Analysis of Airport Taxi Problem based on M/Ek/1 Queuing Model

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Abstract: The abstract should summarize the contents of the paper and should contain at least 70 and at most 200 words. It should be set in 9-point font size, justified and should have a hanging indent of 2-centimenter. There should be a space before of 12-point and after of 30-point. Nowadays, with the growth of the number of private cars and the number of taxis, the problem of taxi queuing in the airport transportation system needs to be solved urgently. This paper combines the M/Ek/1 service model in queuing theory (Qie, Wang 2007), effectively proposed to establish a decision model for the taxi waiting for passengers or no-load return journey, and the actual airport information verification model is more reasonable.

Scheme A and Scheme B decisions were made by comparing the time costs. Under Scheme A, because the taxi arrival time follows the parameter of λ Poisson distribution and the ride service time follows the Erlang distribution of k order, the M/Ek/1 queuing model can be established to list the system equation of state through the relationship between the total exponential service steps j in the system and the probability distribution of k customers in the system. Then the parent function is introduced, and finally the average waiting time of the passenger is $W_q = \frac{(k+1)\rho}{2k\mu(1-\rho)}$, set the time cost of no-load backhaul in scheme B is Q, when Wq <Q, scheme A, and when Wq> Q, scheme B.

Shanghai Pudong Airport and Shanghai taxi data were selected for model test until reasonable. The results were calculated using Lingo to compare the time hours required for schemes A and B. The length of Scheme B can be calculated from the speed of taxi driving on the Shanghai viaduct and the urban speed of Scheme B is 1.1917 hours. When the time required of Scheme A is less than 1.1917 hours, Scheme A is selected, otherwise Scheme B.

The M/Ek/1 queuing model established in this paper avoids the limitations of negative exponential distribution and can be more applicable to multiple serial processes, or if no memory assumption is not significant; queuing theory can not only solve the taxi queuing problem, but also has broad applications in medical and communication fields.

1 INTRODUCTION

A large number of tangible or invisible queuing or crowded phenomenon as a common life problem, such as restaurant dining queuing problems, banking business queuing problems and so on. AS the economic growth increases and the number of private cars increases, the road traffic queuing phenomenon is particularly common. Queuing theory has been widely used in communication systems, storage systems, etc.

Production management and other aspects play an important role, queuing theory for the transportation, especially the taxi queuing problem research.

Through the statistical study of the arrival time and the arrival time of taxi drivers, the statistical law of the waiting time of taxi drivers and passengers and the peak of taxi passengers are obtained. Then, according to these laws, we can improve the taxi queuing problem at the airport terminal or make decisions for taxi drivers, so that not only taxis can carry passengers efficiently, but also solve the problem of passenger retention at the airport.

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In the problem of queuing in the airport terminal, the taxi said that the drivers have two schemes: A and B, to choose. The two schemes compare the waiting time, and the short time, the time cost is low, which is selected as the final decision plan. In the case of scheme A, the taxi is relatively free, can approximate the Poisson distribution, the time of the taxi is independent (Wang, Shi, Wang 2015) so approximately obey the Erlang distribution, from the traditional M/M/1 waiting system queuing model can be optimized to the M/Ek/1 queuing model system, introduce the parent function, using the L'Hopital law to remove the waiting time. In scenario B, the taxi driver will immediately return with an empty load, and the time cost is the return time (or plus the empty load in the urban area Between). Comparing the two-time costs gives the selection strategy. As shown in Figure1:

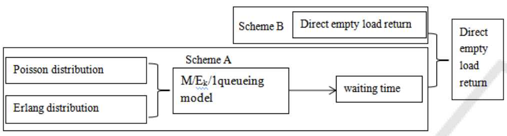

Figure 1: Scheme selection flow chart.

This question selects Shanghai Pudong airport taxi data and airport traffic data, using LINGO and MATLAB software simulation calculation, intercept a day (with peak and stationary period) for the research period, bring data into the problem of the decision results, and analyze the accuracy and rationality of the results, and discuss the correlation.

2 MODEL ESTABLISHMENT

2.1 Model Hypotheses

1. Suppose that taxis and passengers are generally unlimited.

2. Suppose that the passenger arrival is independent of each other (Xue 2004).

3. Taxi and passengers wait (queue without leaving).

4. Suppose that the number of passengers arriving is subject to the Poisson distribution (Tang 2017) (the passengers waiting for the taxi are stable and ineffective, Ordinary), the service time follows the Erlang distribution (can represent the time interval of independent events, good fitting effect).

5. Suppose the driver drives freely, not affected by the weather, and the traffic flow is smooth.

2.2 Representation of Symbol

Table 1: representation of symbol.

symbol	meanings
L_s	Team length: the total number of taxis in the car storage pool
L_q	Length: Number of taxis in the storage pool
W_s	Stay time: the time that the taxi stays in the storage pool
W_q	Line up time: the time the taxi waits in the storage pool
λ	Taxi reach rate per unit of time
μ	Number of taxis completing the service per unit of time
ρ	Average service time per unit of service desk time
Q	Access capacity of taxi point (vehicle / hour)
h	Average taxi stop time (boarding time and getting in and out the sum of points)
N_b	Number of valid berths
R	To offset the reduction coefficient of the docking time fluctuations, it is usually taken $R = 0.833$

2.3 Establishment of Model

The arrival time of the taxi camera follows the Poisson distribution with the parameter, the number of stations of the passenger boarding point is 1 (Geng, Song, Zhao 2013), and the service time is abstract as the time interval of independent random events, so it follows the Erlang distribution of order k; the passenger arrival time can be abstracted as random, thus approximating the Poisson distribution. Using the Poisson distribution and the Erlang distribution, the M/Ek/1 queuing model can also be established to form the queuing system of the airport and make a state transfer diagram to list the probability distribution of the exponential service steps j in the system and the probability distribution of k customers in the system. According to the dynamic transfer diagram, we can list the system transfer equation. It is then solved by introducing the parent function (Baidu Encyclopedia 2019).

3 MODEL SOLUTION

3.1 Model Solution Process

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Taxi arrival time is assumed to obey the Poisson distribution of the parameters, and the probability distribution function is:

$$P(t)=\frac{(\lambda t)^n}{n!}\lambda^{-\lambda t},(t\geq 0)$$

Taxi service time follows the *Erlang* distribution of k, and its probability distribution function is:

$$f(t)=\frac{\mu k(\mu kt)^{k-1}}{(k-1)!}e^{-\mu kt},(t\geq 0)$$

Each taxi in the system leaves the system after the k-step index service and is set in the time t system

The number of car rental is n, and the number of service steps at the time t index is j.

Record $P_j = P$ {The total number of exponential service steps in the system is j} ; $P_k=P$ {There are k cars out of the system}, The relation for these two quantities is:

$$P_k=\sum_{j=(n-1)k+1}^{nk}P_j,(k=1,2,3,\Lambda)$$

The following system equations of state are listed by the state transfer diagram:

$$\begin{cases}\lambda P_0=k\mu P_1\\(\lambda+k\mu)P_j=\lambda P_{j-k}+k\mu P_{j+1}\end{cases},(j=1,2,3,\Lambda)$$

Because the variables in this model are all non-negative integers, the concept of the probability mother function is introduced, let's consider:

$$P(z)=\sum_{j=0}^{\infty}P_j z^j$$

Type (1) is available thus:

$$P(z)=\frac{k\mu P_0(1-z)}{\lambda\mu+\mu z^{k+1}-(\lambda+k\mu)z} \quad (1)$$

Definition is given by the probabilistic mother function $P(z)=1$, we can get（2）:

$$\frac{k\mu P_0}{k\mu-\lambda k} \quad (2)$$

then:

$$P_0=1=1-\frac{\lambda}{\mu}=1-\rho$$

Cut off（1-z）and $k\mu$

Type (3) is available by the partial split method

$$P(z)=\frac{1-\rho}{1-\frac{\lambda}{k\mu}(z+z^2+\Lambda+z^k)}=\frac{1-\rho}{(1-\frac{z}{z_1})(1-\frac{z}{z_2})\Lambda(1-\frac{z}{z_k})}=(1-\rho)\sum_{i=1}^{k}A_i/(1-\frac{z}{z_i}) \quad (3)$$

Among z_1, z_2, Λ, z_k are the root of the denominator polynomial, and have:

$$Ai=\prod_{n=1,n\neq i}^{k}1/(1-\frac{z_1}{z_n})$$

Expand Type (3) and compare the coefficient to get that the taxi reaches the car storage pool and find the car storage pool. The probability of j cars already inside:

$$P_j=(1-\rho)\sum_{i=1}^{k}A_i(z_i)^{-j},(j=1,2,3,\Lambda)$$

The average service time of a taxi is the average pickup time is $1/k\mu$□The average taxi pickup time for j taxis is j/k.

Therefore, the average waiting time required for a new taxi to receive guests is type (4):

$$W_q=\sum_{j=0}^{\infty}\frac{j}{k\mu}P_j=\frac{1}{k\mu}\sum_{j=1}^{\infty}jP_j=\frac{1}{k\mu}\lim_{z\to 1}\left[\frac{dP(z)}{dz}\right] \quad (4)$$

For z to derive formula (5):

$$\frac{dP(z)}{dz}=\frac{-[k\mu+\lambda z^{k+1}-(\lambda+k\mu)z]-(1-z)[(k+1)\lambda z^k-(\lambda+k\mu)]}{[k\mu-\lambda z^{k+1}-(\lambda+k\mu)z]^2}(1-\rho) \quad (5)$$

When $z\to i$, we can use the Lobida rule to get the (6) formula:

$$\lim_{z\to i}\frac{dP(z)}{dz}=\frac{(k+1)\rho}{2(1-\rho)} \quad (6)$$

From types (4) and (6): the average queuing time of the taxi in the storage pool W_q

$$W_q=\frac{(k+1)\rho^2}{2k\mu(1-\rho)}$$

The average number of taxis queuing in the storage pool Lq:

$$L_q=\lambda W_q=\frac{(k+1)\rho^2}{2k(1-\rho)}$$

Therefore, the waiting time for Scheme A is W_s, if the wasted time cost of the taxi going directly empty to the city in Scheme B is Q.

3.2 Results Discussion

Let the waiting time of the taxi in the storage pool be t_1, $t_1=Ws$; In Scheme B, the taxi to return directly to the city is t_2, the cost of no-load loss is $D(D < 0)$.

When $t_1 < t_2$, that is, the taxi waiting time in the storage pool is shorter than the return to the city directly empty load, at this time, in the min (t_1, t_2) period, the income of Scheme A is 0, while Scheme B income D. At this time, the taxi driver chooses Scheme A, that is, the taxi queue to the designated car storage pool and queue into the passengers according to "come first to arrive".

When $t_1 > t_2$, that is, the taxi waits longer in the storage pool than to the city directly on empty load During the max (t_1, t_2) time period, the return of Scheme A is 0, while the return of Scheme B is unknown and requires further discussion:

let $t_3=t_2-t_1$

1.In t_3 period, if the taxi cannot receive the guest, scheme B proceeds to D ($D < 0$) , At this time, scheme A income is 0, so scheme A is selected.

2. In t_3 period, if the taxi receives the customer, set the taxi manned income is Q, then the scheme B income is $(D+Q)$. When $D+Q > 0$, we select Scheme B, and vice versa, Scheme A.

4 MODELLING VERIFICATION

Select Shanghai Pudong Airport and collect the relevant data of Shanghai taxi as shown in Table 1. According to the official website of Shanghai Pudong Airport, the taxi charging standard is as follows:

Table 2: Taxi Charging Standard in Shanghai.

	daytime（05:00-23:00）	**nighttime（23:00-05:00）**
0-3 kilometers	14 CNY	18 CNY
3-10 kilometers	2.5 CNY / km	3.1 CNY / km
More than15 kilometers	3.6 CNY/ km	4.7 CNY / km

Referring to the data of the "Third Comprehensive Traffic Survey Report of Shanghai", we learned that the empty driving rate of Shanghai district in 2004 was 39%, the total daily mileage was 9.5 million kilometers, and the service number was 1.493 million / day.

It is calculated that the average daily mileage per taxi is 6.36 kilometers, and the average daily mileage per hour is 0.265 kilometers, and about 14 CNY and 18 CNY per hour between day and night. Shanghai Pudong Airport is about 55 kilometers away from the city center (Wu, Zheng, Deng 2009). Taxi vehicles use no. 93 gasoline, consuming an average of 8 liters per 100 km and 7.55 CNY per liter. Therefore, the one-way oil cost from downtown to the airport is 33.22CNY. It can accommodate 10 taxis at the same time.

Table 3: Passenger boarding time and frequency table.

Time interval（s）	**Number of Vehicles (Units)**	**Time interval（s）**	**Number of Vehicles (Units)**
3.4-13.4	103	43.5-53.4	8
13.5-23.4	62	53.5-63.4	5
23.5-33.4	27	63.5-73.4	2
33.5-43.4	16	73.5-83.4	2

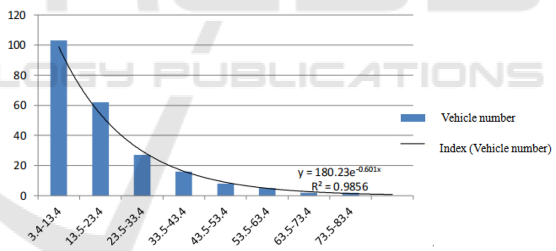

Figure 2: Time and frequency distribution diagram of passengers' boarding time (Sun, Ding, Chen 2017).

The time frequency distribution of passenger boarding is shown in Table 2, the data are analyzed by Excel to approximate the exponential distribution, and the variance of the above 225 random variables is 0.876, and the passenger boarding time approximately follows the exponential distribution of the mean 15, the passenger boarding time frequency distribution diagram is shown in Figure 2.

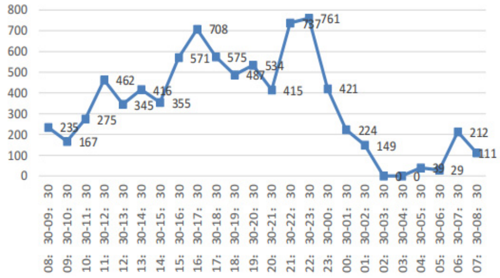

Figure 3: Taxi flow distribution map of Shanghai Pudong International Airport within one day (Yan 2015).

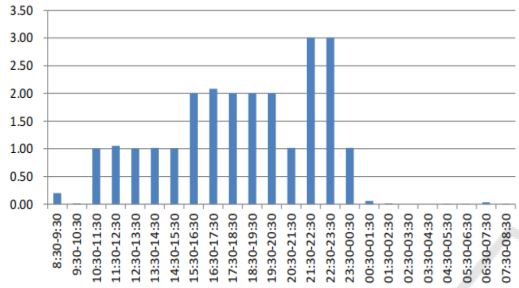

Figure 4: Taxi waiting time in the car storage pool(h).

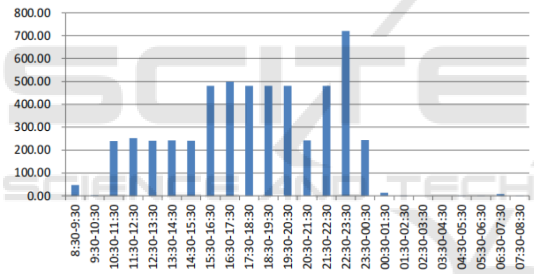

Figure 5: Number of vehicles in the vehicle storage pool.

The actual data of Shanghai Pudong Airport will be brought in Fig. 3, Fig.4 and Figure 5 into the model, with two schemes A and B. The time-consuming scheme of 1.1917 is the priority decision model, and according to the Lingo calculation results, scheme A is the following period:

14:30-15:30, 9:30-10:30, 10:30-11:30, 11:30-12:30, 12:30-13:30, 13:30-14:30, 8:30-9:30, 20:30-21:30, 23:30-00:30, 00:30-01:30, 01:30-02:30, 02:30-03:30, 03:30-04:30, 05:30-06:30, 04:30-05:30, 06:30-07:30, 07:30-08:30.

Select Scheme B for the rest of the period.

5 CONCLUSIONS

The simple negative exponential distribution is broken through in the model establishment, so that the service time of the passengers is basically subordinate to the Erlang Distribution, with a better fit (Lin 2018), reduces the model error. Select the queuing theory in operation research, the statistical rules can be analyzed with the calculated statistical indicators, and then improve the service system structure according to these rules, or reorganize the served object, so that the service system can not only meet the needs of the service object, but also achieve the optimization of some indicators of the organization. The $M/E_k/1$ model of queuing theory can not only compute the statistics, but also optimize the subsequent results (Li 2014), and optimize the overall service level of the model with the marginal analysis method, dynamic planning and other methods.

The longitudinal generalization of the model is extended through the $M/E_k/1$ waiting queuing model, to the queuing model of multi-service desk, M/M/S/∞ model (Wu, Li, Liang 2012), with multiple service desks, and customers can receive service immediately with free service desk. To continue to improve the model complexity, we can build the M/M/S/K hybrid model, which can solve the queuing problem of the loss system, which is more in line with the actual situation.

Horizontal promotion: queuing theory can be widely used in the medical field, banking service system, airport security check system, campus express peak service system, college students' canteen dining problems and so on. For example, through considering the problem of dining in the college students' canteen, deduce the peak period of students during the class, count the number and location of the canteen Windows, and get the rationalization suggestions for the canteen service after the queuing model.

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