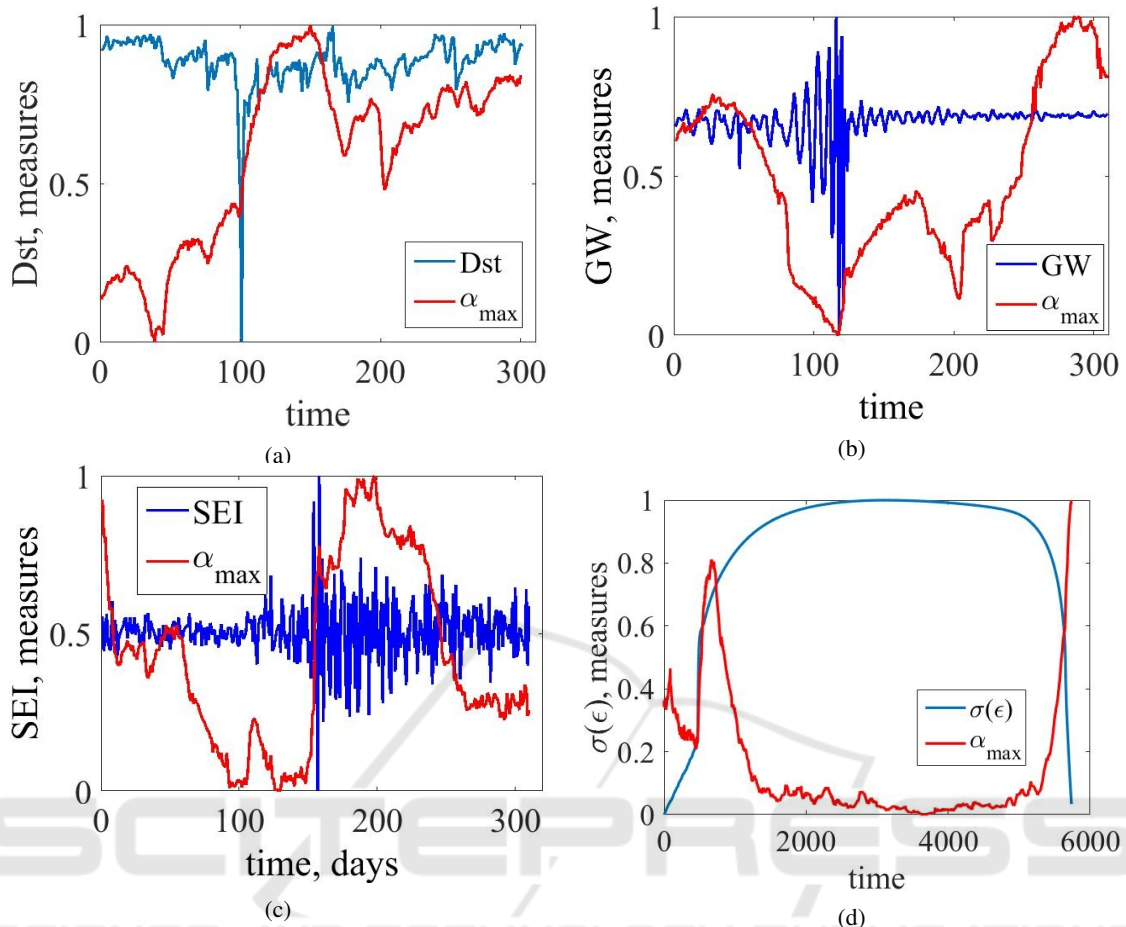


Figure 6: The dynamics of four signals and their  $\alpha_{\max}$  index of multifractality.

### 3.4 Network Analysis

Equally important is the network analysis of complex systems. Today, networks play a central role in modeling complex systems, as they offer a way to describe different types of relationships between agents that act as endpoints in the network. Complex networks can characterize information, social, economic, biological, neural, and other systems (Newman, 2003; Boccaletti et al., 2014, 2006; Baiesi and Paczuski, 2004). For example, a society can be represented as a network, where each individual (university, wealth, city) can be represented as nodes of a graph, and the connection between them through edges. For cities, edges can represent a road, where the possibilities of movement can vary, and therefore a different weight can be determined for each edge.

In general, the computer network model is a random graph, the law of mutual arrangement of edges and vertices for which is defined by the probability distribution.

The simplest of networked objects, so-called

Erdős-Rényi, or random graphs (Erdős and Rényi, 1959). Such graphs can be characterized within the framework of the Poisson distribution, but most complex systems, as already noted, are characterized within the framework of distributions with heavy tails. One of the most interesting characteristics of networks is the vertex degree. The vertex degree distribution for many real-world networks shows a power-law dependence. Such networks are called scale-independent. Scale-free networks are often characterized by very short average distances between randomly chosen pairs of nodes that may have a strong impact on the whole dynamics.

In addition to the topology of graphs, you can also study their quantitative characteristics. In our case, using the window procedure, we get a variable graph representation of our signal over time. For the presented work, we calculated the maximum vertex degree of the graph ( $D_{\max}$ ), since this measure is one of the conceptually simplest measures, although many other measures can be represented. It is worth noting that there are also various algorithms for con-

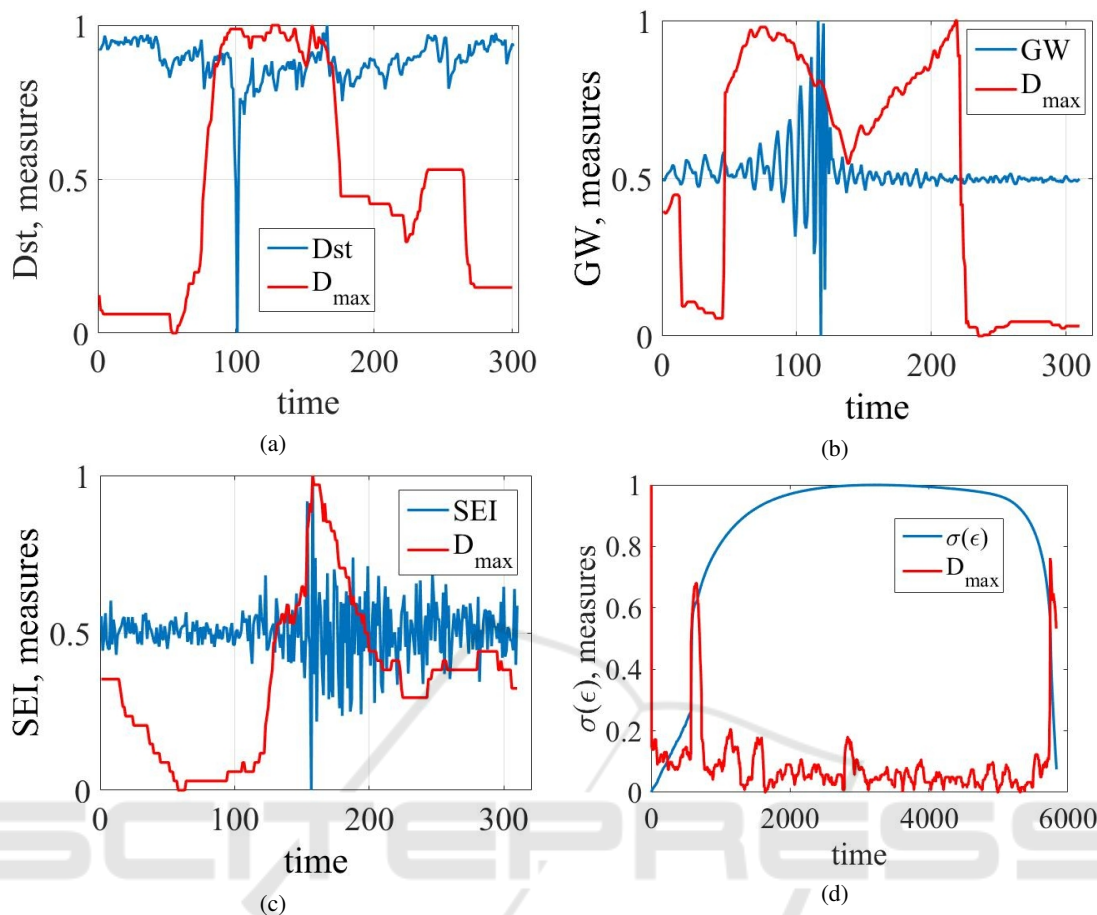


Figure 7: The dynamics of four signals and their  $D_{max}$  in accordance with the visibility graph.

verting a time series to a graph. We would like to emphasize the visibility graph algorithms (Bielinskyi et al., 2021c,b; Juan and Guzmán-Vargas, 2013; Kundu et al., 2021; Soloviev and Belinskiy, 2018; Bielinskyi and Soloviev, 2018) (figure 7) and one based on recurrence analysis (Donner et al., 2011) (figure 8).

The index of maximum degree  $D_{max}$  start to increase during abnormal phenomena. We can make a conclusion that crisis period is presented to be more concentrated in terms of graph comparing to normal dynamics.

### 3.5 Recurrence Analysis

Processes in nature are characterized by pronounced recurrent behavior, such as periodicity or irregular cyclicity.

Moreover, the recurrence (repeatability) of states in the sense of passing a further trajectory quite close to the previous one is a fundamental property of dissipative dynamical systems. This property was noted in the 1880s by the French mathematician Poincaré and

subsequently formulated in the form of the “recurrence theorem”, published in 1890 (Poincaré, 1890).

The essence of this fundamental property is that, despite the fact that even the smallest perturbation in a complex dynamical system can lead the system to an exponential deviation from its state, after a while the system tends to return to a state that is somewhat close to the previous one, and goes through similar stages of evolution.

In 1987, Eckmann et al. (Eckmann et al., 1987) proposed a method for mapping the recurrence of phase space trajectories to  $N \times N$  matrix. The appearance of a recurrence diagram allows us to judge the nature of processes occurring in the system, the presence and influence of noise, states of repetition and fading (laminarity), and the implementation of sudden changes (extreme events) during the evolution of the system. If you look at recurrent diagrams in more detail, you can find small-scale structures (textures) consisting of simple points, diagonal, horizontal, and vertical lines, which in turn correspond to chaotic, repetitive, or laminar states.

Using combinations of these states, Zbilut and

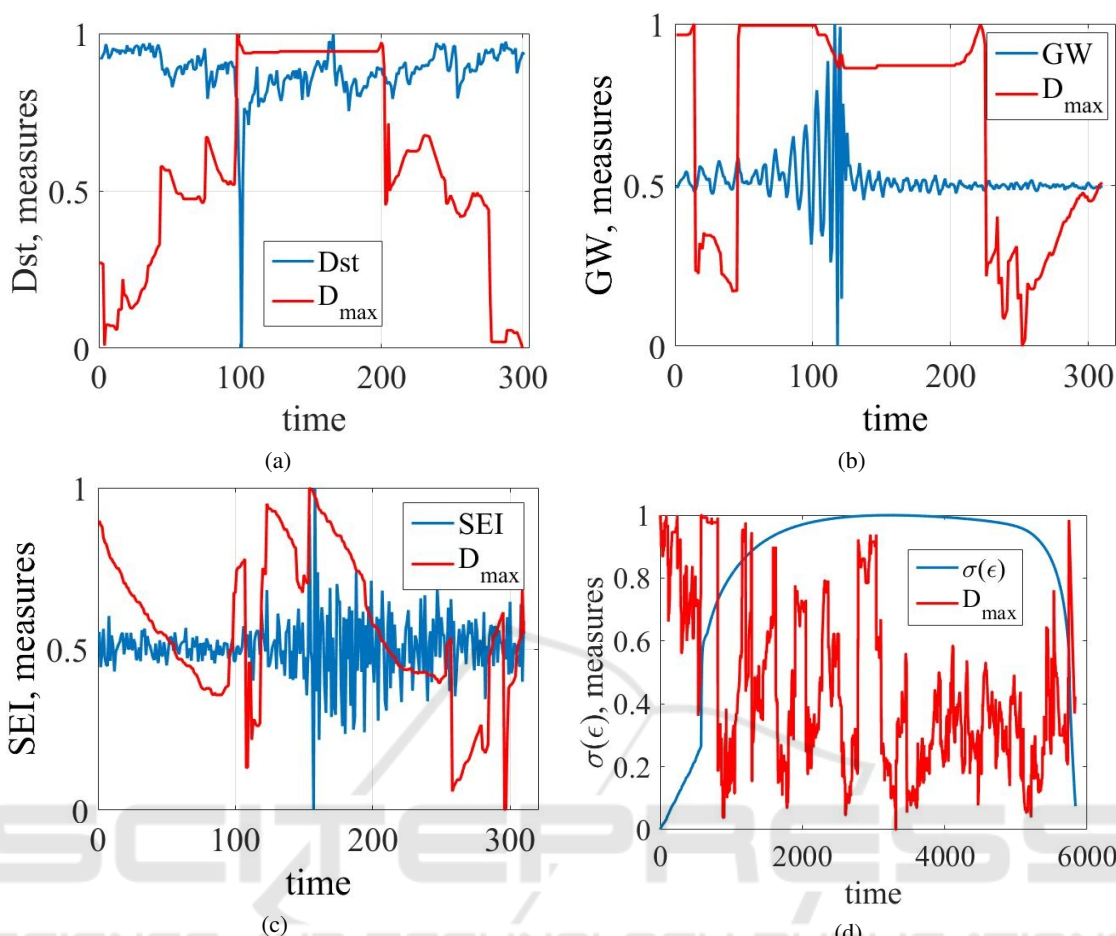


Figure 8: The dynamics of four signals and their  $D_{max}$  in accordance with the algorithm based on recurrence analysis.

Webber (Zbilut and Webber, 1992; Webber and Zbilut, 1994) developed a tool for calculating a series of measures based on the distribution of recurrent points on a recurrence matrix. Later, the toolkit for quantitative recurrent analysis was supplemented by Marwan and Kurths (Marwan and Kurths, 2002). The tools of quantitative recurrent analysis include the recurrence rate, determined by the ratio of recurrent points to the total number of points on the recurrence matrix under study. In addition to the recurrence measure, in the course of analyzing complex systems, it would be possible to present such measures as determinism, divergence, entropy, trend, and so on (Soloviev and Belinskiy, 2018; Soloviev and Belinskiy, 2018a; Derbentsev et al., 2020; Fan et al., 2018; Lin et al., 2015; Banerjee et al., 2021).

In this paper, we will focus on the recurrence rate and present it for the already specified series (figure 9).

Figure 9 demonstrates  $RR$  measure that indicates the probability of finding recurrent (close to each other) points. Our empirical results show that due

to abrupt changes that correspond to crisis state, the probability of finding recurrent points become lower. This indicator starts to decrease even before crash, which makes it as indicator-precursor of such events.

### 3.6 Entropy and Non-Extensive Statistics

The Boltzmann-Gibbs statistical entropy and the classical statistical mechanics associated with it are extremely useful tools for studying a wide range of simple systems that are characterized by a small range of space-time correlations (short memory), the additivity of noise, the presence of intense chaos, the ergodicity of dynamic processes, the Euclidean geometry of phase space, the locality of interaction between elements, the Gaussian probability distributions, etc.

The Boltzmann-Gibbs statistical entropy is a fundamental concept of the school section and the university course of thermodynamics and statistical physics.

In statistical mechanics, entropy denotes the num-

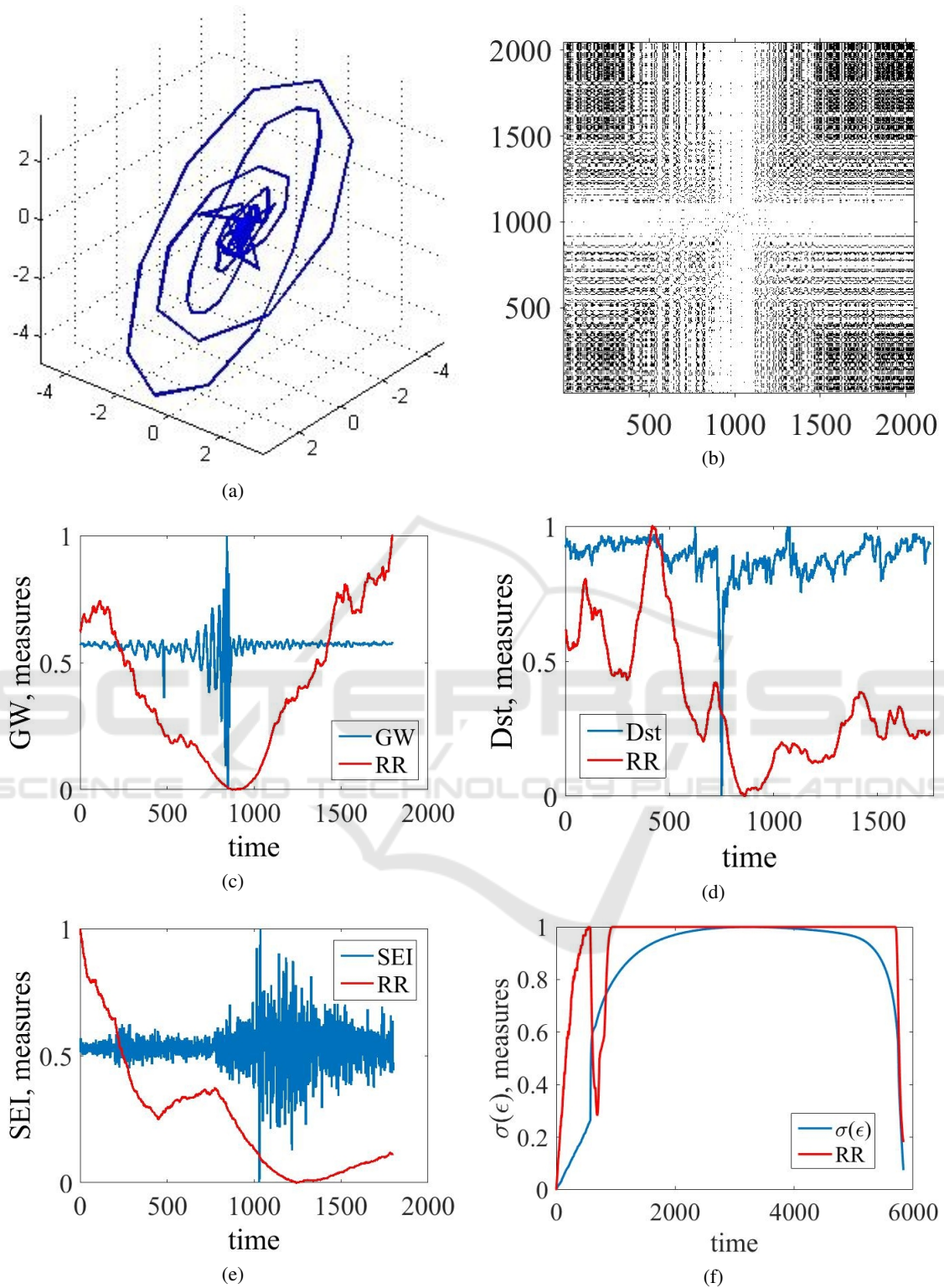


Figure 9: Phase space portrait and recurrence plot of GW (a-b). The dynamics of RR for GW (b), Dst (c), SEI (d), and  $\sigma(\epsilon)$  (f).

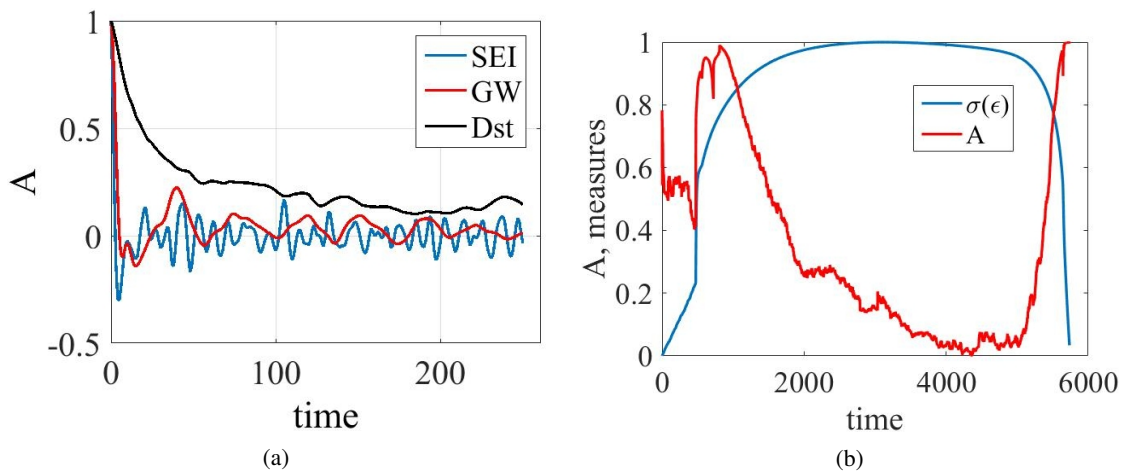


Figure 10: Autocorrelation of SEI, GW and Dst (a), and sliding window autocorrelation for  $\sigma(\epsilon)$  (b).

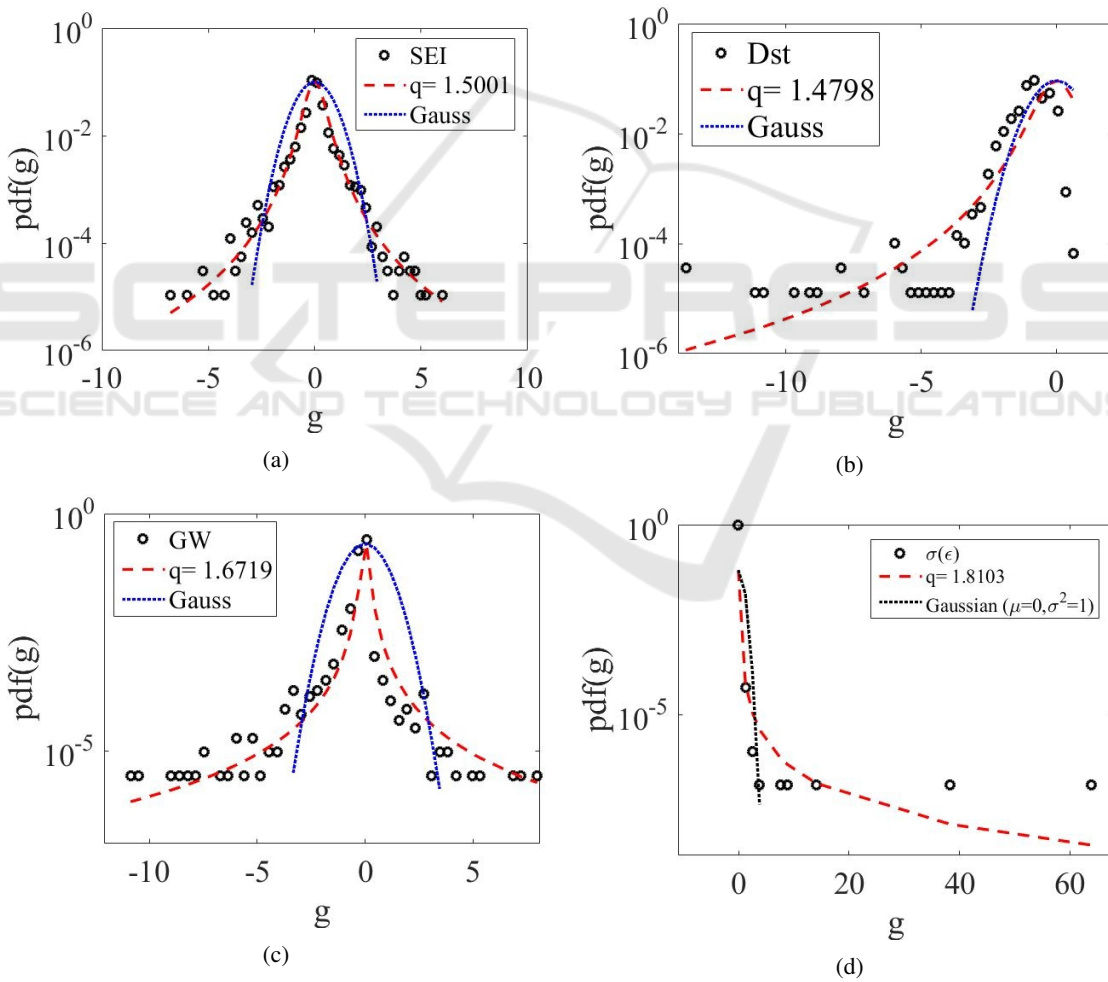


Figure 11: The pdf's of the four signals, Gaussian, and  $q$ -Gaussian functions (a-d).

ber of possible configurations of a thermodynamic system. The notion of entropy can be associated with the uncertainty in the system (Clausius, 1870; Boltz-

mann, 1970). In 1948, Shannon transformed classical statistical entropy to information entropy (Shannon, 1948). Since then, a number of other types of

information entropy have been developed (Karakatsanis et al., 2013; Javaherian and Mollaei, 2021; Litvinenko, 2019; Posadas et al., 2021).

In order to study many real-world systems, it is necessary to go beyond the standard course of thermodynamics, statistical physics, and classical Shannon entropy. A whole range of natural, artificial and social systems, which, unlike those mentioned above, are characterized by a long range of spatio-temporal correlations and non-Gaussian processes.

Since the non-Gaussian and multifractal behavior of the studied systems was presented previously, we will depict the autocorrelation function in the figure 10a, as it should demonstrate an indicator decline. This fact will indicate the dependence of the following values on the previous ones.

It is also worth mentioning that such systems are characterized by multiplicative noise, the presence of weak chaos (vanishing maximum Lyapunov exponent), non-ergodicity of dynamic processes, hierarchy (usually multifractality) of the geometry of the phase space, the presence of asymptotically power-law statistical distributions. A fairly wide class of these complex systems (although not all) it is adequately described by non-additive statistics based on the Tsallis parametric entropy.

Figures 11a to 11c show the  $q$ -Gaussian distribution from the Tsallis statistics for the considered series in comparison with the classical Gaussian one.

Autocorrelation plot (figure 10a) represents that the highest long-range dependence has the signal of magnetic activity, and autocorrelation with the sliding window approach for  $\sigma(\epsilon)$  signal represents how increases dependence between defects during transition from elastic region to the region of plastic flow and hardening.

Figures 11a to 11d present that signals which dynamics exceeds  $\pm 10\sigma$  are described more appropriately in terms of  $q$ -Gaussian distribution. Parameter  $q$  represented the degree of non-extensivity in each system. With the higher  $q$ , we expect more multifractal, chaotic, and dependent dynamics.

### 3.7 Reversibility and Irreversibility

The last characteristic that we would like to mention is time-reversibility. Temporary irreversibility is a key property of non-equilibrium systems.

Again, such systems are characterized by the presence of memory, while reversibility increases with more noisy and unpredictable signals. Thus, by calculating the irreversibility, we determine the degree of nonlinearity and predictability. It is important to note that the significant time reversibility excludes linear

Gaussian processes as a model of generating dynamics. Within the framework of the systems we are considering, we need to think about methods of nonlinear dynamics and non-Gaussian ones (Lawrance, 1991; Stone et al., 1996).

Over the past decade, various methods have been proposed for calculating the degree of irreversibility in systems (Daw et al., 2000; Kennel, 2004; Lacasa et al., 2012; Donges et al., 2013; Flanagan and Lacasa, 2016; Costa et al., 2005; Zanin et al., 2018; Jiang et al., 2016) and we have presented how to use some of them for crises identification (Bielinskyi et al., 2021b). For pedagogical purposes, along with the mentioned concept of multifractality and entropy, we would like to present irreversibility based on the multifractal approach (Jiang et al., 2016) and permutation patterns (Zanin et al., 2018). The last mentioned approach could be taught within the section of entropy approaches if we were teaching students. However, the calculation of irreversibility based on graph theory is also possible (Lacasa et al., 2012; Donges et al., 2013; Flanagan and Lacasa, 2016).

Figures 12a to 12d show the mentioned measures of irreversibility for the studied signals.

In figure 12 we can see that abnormal periods are followed with the increase of irreversibility in signal. In our opinion, permutation-based irreversibility is most stable comparing to the second one. Nevertheless, additional improvements of algorithm for their calculations can be made, and indicators of irreversibility based on graph theory can be studied.

## 4 CONCLUSION

Nowadays real-world challenges and mass integration of information and communication technologies in every sphere of our life demand an evolution of a pedagogical sector. Consequently, increasing complexity of all social structures require multidisciplinary projects, which can provide valuable experience and competencies not only for colleagues from seemingly independent disciplines, but also for students from the same fields.

The analysis of the adaptive nature of many complex systems led to the creation of methods and the development of concepts that were successfully applied to describe formally similar phenomena in chemical, biological, social and other systems of agents of non-physical nature. It is sometimes argued that if physics is the science of the four fundamental forces that matter interacts with.

It is still relevant to create appropriate open innovation laboratories (Cortes et al., 2020) in which will

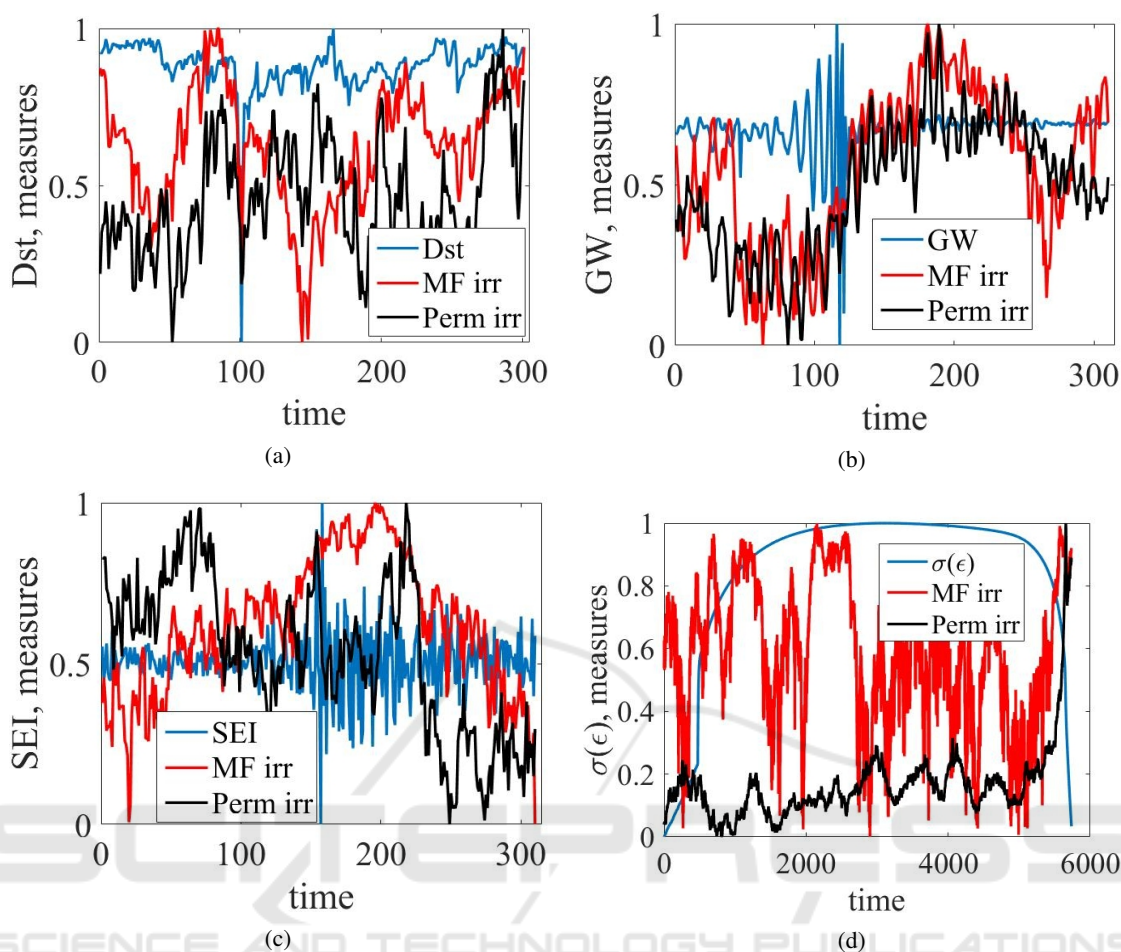


Figure 12: The dynamics of irreversibility measures along with the studied signals.

be possible adaptation of various solutions from the fields of physics, higher mathematics, and computer science to social sciences.

In this paper, we have presented some of the most significant approaches on the example of SEI, GW, Dst, and  $\sigma(\epsilon)$ . Empirical results emphasize that we can not only study critical processes (as in the first three cases), but also physical objects in which these (critical) processes alternate with (quasi)-linear and even catastrophic (destruction). Obviously, today's transformations towards Industry 4.0 gave us a wide range of different indicators and signals to study (Chen et al., 2019; Liu et al., 2019), and the task is to get students interested in learning the appropriate complexity theory tools and developing their complexity thinking.

The theory of complex systems is obviously not limited to the methods presented in this paper. Further, we would like to supplement the presented material with entropy (Soloviev and Belinskyi, 2018b; Soloviev et al., 2019, 2020b), chaotic algorithms

(Soloviev et al., 2020a), and, for example, the tools of random matrix theory (Soloviev and Belinskyi, 2019; Bielinskyi et al., 2021c).

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