Lattice Boltzmann Modelling of MHD Rayleigh-Bénard Convection in a Square Cavity Filled with a Ferrofluid

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Abstract: In this study, heat transfer of ferrofluid $Fe_3O_4 - H_2O$ generated by Rayleigh-Bénard convection in a square cavity is studied numerically in the presence of a vertical uniform magnetic field. The effect of the governing parameters, such as the Rayleigh number ($Ra = 10^3 - 10^5$), the volume fraction of nanoparticles ($\varphi = 0 - 4\%$) and the Hartmann number ($Ha = 0 - 100$), is studied using the Lattice Boltzmann Method (LBM). The results obtained show the existence of up to three different solutions for values of Ha less than some threshold. The obtained solutions have different ranges of existence and generate different amounts of heat transfer.

1 INTRODUCTION

Magnetohydrodynamic convection (MHD) in cavities heated from below is one of the most interesting problems in the literature due to its specificity. The Rayleigh-Bénard (RB) convection in the presence of an external magnetic field has been the object of wide studies by many researchers worldwide, owing to the importance of the field. The presence of an imposed magnetic field engenders the formation of the Lorentz force, whose effect competes with gravity. By this fact, the magnetic field, depending on its strength and orientation, may lead to a substantial modification of the flow structure and its intensity. In a previous study, (Alchaar et al. 1995) conducted a numerical study of RB convection in a shallow cavity filled with a conductive fluid, subject to the effect of an inclined magnetic field. The results of this study show that the vertical magnetic field has a significant impact and may bring back the convective motion to rest. Using a similar approach, (Rudraiah et al. 1995) performed a numerical investigation on free convection in a rectangular chamber confining a conductive fluid under the effect of an external magnetic field. The effect of the inclination angle of a cavity with two opposite sides brought to constant but different temperatures and filled with liquid gallium and exposed to a horizontal magnetic field was investigated by (Pirmohammadi and Ghassemi 2009). They recorded a maximum heat transfer rate for an inclination angle of 45° both in the absence and in the presence of a magnetic field.

Many studies were devoted to investigating experimentally and numerically the effect of the addition of magnetic nanoparticles (like nickel, cobalt, magnetite Fe_3O_4, etc.) in a base fluid (such as water), forming so-called ferrofluids. For instance, (Wang et al. 2016) studied experimentally the magnetic field effect on the viscosity of the ferrofluid $Fe_3O_4 - H_2O$. The results of this experiment show that the ferrofluid viscosity increases with magnetic induction and volume fraction of nanoparticles but decreases with temperature. A numerical study of a periodic magnetic field effect on natural convection and entropy generation in a square cavity filled with the ferrofluid $Fe_3O_4 - H_2O$ has been performed by (Mehryan et al. 2018). Their results show that the increase of the magnetic field period amplifies the vortex intensity inside the cavity. Furthermore, the addition of nanoparticles may lead to an improvement or degradation of the total entropy generation, depending on Ha and the remaining parameters.

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(Kefayati 2014) used the LBM to study a problem dealing with natural convection in a square cavity filled with ferrofluid in the presence of a magnetic force. Recently, (Ghasemi and Siavashi 2020) developed a numerical code based on the MRT-LBM (multi relaxation time Lattice Boltzmann method) to study the flow of $Cu - H_2O$ nanofluid by mixed MHD convection in a 3D enclosure.

The effect of a vertical magnetic field in cavities heated from below and filled with the ferrofluid $Fe_3O_4 - H_2O$ is poorly documented in the literature. Therefore, taking advantage of the simplicity and robustness of the LBM, an appropriate code based on this method was developed to examine the combined effects of numerous control parameters, such as Rayleigh number (Ra), Hartmann number (Ha) and nanoparticles volume fraction (φ) on multiple steady solutions describing the flow of the ferrofluid $Fe_3O_4 - H_2O$.

2 MATHEMATICAL FORMULATION

2.1 Rayleigh Bénard Configuration

The two-dimensional physical model considered is shown in Fig. 1. It consists of a square cavity (L × L) whose horizontal walls are subjected to a vertical destabilizing thermal gradient, while its vertical walls are thermally insulated. The bottom wall temperature (T_h) is higher than that of the upper wall (T_c). The square cavity is filled with the ferrofluid $Fe_3O_4 - H_2O$ and subject to the action of a vertical uniform magnetic field.

The study was conducted considering that the ferrofluid is Newtonian, and the resulting flow is laminar and incompressible. Moreover, the viscous dissipation and the heating due to the Joule effect were neglected. All the ferrofluid properties were considered constant, apart from its density that obeys the Boussinesq approximation in the buoyancy term. The data specific to the ferrofluid used in the present study are listed in Table 1.

2.2 Lattice Boltzmann Method

Essentially, the LBM method is based on two main steps: collision between the fluid particles characterized by the left side of Eq. (1), and streaming to describe the movement of these fluid particles towards the neighbouring nodes, characterized by the right side of Eq. (1). Bhatnagar-Gross-Krook (BGK) approximation was used for both local distribution functions f and g for the momentum and energy equations, respectively. The lattice Boltzmann equation (LBE) with external forces can be expressed at the position r and time t as follows:

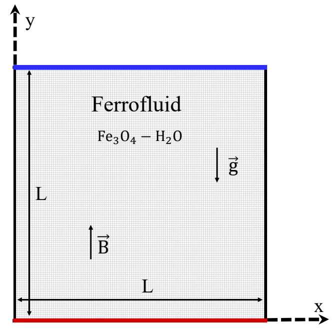

Figure 1: Schematic of the physical problem.

$$f_i(r + c_i\Delta t, t + \Delta t) - f_i(r,t) = \frac{1}{\tau_v}\left(f_i^{eq}(r,t) - f_i(r,t)\right) + F_i \quad (1)$$

$$g_i(r + c_i\Delta t, t + \Delta t) - g_i(r,t) = \frac{1}{\tau_\alpha}\left(g_i^{eq}(r,t) - g_i(r,t)\right) \quad (2)$$

$$f_i^{eq} = \omega_i\rho\left(1 + 3\,\vec{c}_i\vec{u} + \frac{9}{2}(\vec{c}_i\vec{u})^2 - \frac{3}{2}(\vec{u})^2\right) \quad (3)$$

$$g_i^{eq} = \omega_i T\left(1 + 3\,\vec{c}_i\vec{u} + \frac{9}{2}(\vec{c}_i\vec{u})^2 - \frac{3}{2}(\vec{u})^2\right) \quad (4)$$

$$\begin{aligned} F_i &= Fx_i + Fy_i \\ Fx_i &= -3\,\omega_i\,\rho\,A\,uc_{ix} \\ Fy_i &= 3\,\omega_i\,\rho\,g\beta(T - T_m)c_{iy} \end{aligned} \quad (5)$$

Where τ_ν and τ_α are respectively the relaxation coefficients for the momentum and energy equations and $T_m = (T_h + T_c)/2$ is the reference temperature. The quantities f_i^{eq} and g_i^{eq} are the local equilibrium distribution functions for density and temperature, respectively and F_i is the external force, which is composed of two terms (the buoyancy (Fy_i) and Lorentz (Fx_i) forces). The coefficients $\vec{c}_i$ and ω_i are respectively the discrete velocity and the weighting coefficients at direction i, which are defined in D2Q9 lattice arrangement as follows:

$$c_i = \begin{cases} (0,0), & i = 0 \\ \left(\cos\left[\frac{\pi(i-1)}{2}\right], \sin\left[\frac{\pi(i-1)}{2}\right]\right), & i = 1-4 \\ \sqrt{2}\left(\cos\left[\frac{\pi(2i-9)}{4}\right], \sin\left[\frac{\pi(2i-9)}{4}\right]\right), & i = 5-8 \end{cases} \quad (6)$$

Table 1: Thermal physical properties of H_2O (pure water) and Fe_3O_4 (nanoparticles) (Ghaffarpasand 2016).

Properties	$\rho(Kg.m^{-3})$	$c_p(J.Kg^{-1}.K^{-1})$	$k\ (W.m^{-1}.K^{-1})$	$\beta \times 10^5(K^{-1})$	$\sigma(m.\Omega)$
H_2O	997.1	4179	0.613	21	0.05
Fe_3O_4	5200	670	6	1.3	25000

$$\omega_i = \begin{cases} 4/9, & i = 0 \\ 1/9, & i = 1-4 \\ 1/36, & i = 5-8 \end{cases} \tag{7}$$

Finally, the density ρ, velocity $\vec{u}$ and temperature T, are calculated using the local distribution functions as follows:

$$\rho = \sum_{i=0}^{8} f_i \tag{8}$$

$$\vec{u} = \frac{1}{\rho}\sum_{i=0}^{8} f_i \vec{c}_i \tag{9}$$

$$T = \sum_{i=0}^{8} g_i \tag{10}$$

The ferrofluid thermo-physical properties, such as density (ρ_{ff}), coefficient of thermal expansion (β_{ff}) and heat capacity ($(\rho c_p)_{ff}$) that appear in the governing equations, were evaluated using the following equations (Sheikholeslami and Ganji 2014):

$$\rho_{ff} = (1-\varphi)\rho_f + \varphi\rho_s \tag{11}$$

$$\beta_{ff} = (1-\varphi)\beta_f + \varphi\beta_s \tag{12}$$

$$(\rho c_p)_{ff} = (1-\varphi)(\rho c_p)_f + \varphi(\rho c_p)_s \tag{13}$$

The thermal and electrical conductivities are estimated by the Hamilton and Crosser (Hamilton 1962) and Maxwell models, respectively.

$$\frac{k_{ff}}{k_f} = \frac{k_s + 2k_f - 2\varphi(k_f - k_s)}{k_s + 2k_f + \varphi(k_f - k_s)} \tag{14}$$

$$\frac{\sigma_{ff}}{\sigma_f} = 1 + \frac{3\varphi\left(\frac{\sigma_s}{\sigma_f} - 1\right)}{\left(\frac{\sigma_s}{\sigma_f} + 2\right) - \varphi\left(\frac{\sigma_s}{\sigma_f} - 1\right)} \tag{15}$$

The parameter A in equation (5) is obtained as $A = \frac{\sigma_{ff}.\rho_f}{\sigma_f.\rho_{ff}}\mathrm{Ha}^2\nu/L^2$, with $\mathrm{Ha} = LB_0\sqrt{\sigma_f/\mu_f}$ being the Hartmann number.

The boundary conditions adopted in the present study are similar to those used by (Kao and Yang 2007).

The heat transfer by convection is evaluated through the calculation of the Nusselt number that is evaluated locally on the heated wall, Eq. (16), and averaged along this boundary, Eq. (17):

$$Nu_{loc} = -\frac{k_{ff}}{k_f}\frac{\partial T}{\partial Y}\bigg|_{Y=0} \tag{16}$$

and

$$Nu_m = \int_0^1 Nu_{loc}\, dX \tag{17}$$

2.3 Validation of the LBM Code

The validation of the numerical code is an essential step before carrying out the numerical simulations specific to the studied problem. Thus, for the validation tests, the case of natural convection flow of the nanofluid Al_2O_3-water confined in a differentially heated cavity subjected to the action of a magnetic field was considered. This configuration was the object of a previous numerical investigation by (Ghasemi et al. 2011). The results presented in Fig. 2 in terms of mean Nusselt number were obtained with a volume fraction of nanoparticles of 3% and various Rayleigh and Hartmann numbers. Fig. 2 shows a good agreement between our results and those obtained by (Ghasemi et al. 2011), the maximum deviation being within 4.18%. On another hand, preliminary tests were carried out to appreciate the sensitivity of the results by varying the mesh size as illustrated in Table 2 in terms of mean Nusselt numbers for different solutions of the problem obtained with $Ra = 10^5$, Ha $= 30$ and $\varphi = 4\%$. The inspection of these results shows that the selected grid of 120×120 is enough to conduct the present study. In fact, this grid leads to results that differ by about 0.23% (as maximum deviation) from those obtained with the finest grid of 160×160.

Table 2: Grid sensitivity in terms of Nu_m for $Ra = 10^5$, Ha $= 30$ and $\varphi = 4\%$.

	80^2	100^2	120^2	140^2	160^2
MF	3.020	3.018	3.0164	3.0157	3.0153
BF	3.509	3.495	3.486	3.481	3.478
TF	3.481	3.471	3.467	3.464	3.462

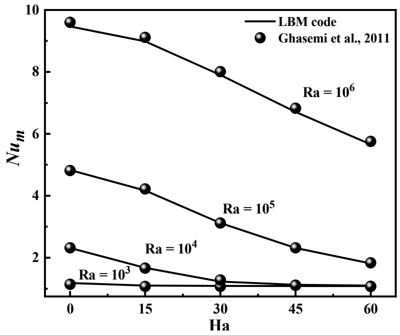

Figure 2: Numerical code validation against results of (Ghasemi et al. 2011) in terms of Nu_m vs. Ha for $\varphi = 3\%$ and various Ra.

3 RESULTS AND DISCUSSION

In the case of cavities heated from below, the literature review shows that heat transfer depends on the type of solution for a problem characterized by a multiplicity of solutions. The present study is part of these problems since multi-steady state solutions have been obtained with different ranges of existence. In fact, the existence of the monocellular, bicellular and tri-cellular flows has been proved numerically; they will be noted MF, BF and TF, respectively. These three types of solution were also obtained by (Mansour et al. 2006) in a square porous cavity heated from below and submitted to a horizontal concentration gradient. The main purpose of this study is to investigate the influence of a uniform vertical magnetic field ($\mathrm{Ha} = 0$ to 100) and nanoparticles volume fractions ($\varphi = 0$ to 4%) on different thermal and dynamic behaviours for a fixed value of Rayleigh number ($Ra = 10^5$).

3.1 Effect of Hartmann Number

The effect of Hartmann number on the thermal and dynamic behaviours of the base fluid (solid lines) and the $\mathrm{Fe_3O_4 - H_2O}$ ferrofluid (dashed lines) is illustrated in Figs. 3(a) and 3(b) for $Ra = 10^5$. The inspection of Fig. 3a shows that for $\mathrm{Ha} = 0$ and 25, the three types of solutions previously mentioned are obtained. The bicellular solution is characterized by a symmetry regarding the vertical axis passing by the centre of enclosure, while the monocellular and tricellular solutions show a symmetry with respect to the centre of the cavity. The increase of Ha to 25 leads to a substantial reduction of the flow intensity characterized by a division by factors of 2.87 / (1.65) / (1.28) in the case of the MF / (BF)/ (TF). These important reductions that accompany the increase of Ha are expected knowing the damping role engendered by the increase of the intensity of the magnetic field. The increase of Ha from 25 to 50 leads to the disappearance of the MF solution and severely reduces the intensities of the remaining structures that become 2.08 and 1.80 times less intense for the BF and TF solutions, respectively. The addition of the nanoparticles promotes the flow intensity since the effect of the global improvement of the ferrofluid conductivity outweighs the increase of viscosity for the small fraction of nanoparticles added. It is also observed that the impact of nanoparticles on the MF flow is more important compared to the other flow types (BF and TF). More specifically, for $\mathrm{Ha} = 0$, the flow intensity increases by about 4.2% for the MF, while this increase does not exceed 2.4% and 1.3% for the BF and TF flows, respectively. On another side, the flow intensity is influenced differently by adding nanoparticles in the presence of a magnetic field, and this influence is more attenuated in comparison with $\mathrm{Ha} = 0$. In fact, the addition of nanoparticles for $\mathrm{Ha} = 25$ leads to an improvement of the MF and BF flows intensities, respectively, by about 2.5% and 1.7%, while the intensity of the TF stays unchanged. By increasing progressively Ha, the MF flow transits toward the TF from a threshold value Ha_c of this parameter. This critical value depends strongly on the volume fraction of the nanoparticles. More exactly, Ha_c drops from 45 (case of pure fluid) to 40 (case of ferrofluid with $\varphi = 4\%$).

By considering the thermal aspect of the problem, Fig. 3b shows clearly that the temperature fields undergo strong changes accompanying the change in the flow structure, particularly in the central region of the cavity due to the interaction between the neighbouring cells. Thus, the number of ripples revealed by the isotherms increases horizontally as the flow cells number increases. This behaviour results from the fact that each cell has one cold vertical side and one hot vertical side. The presence of the ripples attests that the changes of temperature gradients prevail horizontally following the increase of the number of cells. For the three types of solutions, the thicknesses of the thermal boundary layers developed near the horizontal active walls increase by incrementing Ha.

3.2 Heat Transfer

The effect of the magnetic field on the mean Nusselt number calculated along the heated wall is exemplified in Fig. 4 for $\varphi = 0$ and 4%. This figure shows that the addition of nanoparticles loses its

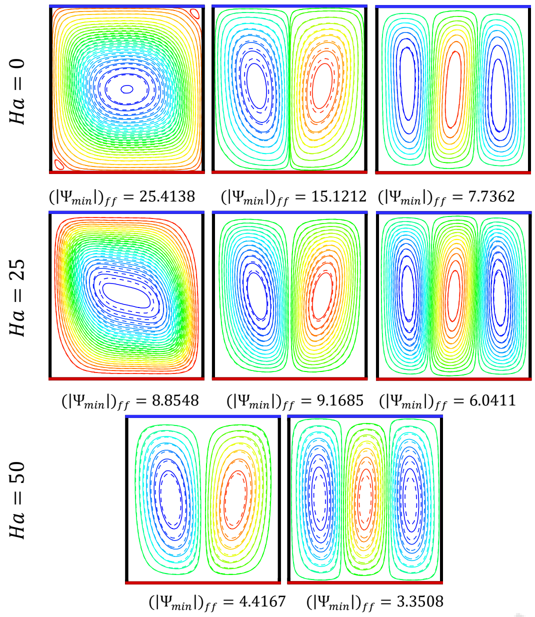

Figure 3a: Streamlines obtained with $\varphi = 0$ (solid lines) and $\varphi = 4\%$ (dashed lines), Ra $= 10^5$ and different Ha.

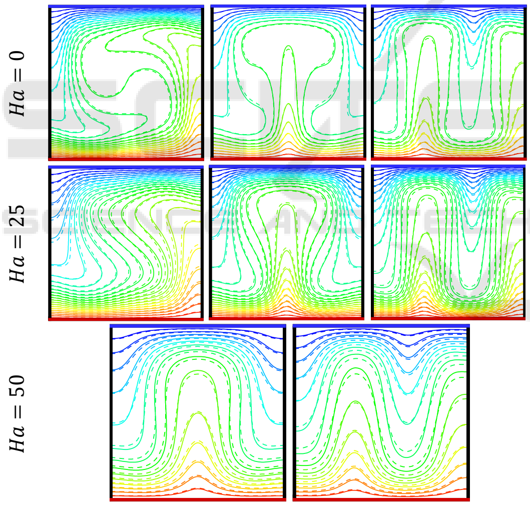

Figure 3b: Isotherms obtained with $\varphi = 0$ (solid lines) and $\varphi = 4\%$ (dashed lines), Ra $= 10^5$ and different Ha.

advantage from some thresholds of Ha depending on the type of solution. Moreover, the increase of the magnetic field strength (which leads to the increase of Ha) has a negative effect on heat transfer rates within the cavity since it is accompanied by a continuous deterioration of the Nusselt numbers corresponding to each type of solution. The MF is the least favourable to heat exchange for this configuration and the most unstable in the sense that it transits first toward the TF from Ha $= 40/45$ for $\varphi = 0/4\%$. This transition leads to a substantial improvement of the heat transfer rate (with an

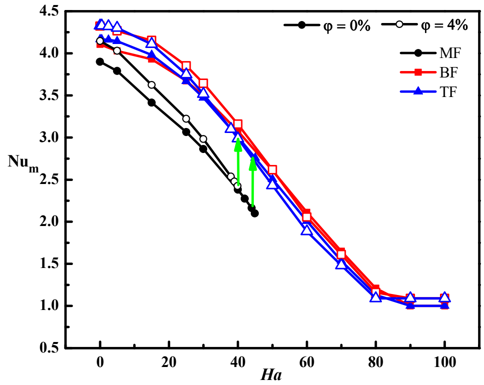

Figure 4: Mean Nusselt number vs. Ha for Ra $= 10^5$ and different solutions.

increase of about 24%). Globally, the BF dethrones the TF in terms of heat exchange from some thresholds values of Ha that depend on φ, and this limited advantage persists in the whole range of Ha. Note also that the TF resists to the increase of Ha, and its transition towards the BF is delayed and occurs in a regime dominated by conduction.

4 CONCLUSION

In the present study, natural convection heat transfer of ferrofluid $Fe_3O_4 - H_2O$ in a Rayleigh-Bénard square cavity was investigated numerically using the LBM. The results presented show that up to three steady-solutions were obtained for relatively low Hartmann numbers (Ha ≤ 19). The MF is less favourable to heat transfer and transits towards the TF from this threshold value of Ha. Both BF and TF resist to the increase of Ha even when the role of convection vanishes (Ha > 80). Finally, the heat transfer rates generated by the BF and TF remain comparable with a slight advantage in favour of the BF for relatively high values of Ha.

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