Contribution to the Study of the Numerical Simulation of Compressible Flow in a Convergent-divergent Nozzle

A. Ederouich[1][a], M. Essahraoui [1][b], D. Zejli [1][c] and A. Saad [2][d,*]
[1] *SEALAB, Advanced Systems Engineering Laboratory. Kenitra, Morocco*
[2] *National School of Applied Sciences, Ibn Tofail University, Kenitra, Morocco*

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Abstract: The use of a convergent-divergent nozzle meets several needs, namely the study of the performance of wind turbines, turbine engine blades, aeroplane wings, and propellants. In this work, we have been studying compressible flow of an ideal gas in a one-dimensional convergent-divergent nozzle essentially. The study aims at determining the evolution of all the parameters of the flow along with the Nozzle (the pressure, temperature, Mach number, and density). Thus, we studied the influence of the nozzle geometry on the flow by the variation of the diverging angle. The numerical simulation of the flow has been carried out using the ANSYS Fluent software, which uses the finite volume method to solve the various partial differential equations modelling the physical phenomenon. The results obtained from this model have been compared with the theoretically calculated results. Good agreement was observed.

1 INTRODUCTION

The nozzle is widely used in various areas, from rocket propulsion to fuel sprayer. It has been applied in industrial, aerospace, automobile, and other sectors. The nozzle is a major part of any high-performance engine or rocket motor. It is used to control the velocity, direction, and required parameters of the flow. Nozzles are designed to operate in all flow regions like subsonic, sonic, supersonic, and hypersonic. The design of the supersonic nozzle remains a challenging task in fluid mechanics. In a supersonic nozzle, not only do the physical parameters of the nozzle play an essential role, but the thermodynamic parameters of the flow also play a crucial role in defining the design of a nozzle. The Converging-Diverging Nozzle known as de Laval nozzle is the most common and converts high pressure, high temperature, and low velocity (subsonic) gases into low pressure, low temperature, and high velocity (supersonic) gases, hence producing high thrust (Khalid and Ahsan, 2020).

CFD (computational fluid dynamics) a branch which is widely used for solving governing equations of fluid dynamics. Today we can find its applications for all disciplines such as heat transfer, fluid dynamics and even for natural science etc. Problems which are very complicated to solve by means of general analytical method can be easily solved by CFD. Since the set of equations of continuity, momentum, energy of fluid dynamics is called as Navier-stokes equation.

There are many approaches in CFD through which we can obtain the appropriate result, but the standard method used is finite volume method. For every bit of volume, the equations are solved and results are obtained. Thus after the completion of iterations each point specific some value. Thus through these results we can make a point on the behavior of fluid flow (Maddu et al., 2018).

For the present study, we are using ANSYS to determine the evolution of the flow parameters (the pressure, temperature, Mach number, and density) in the convergent-divergent nozzle. Thus, we put focus on the influence of the nozzle geometry on the flow by the variation of the diverging angle.

2 GOVERNING EQUATIONS OF FLUID FLOW

To understand the physics of the fluid in motion related to any engineering problem, it's important that we develop a accurate relationship among the

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variations of the fluid flow properties such pressure, temperature, velocity, density etc at discrete points in space and time. The fluid governing equations proves a theoretical solution to how these flow properties are related to each other by either integral, differential or algebraic equations. The following three fundamental laws known as the conservation laws are used to establish the governing equations of the fluid flow [3].

- Conservation of Mass [4]:

$$\frac{\partial\rho}{\partial t} + \nabla(\rho\vec{v}) = S_m \tag{1}$$

- Conservation of momentum equation is:

$$\frac{\partial\rho}{\partial t}(\rho\vec{v}) + \nabla(\rho\vec{v}\vec{v}) = -\nabla p + \nabla(\bar{\bar{\tau}}) + \rho\vec{g} + \vec{F} \tag{2}$$

- Conservation of energy equation is :

$$\frac{\partial\rho E}{\partial t} + \mathrm{div}(\rho E\vec{v} - \sigma\vec{v} + \vec{q}) = \rho\vec{f}\vec{v} + \rho w \tag{3}$$

To study compressible flow in CD nozzles, the conservation equations for mass, momentum and energy must be solved. Note that these equations are partial differential equations whose resolution is not known a priori. Put as such, the calculation is done based on the following assumptions:

✓ Perfect gas.
✓ Flow is unidirectional and steady.
✓ Flow is adiabatic and reversible.

3 DESCRIPTION OF THE PROBLEM

We propose to study the flow of air assumed to be ideal gas in a convergent -divergent nozzle (De Laval), the basic dimensions of which are given in the following table:

Table 1 : Nozzle Dimensions.

Parameters	Dimensions (mm)
Nozzle length	200
Nozzle exit radius	70
Nozzle inlet radius	50
Convergent angle	40°
Divergent angle	20°

The convergent and the divergent are connected by an arc of a circle.

4 CFD ANALYSIS

Designing, modelling, and mesh generation of the nozzle were carried out in Design Modeler and ANSYS Mesh.

4.1 Geometry and Modelling

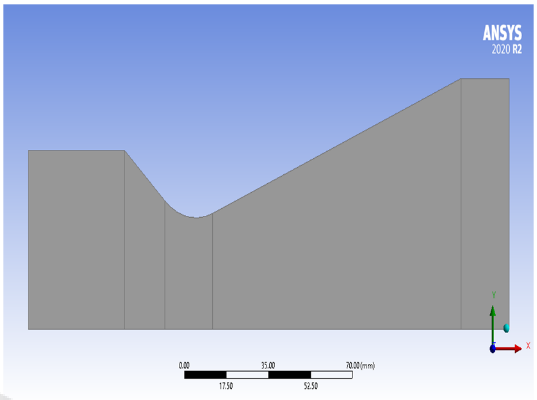

Figure 1: The Geometry of the Half-Nozzle Studied.

4.2 Meshing and Boundary Conditions

One of the most important steps in numerical simulations is to achieve a mesh appropriate to the problem being dealt with.

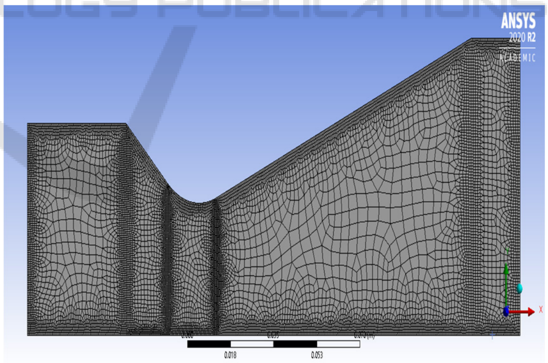

Figure 2: Mesh of the Half-Nozzle (Unstructured).

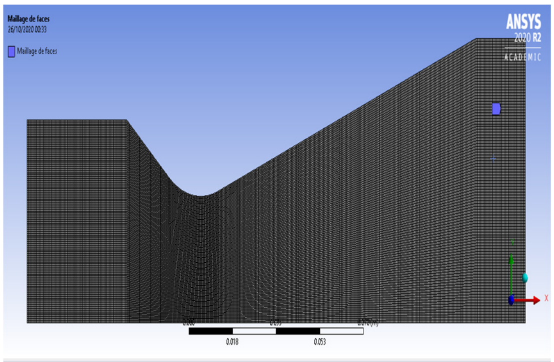

Figure 3: The Final Mesh of the Half-Nozzle (Structured).

Figure 4:Limit Boundaries.

4.3 "Fluent" Solver

Table 2:General Setup.

Setup	
Solver	Density-based
2D Space	axisymmetric
Time	steady
Pressure at inlet	100000 Pa
Temperature at inlet	300 K
Pressure at outlet	2814 Pa
Temperature at outlet	300 K

5 RESULTS AND INTERPRETATIONS

5.1 Evolution of Flow Parameters

5.1.1 Variation of Static Pressure

In the convergent-divergent nozzle the gas undergoes a large expanding operation to transform the thermal energy and the pressure energy of the gases into kinetic energy. Figures (5 and 6) show the drop in static pressure in the nozzle. The gas expands from pressure 100911.4 (Pa) to pressure 799.6063 (Pa) at the outlet of the nozzle. This given fact is logical since in a supersonic flow the pressure is inversely proportional to the section (referring to the formula of Hugoniot), and the pressure graph does not represent any disturbance or fluctuating, which corresponds to a completely isentropic flow along of the divergent.

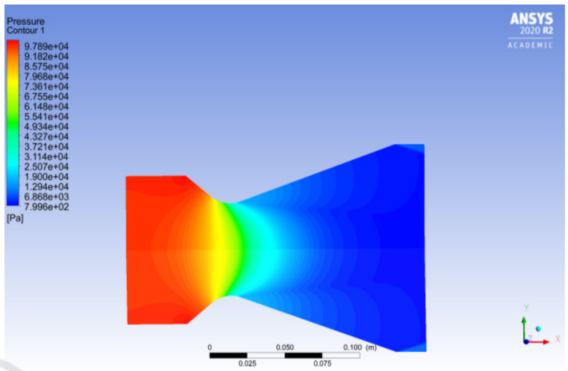

Figure 5: Contours of Static Pressure.

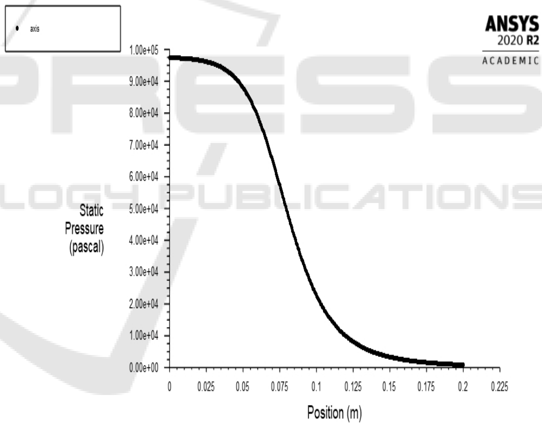

Figure 6: Evolution of the Static Pressure along the Axis of the Nozzle.

5.1.2 Variation of Static Temperature

As the flow is completely isotropic in the nozzle, then the temperature change is proportional to the pressure, referring to the ideal gas law. This can be seen in Figures (7 and 8); the temperature in the middle of the nozzle is subjected to a uniform and continuous descent of the outlet. It varies from 299,667 K at the inlet of the nozzle to 75,383 K at the outlet.

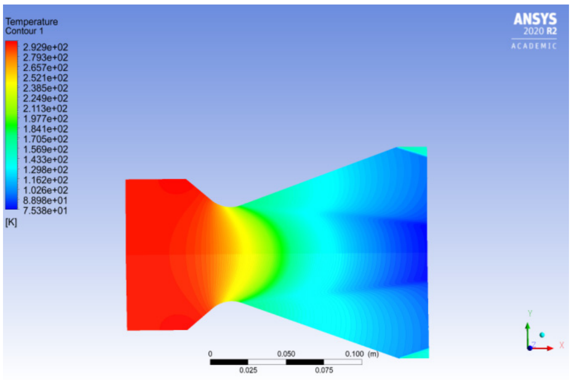

Figure 7: Contours of the Static Temperature.

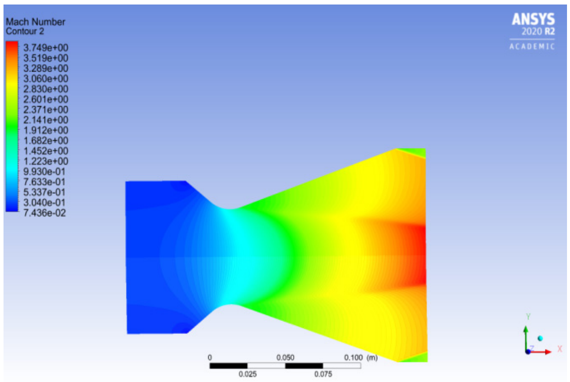

Figure 9: contours of Mach number.

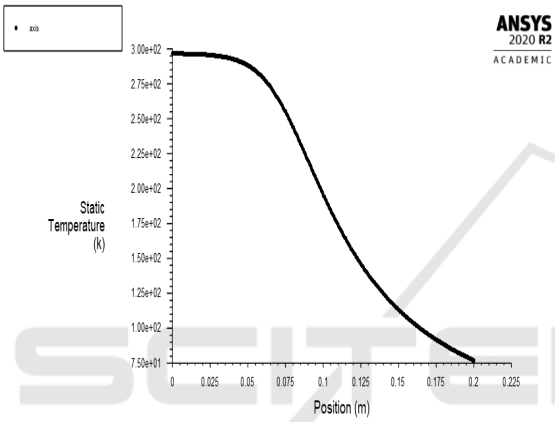

Figure 8: Evolution of the Static Temperature along the Axis of the Nozzle.

5.1.3 Variation of Mach Number

Figures (9 and 10) display the distribution of the Mach number in the nozzle. It is observed that the speed at the inlet of the nozzle remains almost constant or invariable up to its throat. We can say that the regime is subsonic (Ma 1). We can neglect the compressibility of air for Mach numbers less than 0.3 because the value of the Mach number is strictly less than unity. The flow in the throat is transonic (0.8 Mach 1.2). In the divergent flow, the flow becomes supersonic and reaches a maximum value equal to 3.863 at the outlet of the nozzle. Thus, the convergent-divergent profile of the nozzle allows the gas to accelerate from a subsonic speed to a supersonic speed.

This evolution follows Hugoniot's law which states that the speed is proportional to the section of a supersonic flow, and varies in the opposite direction of the section for a subsonic flow.

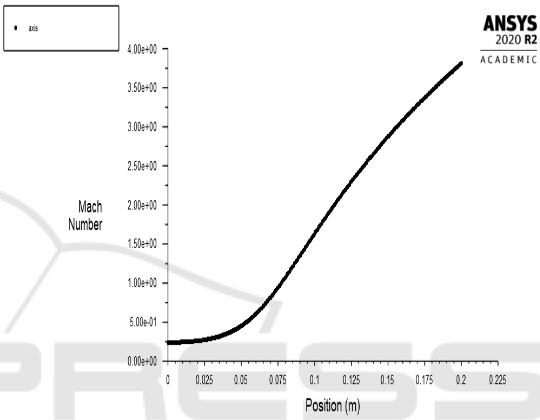

Figure 10: Evolution of the Mach number along the Axis of the Nozzle.

5.1.4 Variation of Density

Figures (11 and 12) show the variation of the density along the nozzle. We observe that the density almost constant at the convergent part at its maximum value (1.173 Kg/m^3).On the divergent part the density undergoes a decrease until the exit of the nozzle (0.037Kg/m^3).This is normal since the pressure decreases.

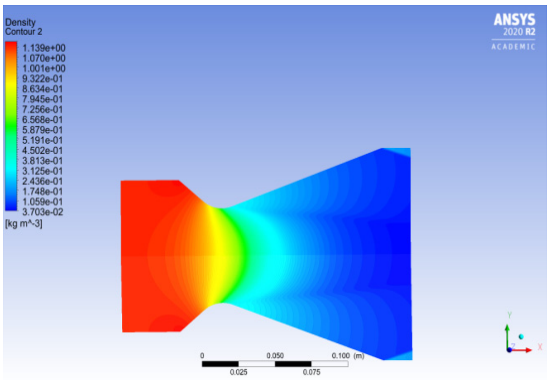

Figure 11: Contours of Density.

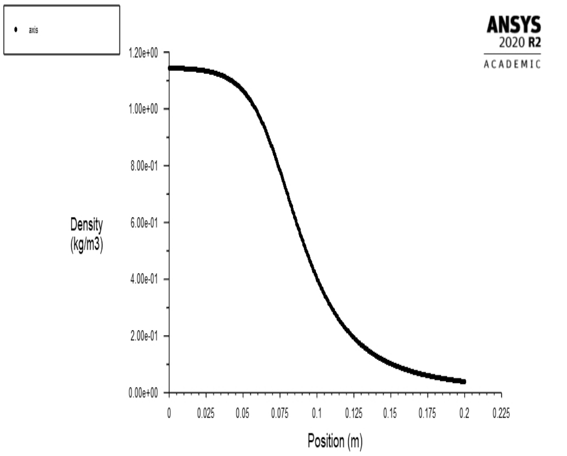

Figure 12: Evolution of the Density along the Axis of the Nozzle.

5.2 Study of the Influence of the Nozzle Geometry on the Flow

In this part, the numerical study is carried out at different angles of the divergent to find out the effect of the diverging angle on the Mach number and the static pressure. The different diverging angles used for the analysis are 17 °, 20 °, 23 ° and 26 °. The boundary conditions are the same in the four cases.

Four different dimensional designs are modelled by changing the diverging angle, then the CFD analysis is performed for four different models and the variation in Mach number and static pressure is observed in each case.

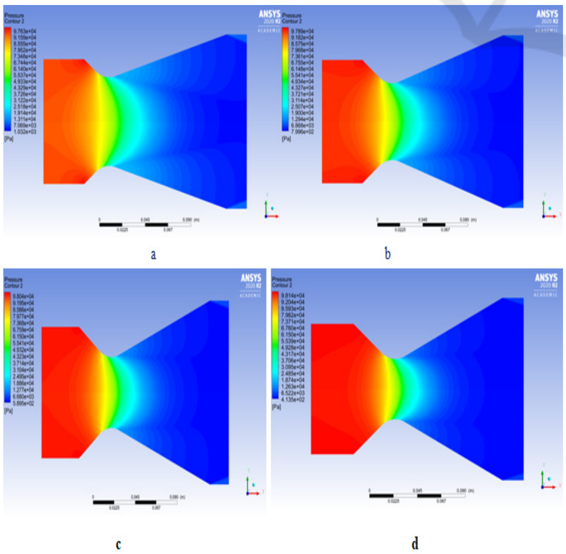

Figure 13: Static pressure Contours for Different Angles of the Diverging Part: a (17 °), b (20 °), c (23 °) and d (26 °).

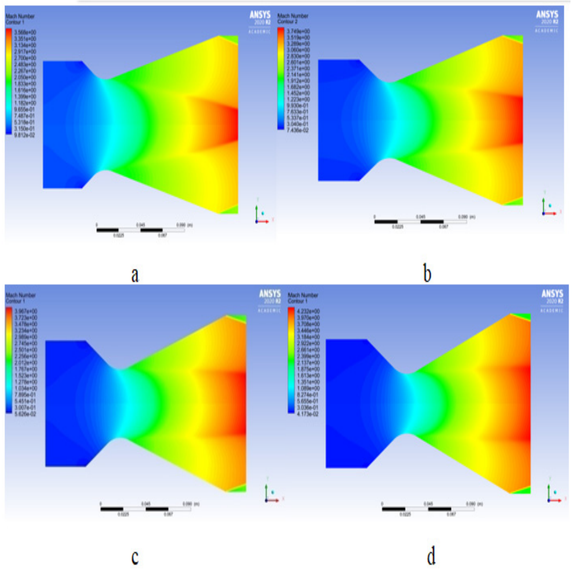

Figure 14: Mach number Contours for Angles a, b, c and d.

5.2.1 Exit Conditions

Table 3: The Conditions at the Outlet of the Nozzle for the Four Cases.

Case	Diverging angle	Mach number	Static pressure (Pa)
1	17 °	3 .676	1032.146
2	20 °	3.863	799.606
3	23 °	4.089	589.508
4	26 °	4.362	413 .517

The results of the analysis on the nozzle with a variable diverging angle are as follows:

- ✓ In the divergent section, the velocity distribution increases with increasing divergent angle.
- ✓ Static pressure decreased with increasing divergent angle.

5.3 Comparison with Analytical Results

We choose case 2 (the diverging angle is 20 °).Table (4) shows the comparison of the values of pressure, temperature and velocity at the throat obtained from this simulation and the analytical results calculated from the following relations:

$$\frac{T_c}{T_0} = \frac{2}{\gamma + 1} = 0.8316 \tag{4}$$

$$\frac{p_c}{p_0} = \left(\frac{T_c}{T_0}\right)^{\gamma/(\gamma-1)} = \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)} = 0.5275 \quad (5)$$

$$a_c = \sqrt{\gamma r T_c} = \sqrt{\frac{2\gamma}{\gamma+1} r T_0} \quad (6)$$

The simulation results obtained from this model have been showed a good agreement with the analytical results.

Table 4: The Parameters of the Flow at the Throat.

	Analytical	This model
Static pressure at throat (Pa)	53 448.94	53500
Static temperature at throat (K)	249.48	250
velocity at throat (m/s)	316	318.75

6 CONCLUSION

This study project allowed us to discover a topical and research topic that concerns the field of turbomachines, aeronautics and space. The primary motivation for this work has been the study and understanding of physical phenomena encountered in practical fields such as turbines, supersonic wind tunnels, supersonic airplanes and space launchers. Our project is mainly interested in the processing by numerical simulation of compressible flows in convergent-divergent nozzles.

The results obtained from this model were compared with the theoretically calculated results. Good agreement was observed.

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