Forecasting Stock Returns with Fuzzy HEAVY-r Model using Genetic Algorithm

Youssra Bakkali, Mhamed El Merzguioui and Abdelhadi Akharif
Laboratory of Mathematics and Applications, Abdelmalek Essaadi University, Faculty of Science and Technology of Tangier, Morocco

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Abstract: Financial returns expose complex dynamics that are difficult to capture with classical econometric models, the most common feature in financial series is volatility clustering. We propose the Fuzzy HEAVY-r model for modelling and predicting returns of the CAC40 stock market index. This model has been developed by a combination of the fuzzy inference system and the HEAVY-r model. A Genetic Algorithm (GA) based parameters estimation algorithm is suggested to obtain the optimal solution for the fuzzy membership function and the HEAVY-r model. We apply these models to the high-frequency financial data regularly spaced in time (every minute) and (every five minutes), and we compared it with the Fuzzy GARCH model and the classical models. The results indicate that the Fuzzy HEAVY-r model outperforms other models in out of sample evaluation according to RMSE.

1 INTRODUCTION

In econometrics, volatility has been one of the most active research subjects. The autoregressive Conditional Heteroscedasticity Models (ARCH) introduced by Engle (1982) and their extensions GARCH (generalized ARCH) introduced by (Bollerslev, 1986) are essentially based on the concept of conditional variance and play an effective role in modelling the dynamic features of volatility. The GARCH family models are ineffective in cases where volatility changes rapidly to a new level.

With the arrival of high-frequency data in the world of finance, a large number of studies have been recently published. Research on realized measures of volatility is becoming popular in studies, including realized variance introduced by Andersen et al. (2001a) and Barndorff-Nielsen (2002), the realized kernel introduced by Barndorff-Nielsen et al. (2008), and many related quantities. These measures are more precise and effective than the squared return in determining the current level of volatility.

The HEAVY model (SHEPHARD & SHEPPARD, 2010) blends the intellectual lessons of the GARCH model with modern higher frequency data literature and shows that the HEAVY models are more resilient than traditional GARCH models to level breaks in the volatility that adjust much faster to the new level.

Given that financial series present complex and nonlinear behaviours that make modelling difficult, various artificial intelligence techniques have been tested for prediction problems and have shown better performance.

Artificial Neural networks (ANNs) have been used successfully, but the weak point is that the (ANNs) are black boxes, and it is not possible to explain the links between inputs and outputs.

In order to compensate for this weakness of (ANNs), studies insist on the interest of systems combining the aspect connectionist of (ANNs) to reasoning techniques. In this objective, neuro-fuzzy systems are particularly indicated.

Current research on prediction problems of nonlinear time series shows that the neuro-fuzzy performs better than ANNs (Wang, Golnaraghi, & Ismail, 2004).

Hung (2009b) proposed a hybrid Fuzzy-GARCH model. The model was combining a functional fuzzy inference system to analyze clustering with a GARCH model using genetic algorithms to estimate the parameters.

We propose the Fuzzy HEAVY-r model that combines the heavy model in order to capture conditional volatility and the fuzzy approach offers

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the ability to simulate stock movements with volatility clustering.

For the Fuzzy HEAVY-r model estimation problem, the GA method (Holland, 1984) aims to achieve an optimal solution.

Our study will be based on daily financial returns (r_t) and a sequence of daily realised variance (RV_t) $t = 1,2 \ldots N$.

The rest of this paper is organised into six sections. The HEAVY models are presented in section 2. The Fuzzy HEAVY-r is presented in section 3. In section 4, the Genetic algorithm. In section 5, We have highlighted the properties observed empirically in high-frequency financial data, and we apply the Fuzzy HEAVY-r model to the real data. Concluding remarks are given in section 6.

2 HEAVY MODELS

The structure of the HEAVY models (SHEPHARD & SHEPPARD, 2010) is given by:

$$\mathrm{Var}(r_t/F_{t-1}^{HF}) = h_t = \alpha_0 + \alpha\, h_{t-1} + \beta\, RM_{t-1}, \quad \alpha \geq 0, \beta \in [0,1]. \tag{1}$$

$$E(RM_t/F_{t-1}^{HF}) = \mu_t = \gamma + \alpha_R\, \mu_{t-1} + \beta_R\, RM_{t-1}, \quad \alpha_R + \beta_R \in [0,1] \tag{2}$$

Where r_t denote the daily return, F_{t-1}^{HF} is the information set generated by high-frequency past data, and RM_t, that is, the high-frequency volatility estimators. Note that (1) is called the HEAVY-r model and (2) the HEAVY-RM model.

The volatility estimator that we adopt in our model is the realized volatility (Barndorff-Nielsen & Shephard, 2002) , as a realised measure:

$$RV_t^{\Delta} = \sum_{i=1}^{N} \left(\log(P_{t-1+i\Delta}) - \log\left(P_{t-1+(i-1)\Delta}\right)\right)^2,$$

The intra-day time subscripted as i = 0, 1, 2, ..., N and Δ is the frequency. Note that $P_{t-1+i\Delta}$ is the closing price at the i-th interval of day t.

3 THE FUZZY HEAVY-R MODEL

Neuro-fuzzy systems are suitable tools for solving the prediction problems of nonlinear time series. Since the most common feature in financial series is volatility clustering, we combine the HEAVY-r model and fuzzy systems to capture the accumulation of volatility.

Fuzzy set theory is similar to human reasoning. The capacity of fuzzy logic to imitate human reasoning is one of the reasons why fuzzy systems are being considered in this study.

The fuzzy inference system is a computational framework that is used to examine and evaluates the output of fuzzy systems in three steps: the fuzzification (i.e. partitioning of the input data to the antecedent of the fuzzy rules), then IF-THEN rules, and finally the defuzzification.

The use of language rules IF-THEN reflects knowledge about a system's dynamics, which makes creating prediction systems with fuzzy inference systems interesting.

Fuzzy sets are defined through a membership function (denoted by φ) which converts data into scale inputs ranging from 0 to 1.

$Rule^{(l)}$: IF r_{t-1} is φ_l THEN:

$$r_{lt} = \sqrt{h_{lt}}\varepsilon_{lt},$$
$$h_{lt} = \alpha_0^l + \sum_{i=1}^{q} \alpha_i^l\, h_{t-i} + \sum_{j=1}^{p} \beta_j^l\, RM_{t-j}$$

Such that: r_{t-1} the previous value of the stock returns, φ_l is the fuzzy set for rule l (l = 1,2…L), with L the number of rules IF_THEN.

The function that we adopt in our model is the Gaussian membership function:

$$\varphi^l(r_{t-1}) = \exp\left(-\frac{1}{2}\left(\frac{r_{t-1}-c_l}{\sigma_l}\right)^2\right),$$

With c is the center of the Gaussian function and σ is a positive constant determines the zone of influence of the cluster in question.

The output y of the inference system is determined by taking a weighted average of the outputs of the different rules.

$$r_t = \sqrt{h_t}\varepsilon_t,$$

$$h_t = \frac{\sum_{l=1}^{L} \varphi^l(r_{t-1})\ \left[\alpha_0^l + \sum_{i=1}^{q} \alpha_i^l\, h_{t-i} + \sum_{j=1}^{p} \beta_j^l\, RM_{t-j}\right]}{\sum_{l=1}^{L} \varphi^l(r_{t-1})}$$

4 GENETIC ALGORITHM

Genetic algorithms are a family of techniques draws on the Darwinian theory of evolution to solve optimization problems (Holland, 1984) based on three basic genetic operators: reproduction, crossover and mutation.

Genetic algorithms belong to the family of evolutionary algorithms. They are used to optimize complex problems in order to find an optimal solution. It is an effective approach for nonlinear functions (Zhou, Khotanzad, & Alireza, 2007).

5 EMPIRICAL STUDY

This paper focus on daily financial returns (r_t) of CAC40 obtained by taking the logarithmic difference of the daily price and multiplying it by 100, the resulting stock return: r(t) = (log P(t) –log P(t-1)) *100, where P(t) is the closing price for day t over the period from 01/11/2017 through 09/10/2020 and a sequence of daily realised variance RV obtained by the sum of the N intra-day squared returns at frequency 1 min for the first series and 5 min for the second series. Where the intra-day time subscripted as i = 0, 1, 2…N.

The first series is uniformly sampled at 1-minute scales. This series includes exactly 388,074 observations of 01/11/2017 09: 00 until 09/10/2020 18: 00 or 520 points per day.

The second series is uniformly sampled at 5-minute scales. This series includes exactly 77,657 observations of 01/11/2017 09: 00 until 09/10/2020 18: 00 or 105 points per day.

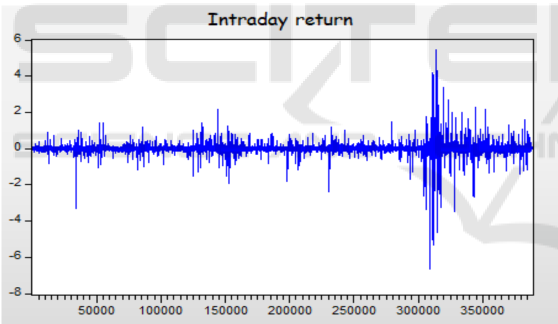


Figure 1: The intraday return at frequency 1 min

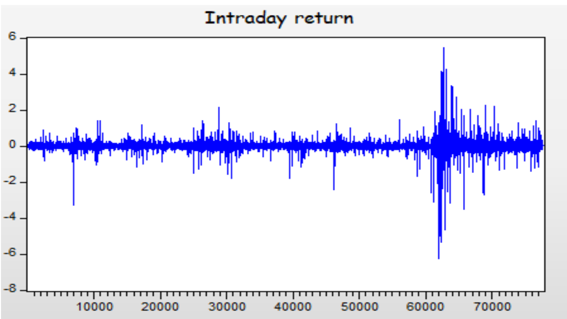


Figure 2: The intraday return at frequency 5 min

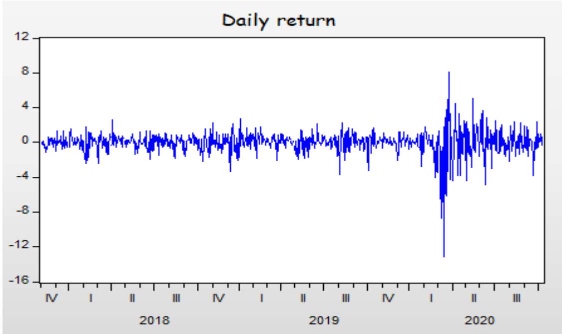


Figure 3: The daily return

The graphics (figures 1, 2, 3) represent the Time series plots of the intraday returns and daily returns. We can notice that the returns appear to be stationary around a constant. The evolution of returns indicates that the series is highly volatile.

Figures 4 and 6 below, presented the realized variance (RV) of high-frequency logarithmic returns.

We observe the clustering of volatilities of the realized variance obtained by the sum of the N intra-day squared returns.

The correlograms (figures 5, 7) show the presence of a significant correlation between the RV_t and RV_{t-1}.

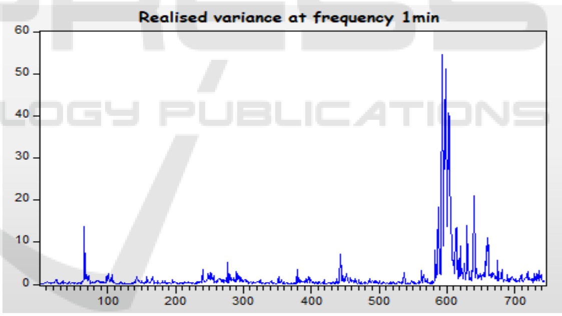
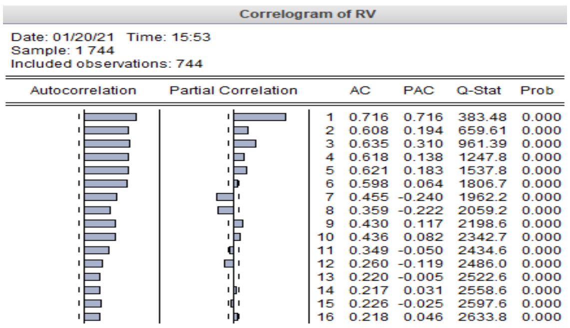


Figure 4: Realized volatility (RV) at frequency 1 min.


Correlogram of RV

Date: 01/20/21 Time: 15:53
Sample: 1 744
Included observations: 744

	AC	PAC	Q-Stat	Prob
1	0.716	0.716	383.48	0.000
2	0.608	0.194	659.61	0.000
3	0.635	0.310	961.39	0.000
4	0.618	0.138	1247.8	0.000
5	0.621	0.183	1537.8	0.000
6	0.598	0.064	1806.7	0.000
7	0.455	-0.240	1962.2	0.000
8	0.359	-0.222	2059.2	0.000
9	0.430	0.117	2198.6	0.000
10	0.436	0.082	2342.7	0.000
11	0.349	-0.050	2434.6	0.000
12	0.260	-0.119	2486.0	0.000
13	0.220	-0.005	2522.6	0.000
14	0.217	0.031	2558.6	0.000
15	0.226	-0.025	2597.6	0.000
16	0.218	0.046	2633.8	0.000

Figure 5: correlogram of RV at frequency 1min.

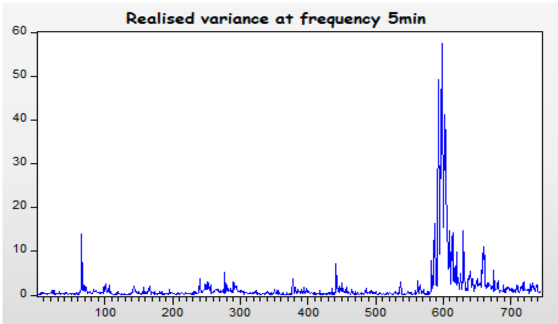


Figure 6: Realized volatility (RV) at frequency 5 min

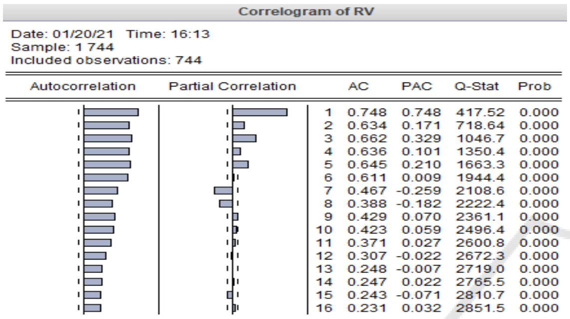


Figure 7: correlogram of RV at frequency 5 min

The data sample was subdivided into two sets, the first set contains 700 observations for the training model, and the second set contains 44 observations for the testing model.
Using the K-means algorithm, the number of rules was specified using the "gap" method for estimating the optimal number of clusters.

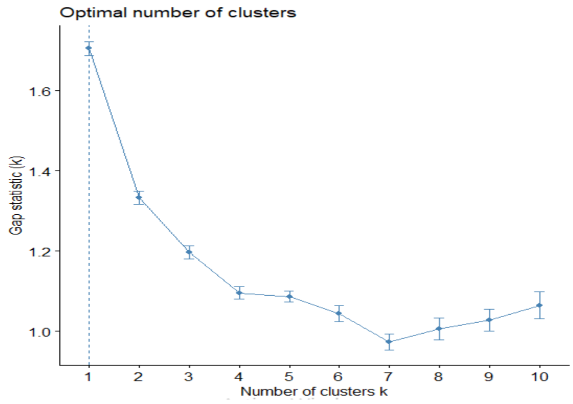


Figure 8: Gap curve

Our system has three inference rules, and the membership function chosen is the Gaussian function.

$Rule^{(l)}$: IF r_{t-1} is φ_l THEN
$h_{lt} = \alpha_0^l + \alpha_1^l\, h_{t-1} + \beta_1^l\, RV_{t-1}$, l = 1, 2, 3.

The parameters are significant at a 0.05 significance level.

In order to evaluate the performance of models in forecasting returns, a loss function is considered: Root mean square error (RMSE): $RMSE = \sqrt{\left(\frac{1}{n}\sum_{i=1}^{n}(r_{i\,obs} - r_{i\,Pred})^2\right)}$

Table1: Parameters estimation of Fuzzy HEAVY-r using GA.

Parameters	$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$	$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{pmatrix}$	$\begin{pmatrix} \alpha_0^1 \\ \alpha_0^2 \\ \alpha_0^3 \end{pmatrix}$	$\begin{pmatrix} \alpha_1^1 \\ \alpha_1^2 \\ \alpha_1^3 \end{pmatrix}$	$\begin{pmatrix} \beta_1^1 \\ \beta_1^2 \\ \beta_1^3 \end{pmatrix}$
Fuzzy HEAVY-r (RV at frequency 1min)	$\begin{pmatrix} 0.014 \\ 0.012 \\ 0.007 \end{pmatrix}$	$\begin{pmatrix} 0.032 \\ 0.002 \\ 0.013 \end{pmatrix}$	$\begin{pmatrix} 0.005 \\ -0.003 \\ -0.04 \end{pmatrix}$	$\begin{pmatrix} 0.058 \\ 0.02 \\ 0.024 \end{pmatrix}$	$\begin{pmatrix} 0.043 \\ 0.058 \\ 0.025 \end{pmatrix}$
Fuzzy HEAVY-r (RV at frequency 5 min)	$\begin{pmatrix} 0.024 \\ 0.030 \\ -0.046 \end{pmatrix}$	$\begin{pmatrix} 0.004 \\ 0.02 \\ 0.05 \end{pmatrix}$	$\begin{pmatrix} 0.04 \\ -0.05 \\ -0.008 \end{pmatrix}$	$\begin{pmatrix} 0.019 \\ 0.018 \\ 0.057 \end{pmatrix}$	$\begin{pmatrix} 0.022 \\ 0.035 \\ 0.051 \end{pmatrix}$
Fuzzy GARCH(1,1)	$\begin{pmatrix} -0.027 \\ -0.001 \\ 0.038 \end{pmatrix}$	$\begin{pmatrix} 0.015 \\ 0.023 \\ 0.043 \end{pmatrix}$	$\begin{pmatrix} 0.043 \\ 0.015 \\ 0.017 \end{pmatrix}$	$\begin{pmatrix} 0.041 \\ 0.047 \\ 0.044 \end{pmatrix}$	$\begin{pmatrix} 0.021 \\ 0.016 \\ 0.051 \end{pmatrix}$

Table2: The results of forecasting returns in out-of-sample evaluation.

Loss function	Fuzzy HEAVY-r (at frequency 1min)	Fuzzy HEAVY-r (at frequency 5min)	Fuzzy GARCH(1,1)	GARCH(1,1)
RMSE	1.33	1.32	1.44	1.34

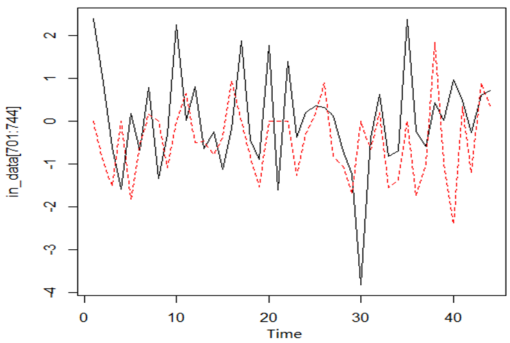


Figure 9: Predicted return by the Fuzzy HEAVY-r model (RV at frequency 1min)

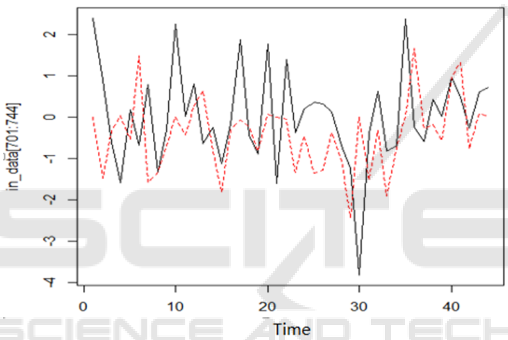


Figure 10: Predicted return by the Fuzzy HEAVY-r model (RV at frequency 5min)

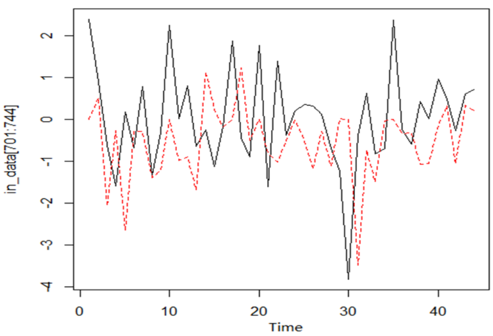


Figure 11: Predicted return by the FUZZY GARCH model.

6 CONCLUSION

We found that the correlation and the clustering of volatilities observed empirically better captured by the Fuzzy HEAVY-r model.

The result shows that the Fuzzy HEAVY-r models outperform the Fuzzy GARCH model and GARCH model in out-of-sample evaluation according to RMSE.

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