Sparse Decomposition as a Denoising Images Tool

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Abstract: The sparse representation and Elimination of image noise has been largely used successfully by the signal processing community. In this work, we present its benefits particularly in image denoising applications. The general purpose of sparse representation of data is to find the best approximation of a target signal applying a linear combination of a few elementary signals from a fixed collection. Several methods have been found for sparse decompositions to remove noise from the image, and there are other problems, like How to decompose a signal with a dictionary, which dictionary to use, and learning the dictionary.

1 INTRODUCTION

The adopted approach of image denoising is based on sparse redundant representations compared to trained dictionaries. Several algorithms are proposed to build this type of dictionaries. Among them, the K-SVD algorithm is used to obtain a dictionary that can effectively describe the image. In addition, some greedy algorithms are used to perform sparse coding of the signal.

Since the K-SVD is limited in handling small image fixes, we are expanding its deployment to arbitrary image sizes by defining a global front image that forces sparse fixes at each location in the image. We show how these methods lead to a simple and efficient denoising algorithm. This leads to a denoising performance equivalent to and sometimes better than the most recent alternative denoising methods.

The first problem is divided according to the type of imagery The first problem is divided according to the type of imagery, then which dictionary we are going to use then the sparse coding task, i.e. which algorithm we are going to use, that's our goal, we are looking for the most parsimonious algorithm possible, ie the closest solution to the problem.

2 FORMULATION

The general objective of the sparse representation is to seek an approximate representation of a signal chosen by applying a linear combination of some elementary signals of a fixed collection. In practice, there are several sparse decomposition algorithms used to solve this type of problem.

The problem is to find the exact decomposition which minimizes the number of non-zero coefficients:

$$\min_{x} \|x\|_0 \quad s.t \quad y = Dx \tag{1}$$

$x \in \mathbb{R}$ and K is the sparse representation of y.

And $\|x\|_0$ the norm l_0 of x and corresponds to the number of non-zero values of x.

The dictionary D is made up of K columns dk, $k = 1, \dots, K$, called atoms, each atom supposed to be normalized.

In theory, there is an infinity of solutions to the problem, and the goal is to find the possible sparseness solution, that is to say the one with the lowest number of non-zero values in x.

In practice, we seek an approximation of the signal and the problem becomes (2.1):

$$\min_{x} \| y - Dx \|^2 \; s.t \| x \|_2 \leq L \tag{2}$$

with L > 0 the constraint of sparsity, that is to say an integer representing the maximum number of non-zero values in x .

We can use a $\tau > 0$ parameter to balance the dual purpose of minimizing error and sparsity :

$$\min_{x} \frac{1}{2} \|y - dx\|_2^2 + \tau \| x \|_0 \tag{3}$$

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Solving this problem is NP-hard, which precludes any exhaustive search for the solution. This is why sparce decomposition algorithms have emerged in order to find an approximation of the solution.

3 SPARSE DECOMPOSITION ALGORITHMS

Many approximation techniques have been proposed for this task. We have proposed the following algorithms:

Matching pursuit (MP), Orthogonal Matching Pursuit algorithm (OMP), LASSO algorithm, and least angle regression LARS.

who find approximate solutions:

3.1 Matching Pursuit (MP)

Algorithm 1: Matching pursuit *(MP)*

$$\min_{\alpha\in\mathbb{R}^m} \frac{1}{2} \|\mathbf{x} - \mathbf{D}\alpha\|_2^2 \quad \text{s.t.} \quad ||\alpha||_0 \le \mathbf{L}$$

1. Initialization: $\alpha = 0$; residual $\mathbf{r} = \mathbf{x}$
2. while $||\alpha||_0 \le \mathbf{L}$
3. Select the element with maximum correlation with the residual

$$\hat{\imath} = \arg\max_{i=1,\dots,m} |\mathbf{d}_i^T \mathbf{r}|$$

4. Update the coefficients and residual

$$\alpha_{\hat{\imath}} = \alpha_i + \mathbf{d}_i^T \mathbf{r}$$
$$\mathbf{r} = \mathbf{r} - (\mathbf{d}_i^T \mathbf{r})\, \mathbf{d}_i$$

5. End while.

3.2 Orthogonal Matching Pursuit

Algorithm 2: Orthogonal matching pursuit *(OMP)*

$$\min_{\alpha\in\mathbb{R}^m} \frac{1}{2} \|\mathbf{x} - \mathbf{D}\alpha\|_2^2 \quad \text{s.t.} \quad ||\alpha||_0 \le \mathbf{L}$$

1. Initialization: $\alpha = 0$ residual $r = x$ active set $\Omega = \emptyset$
2. while $||\alpha||_0 \le \mathbf{L}$
3. Select the element with maximum correlation with the residual

$$\hat{\imath} = \arg\max_{i=1,\dots,m} |\mathbf{d}_i^T \mathbf{r}|$$

4. Update the active set, coefficients and residual

$$\Omega = \Omega \cup \hat{\imath}$$
$$\alpha_\Omega = (d_\Omega^T d_\Omega)^{-1} d_\Omega^T\, r$$
$$r = x - d_\Omega\, \alpha_\Omega$$

5. End while.

3.3 The LASSO Algorithm

This approach consists in replacing the combinatorial function l_0 in the formul (1) by the norm l_1. The norm l_1 is the closest convex function to the function l_0, which gives convex optimization problems admitting exploitable algorithms.

The convex relaxation of problem (1) becomes:

$$\min_x \|x\|_1 \quad s.t \quad y = Dx \tag{4}$$

The mixed formulation (3) becomes

$$\min_x \frac{1}{2}\|y - dx\|_2^2 + \tau||\, x\, ||_1 \tag{5}$$

Here, $\tau > 0$ is a regularization parameter whose value determines the sparcity of the solution, high values generally produce clearer results.

3.4 Least Angle Regression Algorithm (LARS)

A fast algorithm known by (LARS) can make a small modification to solve the LASSO problem, and its computational complexity is very close to that of greedy methods. However, the LARS algorithm only permits us to choose one atom in the atom selection process, that why strongly encourages us to select more atoms in each iteration to speed up convergence.

We note another common formulation

$$\min_x ||\, x\, ||_1\; s.t\; ||\, Dx - y\, ||_2 \le \varepsilon \tag{6}$$

which explicitly sets the error constraint.

LARS also only allows one atom to be chosen in the atom selection process, which provides a strong incentive to select more atoms with each iteration in order to speed up convergence.

4 DICTIONARY LEARNING

It is important to take consider that the quality of sparse representation of a signal depends on the space in which it is represented. Learning the dictionary is a key point to make atoms as efficient as possible for a particular type of data. It has been shown that a learned dictionary has the power to provide better reconstruction quality than a predefined dictionary. This section addresses the problem of dictionary learning. Several algorithms are used, learning dictionaries without constraint, dictionaries themselves sparse, or dictionaries with a constraint of non-negativity.

For dictionary learning, we choose the K-SVD algorithm:

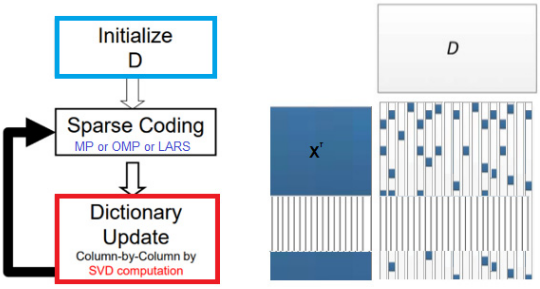

Figure 1: Principle of the K-SVD algorithm

5 SIMULATION

Image denoising is a difficult and open problem. Mathematically, the nature of image denoising is an inverse problem, and its solution is not unique. Thus, additional assumptions must be made in order to obtain a practical solution. since it is difficult to find and remove noise for all types of images, much research is carried out and various techniques are developed to promote the performance of denoising algorithms, In the following, we have presented tests to compare methods which give the best approximation in the context of the image denoising problem.

We used several approaches for the simulations, for the first test, we used the OMP algorithm for an image by fixing the number of atoms, and changing the pixel number values, and for the second test, we used the OMP algorithm for the same image by setting the pixel number and changing the atom number values.

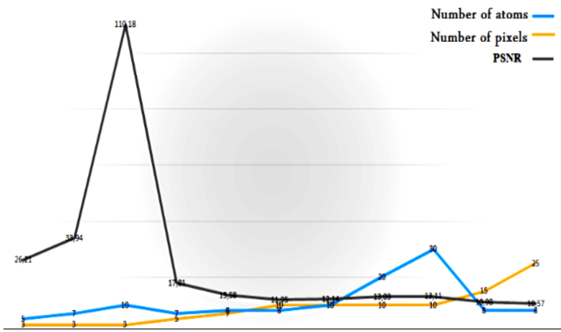

Figure 2: Comparison at PSNR level

5.1 The Principle of Dictionary Learning

In the denoising application, the objective is to restore an image degraded by noise (often an additive Gaussian white noise), abrod we must put the image in white and black then we make a simulation where half of the image is affected by a white Gaussian noise, We use half of the received image to reconstruct the image using dictionary decomposition.

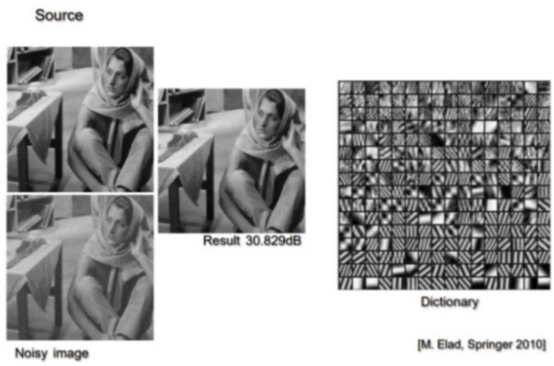

Figure 3: The principle of dictionary learning

We have chosen different simulations for learning the dictionary for the same image:
number of pixels 20, number of atom 2, we find:

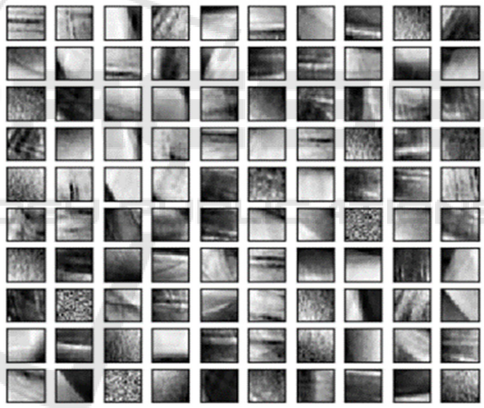

Figure 4: Example of dictionary

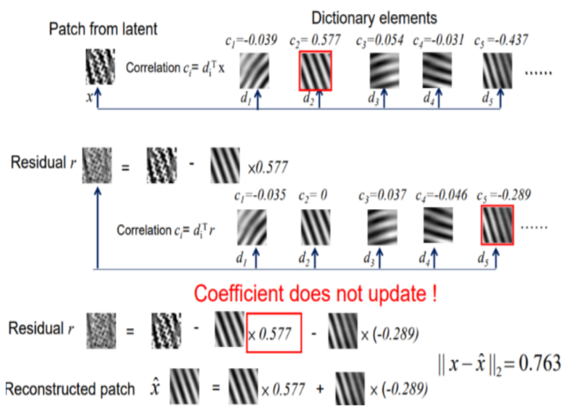

Figure 5: Dictionary elements

5.2 Comparison between OMP and LARS

Based on simulations, it is clear that LARS is better than OMP in terms of the efficiency and PSNR of the results, but the drawback is that the computation time is very slow compared to the first method. These diagrams clearly show the difference and comparison between the two methods at PSNR level and the calculation time:

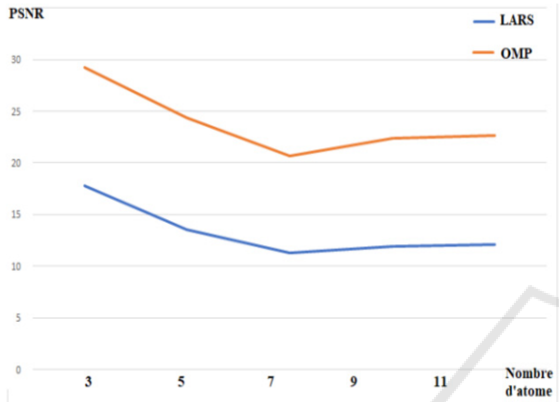

Figure 6: Comparison between OMP and LARS at the number of atoms

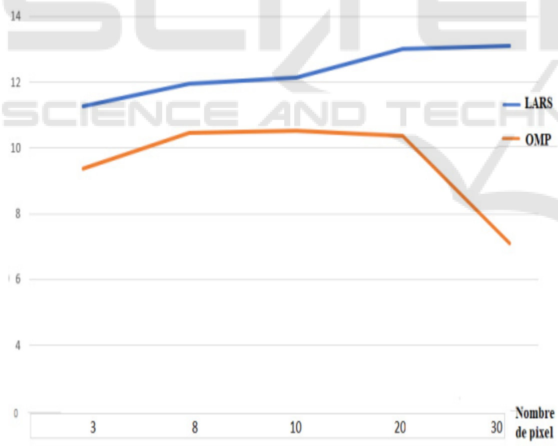

Figure 7: Comparison between OMP and LARS at pixel number level

6 CONCLUSION

Finally, we note that the KSVD is a fast approximation tool for updating the dictionary, which depends on the dictionary learning algorithm. The results obtained demonstrate the best performance of the proposed method in terms of training. This learning algorithm is therefore perfectly suited to certain signal processing applications.

In addition, there are several methods of reducing image noise by sparse decomposition, and since greedy algorithms such as MP or OMP, are capable of offering good reconstruction performance, are relatively complex because of the comparisons necessary to each iteration with each atom of the dictionary. so do OMP and LARs remain the most efficient, and KSVD also remain the best approximation for dictionaries?

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