Decomposition of Classification Context as a Tool for Big Data Management

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Abstract: The paper considers a problem of generating all classification Good Maximally Redundant Tests (GMRTs) as the set of all maximal elements of the formal concept lattice generated over a classification context. The number of concepts is exponential in the size of input context and decomposing contexts is one of the methods to decrease the computational complexity of inferring GMRTs. Three kinds of sub-contexts are defined: attributive, object and object-attributive ones. The rules of reducing sub-contexts are given. The properties of the sub-contexts are analysed related to the fact that the set of all GMRTs in a classification context is a completely separating system. Some strategies are considered for choosing sub-contexts based on the definition of essential objects and attribute values. The rules of the decomposition proposed imply constructing some incremental procedures to construct GMRTs. Two methods of pre-processing the formal contexts greatly decreasing the computational complexity of inferring GMRTs are proposed: finding the number of subtasks to be solved (the number of essential values) and the initial content of the set of GMRTs. Some unsolved problems difficult for analytical investigations have been formulated. The decomposition proposed can be fruitful in processing big data based on machine learning algorithm.

1 INTRODUCTION

The paper considers a symbolic machine learning problem of generating all classification Good Maximally Redundant Tests (GMRTs) as the set of all maximal elements of the formal concept lattice generated over a classification context. GMRTs provides a basis for mining logical rules from data. The number of concepts is exponential in the size of input context and decomposing contexts is one of the methods to decrease the computational complexity of inferring GMRTs.

Unfortunately, not enough attention has been paid to the methods of formal context decomposition due to its analytical difficulty and, at least in part, by the consideration that having good algorithm for lattice construction is more important than decomposing formal contexts in sub-contexts.

Our attention has been attracted to the following methods of decomposing formal contexts described in literature. The first one has been developed by Ch. Mongush and V. Bykova, 2019. In this method, some fragments of the initial context are partitioned into the so-called boxes. The division of context into boxes is "safety", i. e. the formal concepts are not lost and new formal concepts do not arise during the decomposition. It is proved that the number of boxes arising at each iteration of the decomposition is equal to the number of unit elements of the 0,1-matrix representing the initial formal context. The number of boxes at each iteration can be reduced by constructing mutually disjoint chains of boxes.

The second method of decomposition has been proposed by T. Qian, L. Wei, J.-J. Qi, 2017. This method is based on sub-contexts, closed relation and pairwise non-inclusion covering on the attribute set. The authors provide the method and algorithm of constructing the concept lattice based on a decomposition theory proposed. They also consider the similar decomposition theory based on the object set. Combining the above two decompositions is used.

In our paper, three kinds of sub-contexts are defined: attributive, object and object-attributive ones. The rules of forming and reducing sub-contexts are given. The properties of the determined sub-

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contexts are analyzed related to considering all the GMRTs for a given formal context as a completely separating system of the subsets of a finite set (Dickson, 1969). The definition of GMRTs is given via two interrelated Sperner systems (Sperner, 1928): the family of test's intents and the family of test's extents. Some strategies are considered for choosing sub-contexts in inferring GMRTs based on the definition of essential object and value of attribute. We formulate, in conclusion, some very important but not yet investigated problems connected with the formal context decomposition considered in this paper.

The paper is organized as follows: Section 2 describes GMRT as a formal concept, Section 3 gives the rules of decomposing classification contexts, Section 4 gives the rules for reducing classification contexts, Section 5 represents some strategies for decomposing contexts. We complete with a short conclusion.

2 DEFINITION OF GMRT

A classification context is a set (G, M, I, AClass), where G is a set of objects, M is a set of attribute values (values, for short), $I = G \times M$ is a binary relation between G and M, and AClass is a set of additional attributes by values of which the given set of objects is partitioned into disjoint classes.

Denote a description of object $g \in G$ by $\delta(g)$, and descriptions of positive and negative objects by $D+ = \{\delta(g) | g \in G+\}$ and $D- = \{\delta(g) | g \in G-\}$, respectively. The Galois connection (Ore, 1944) between the ordered sets $(2^G, \subseteq)$ and $(2^M, \subseteq)$, is defined by the following mappings called derivation operators (Ganter & Wille, 1999): for $A \subseteq G$ and $B \subseteq M$, val(A) $= \bigcap\delta(g), g \in A$, and obj(B) $= \{g | B \subseteq \delta(g), g \in G\}$.

In our approach, there are two closure operators: generalization_of(B) = val(obj(B)) and generalization_of(A) = obj(val(A)). A is closed if A = obj(val(A)) and B is closed if B = val(obj(B)). If (val(A) = B) & (obj(B) = A), then a pair (A, B) is called a formal concept (Ganter & Wille, 1999), subsets A and B of which are called concept extent and intent, respectively. According to the values of a goal attribute K from AClass, we get some possible forms of the formal contexts: $K\epsilon := (G\epsilon, M, I\epsilon)$ and $I\epsilon := I \cap (G\epsilon \times M)$, where $\epsilon \in$ rng(K), rng(K) = {+, −}. A classification context K± (G±, K, G± × K) is formed after adding the classification attribute. A context K± is illustrated by Table 1.

Definition 1. A diagnostic test (DT) for K+ is a pair (A, B) such that $B \subseteq M$, A = obj(B) $\neq \emptyset$, $A \subseteq G+$, and obj(B)$\cap$G− = $\emptyset$.

Table 1: An example of classification context

No	Height	Color of hair	Color of Eyes	K1	K2
1	Small	Blond	Blue	+	+
2	Small	Brown	Blue	−	+
3	Tall	Brown	Hazel	−	+
4	Tall	Blond	Hazel	−	−
5	Tall	Brown	Blue	−	−
6	Small	Blond	Hazel	−	−
7	Tall	Red	Blue	+	−
8	Tall	Blond	Blue	+	−

Definition 2. A diagnostic test (A, B) for K+ is said to be maximally redundant if obj(B $\cup$ m) $\subset$ A for all $m \in M \setminus B$.

Definition 3. A diagnostic test (A, B) for K+ is said to be good iff any extension A1 = A $\cup$ i, $i \in G+ \setminus A$, implies that (A1, val(A1)) is not a DT for K+.

A maximally redundant DT which is simultaneously good is called a good maximally redundant DT. Any object description $\delta(g)$ is a maximally redundant collection of values because for any value $m \notin \delta(g)$, $m \in M$, obj($\delta(g) \cup m$) = $\emptyset$.

Definitions of tests (as well as other definitions), associated with K+, are applicable to K−. In general, a set B is not closed for DT (A, B), consequently, DT is not necessarily a formal concept. A GMRT can be regarded as a special type of formal concept [Naidenova, 2012].

An example in Table 1: ({1, 8},{Blond, Blue}) is a GMRT for K1 = + (K1+), ({4, 6},{Blond, Hazel}) is a DT for K1 = − (K1−) but not a good one, and ({3, 4, 6}, {Hazel}) is a GMRT for K1−.

2.1 GMRT as a Sperner System

It is clear that the set of intents of all diagnostic tests for K+ (call it 'DT(+)) is the set of all the collections t of values for which the condition obj(t) $\subseteq$ G+ is true. The set DT(+) is the ordered set w. r. t. inclusion relation. This consideration leads to the next definition of good diagnostic test.

Definition 4. A diagnostic test (A, B) for K+ is said to be good iff obj(B) $\subseteq$ G+ and, simultaneously, the condition obj(B) $\subset$ obj(B*) $\subseteq$ G+ is not satisfied for any B*, B* $\subseteq$ M, such that B* $\neq$ B.

This definition means that the family of the extents of all good tests for K+ is a family of maximal elements of DT(+) and it is therefore a Sperner system

(Sperner, 1928). On this basis, we can give the following definition for the GMRTs.

Definition 5. To find all the GMRTs for a given K+ means to construct a family PS of subsets s1, s2, …, sj, …, snp of G+ such that:

1) PS is a Sperner system;

2) each sj is a maximal set in the sense that adding to it any object g such that $g \notin sj$, $g \in G+$ implies that $obj(val(sj \cup g)) \not\subseteq \delta(g)$, $\forall g \in G+$.

3) The set of all GMRTs is determined as follows: {(sj, val(sj)), $sj \in PS$, $\forall j = 1, \dots, np$}, where {val(sj)} is also a Sperner System.

Some algorithms NIAGaRa and DIAGaRa to find all the GMRTs in a classification context are described in (Naidenova, 2006). The Diagnostic Test Machine (DTM) is given in (Naidenova & Shagalov, 2009). The experiment conducted with the publicly available database (Schlimmer, 1987) of 8124 mushrooms showed that the result of the DTM turned out to be 97,5% w.r.t. classification accuracy.

3 RULES OF DECOMPOSING CLASSIFICATION CONTEXT

To transform inferring GMRTs into an incremental process, we introduce three kinds of subtasks for K+ (K−), called subtasks of the first, second and third kind, respectively:

1. Given a positive object g, find all GMRTs (obj(B), B) for K+ such that B is contained in δ(g). In the general case, instead of δ(g) we can consider any subset of values B1, such that $B1 \subseteq M$, $obj(B1) \neq \emptyset$, $B1 \not\subseteq \delta(g)$, $\forall g \in G-$.

2. Given a non-empty set of values $B \subseteq M$ such that (obj(B), B) is not a DT for positive objects, find all GMRTs (obj(B1), B1) such that $B \subset B1$.

3. Given a value $m \in M$ and object $g \in G+$, find all the GMRTs (X, val(X)) such that $X \subseteq obj(m)$, $val(X) \subseteq \delta(g)$.

One can easily see that each subtask of the first, second or third kind is simpler than the initial one, because each object description contains only some subset of values from M and each subset $B \subseteq M$ appears only in a part of the set of objects descriptions.

Accordingly, we define three kinds of sub-contexts of a given classification context called the object, attribute value and attribute value-object (or object-attribute value) projections, respectively. If (G, M, I) is a context and if $N \subseteq G$, and $H \subseteq M$, then $(N, H, I \cap N \times H)$ is called a sub-context of (G, M, I).

Definition 6 (Naidenova & Parkhomenko, 2020). The object projection ψ(K+, g) returns the sub-context (N, δ(g), J), where $N = \{n \in G+ \mid$ n satisfies $(\delta(n) \cap \delta(g)$ is the intent of a test for K+)}, $J = I+ \cap (N \times \delta(g))$.

Definition 7 (Naidenova & Parkhomenko, 2020). The attribute value projection ψ(K+, B) returns the sub-context (N, B, J), where $N = \{n \in G+ \mid$ n satisfies $(B \subseteq \delta(n))\}$, $J = I+ \cap (N \times B)$.

Definition 8. The attribute value-object projection ψ(K+, m, g) is the intersection of two projections: attribute value projection ψ(K+, m) and object projection ψ(K+, g).

In the case of negative objects, symbol + is replaced by symbol − and vice versa.

The decomposition of inferring GMRTs into the subtasks requires the following actions:

1. Select an object, attribute value or a pair of attribute value - object to form a subtask.
2. Form the subtask (projection).
3. Reduce the subtask (projection).
4. Solve the subtask.
5. Reduce the parent classification context when the subtask is over.

4 RULES OF REDUCING CLASSIFICATION CONTEXT

It is essentially that the projection is simply a subset A* of objects defined on a certain restricted subset B* of values.

Let objϵ(m) be a set of positive or negative objects {obj(m) ∩ Gϵ}, where $\epsilon \in rng(K)$. Then for any $B \subseteq M$ $obj\epsilon(B) = \bigcap_{m \in B} obj\epsilon(m)$, where $\epsilon \in rng(K)$.

Let Sgood+ be the partially ordered set of obj+(m), $m \in M$ satisfying the condition that (obj+(m), val(obj+(m))) is a current GMRT (in any algorithm of inferring GMRTs) for K+. Sgood− for K− is defined based on obj− (m).

Essentially, the process of forming Sgood is an incremental procedure of finding all maximal elements of a partially ordered (by the inclusion relation) set. It is based on topological sorting of partially ordered sets. Thus, when the algorithm is over, Sgood contains the extents of all the GMRTs for K+ (for K−) and only them (Naidenova & Parkhomenko, 2020). The operation of inserting an element A∗ into Sgood (in the algorithm formSgood (Naidenova & Parkhomenko, 2020) under the lexicographical ordering of these sets is reduced to lexicographically sorting a sequence of k-element collections of integers. A sequence of n-collections whose components are represented by integers from 1 to |M|, is sorted in time of O(|M| + L), where L is the sum of lengths of all the collections of this sequence (Hopcroft et al., 1975). Consequently, if Lgood is the sum of lengths of all the collections A of Sgood, then the time complexity of inserting an element A∗ into Sgood is of order O(|M| + Lgood). The set Tgood of all the GMRTs is obtained as follows: Tgood = {t | t = (A, val(A)), A ∈ Sgood}

It is useful to introduce the characteristic W(m), m ∈ B* named by the weight of m in the projection: W(m) = ||obj+(m)|| or W(m) = ||obj− (m)|| is the number of positive (negative) examples of the projection containing m. Let WMIN be the minimal permissible value of the weight.

The following reduction rules are determined:

Rule 1. For each value m in the projection, the weight W(m) is determined and if the weight is less than WMIN, then the value m is deleted from the projection.

Rule 2. We can delete the value m if W(m) is equal to WMIN and (obj+(m), val(obj+(m)) is not a test; in this case m will not appear in a GMRT with the weight of its intent equal to or greater than WMIN.

Rule 3. The value m can be deleted from the projection if obj+(m) ⊆ s' for some s' ∈ Sgood+.

Rule 4. If obj(val(obj+(A)) = obj+(A), then the value A is deleted from the projection and obj+(A) is stored in SGOOD+ if obj+(A) corresponds to a GMRT at the current step.

Rule 5. If at least one value has been deleted from the projection, then the following its reduction is necessary. The reduction consists of deleting the elements of projection that do not correspond to tests (as a result of previous eliminating values). If, under reduction, at least one element has been deleted from the projection, then applying Rule 1 – Rule 5 are repeated.

Algorithms for GMRTs inferring based on these rules have been described in (Naidenova, 2006; Naidenova & Parkhomenko, 2020].

Rule 1 is based on the following Theorem 1 (Naidenova, 2006):

THEOREM 1.

Let m ∈ M, (Y, X) be a maximally redundant test for G+ and obj(m) ⊆ obj(X) =Y. Then m does not belong to the intent of any maximally redundant good test for G+ different from X.

Consider an example of reducing a sub-context for K−, where − is the value of K2 in Table 1. The result of the attribute value projection ψ(K−, Tall) is in Table 2. In Table 2, obj−(Blue) = {5, 7, 8}, but obj(Tall, Blue) = {5, 7, 8}, and, consequently, (obj(Tall, Blue),{Tall, Blue}) is a DT for K2 = −. We have also obj−(Brown) = {5} and obj−(Red) = {7}, but both {5} ⊂ {5, 7, 8} and {7} ⊂ {5, 7, 8}, and, consequently, there does not exist any good test which contains simultaneously the values ''Tall'' and ''Brown''. ''Red'' is not a good test for K−. Then one can delete ''Blue'', ''Red'' and ''Brown'' from the sub-context. The result is shown in Table 3. Note, that the descriptions of objects 5 and 7 are included in the description of object 3 for K+ (see Table 1) and these objects are deleted. Objects 4 and 8 form the extent of a test for K− equal to (obj(Tall, Blond),{Tall, Blond}).

Table 2: Attribute-value projection for K2 = − in Table 1.

No	Height	Color of hair	Color of Eyes
4	Tall	Blond	Hazel
5	Tall	Brown	Blue
7	Tall	Red	Blue
8	Tall	Blond	Blue

Table 3: The projection of Table 2 after reducing.

No	Height	Color of hair	Color of eyes
4	Tall	Blond	Hazel
5	Tall		
7	Tall		
8	Tall	Blond	

5 STRATEGIES OF DECOMPOSING

The advantage of a projection-forming operation is to increase the likelihood of finding all the GMRTs (contained in the projection) by only one passing of it. By limiting the number of tests contained in the projection, we increase the probability of their separation, that is, the probability of finding exactly those attributes (values) or objects that will enter only one test in the projection considered. Let's explain this idea.

Any subset t1,…, ti, tj,.. , tk of GMRTs and corresponding to it subset val(t1), val(t2), …, val(ti), val(tj),...., val(tk), where t1,…, ti, tj,.. , tk are intents of GMRTs are two systems of completely separating subsets. It means that for any pair (ti, tj) there is such a pair of values (mq, mf) that mq occurs in ti and does not occur in tj, and mf occurs in tj and is not found in ti. Analogously, for any pair of val(ti), val (tj), there is such a pair of objects (gq, gf), that gq is found in val(ti) and is not found in val(tj), and gf is found in val(tj) and is not found in val(ti).

The phenomenon of a completely separating system of GMRTS can be illustrated in a projection in Table 4 (this example is extracted from a real task).

In this example, the projections t9, t12 of objects 9 and 12 do not contain any intents of tests, we can delete the corresponding lines. The result is in Table 5, where:

Table 4: Example of a projection.

t\A	A 3	A 6	A 7	A *	A 13	A +	A 19	A 20	A 21	A 22
8	1	1	1	1	1	1	1	1	1	1
6	0	0	1	0	0	0	0	1	1	0
4	0	1	1	0	0	1	0	1	1	0
7	1	1	0	0	0	1	0	1	0	1
9	0	0	0	0	0	0	1	1	1	1
11	1	0	1	0	0	0	1	1	1	1
10	1	1	0	1	1	0	0	1	1	0
12	1	0	0	0	0	0	0	1	1	0

Obj+(m7) = {4,6,8,11}, val({4,6,8,11}) = {m7 m20 m21 } corresponds to a test,

Obj+(m22) = {7,8,11}, val({7,8,11}) = {m3 m20 m22} corresponds to a test,

obj(m13) = obj(A*) = {8,10}, valt({8,10}) = {m3 m6 m* m13 m20 m21 } corresponds to a test.

Delete m22, m7, m13, and m* and reducing the projection. After reducing, this subtask is over.

Table 5: The projection after reducing.

t\A	A 3	A 6	A 7	A *	A 13	A +	A 19	A 20	A 21	A 22
8	1	1	1	1	1	1	1	1	1	1
6	0	0	1	0	0	0	0	1	1	0
4	0	1	1	0	0	1	0	1	1	0
7	1	1	0	0	0	1	0	1	0	1
11	1	0	1	0	0	0	1	1	1	1
10	1	1	0	1	1	0	0	1	1	0

All the GMRTs in this projection have been revealed by only one passing.

Before entering into the details of choosing projections when decomposing the classification contexts, we need the following definitions of essential value and essential object.

Definition 9. Let B be a set of values such that (obj(B), B) is a DT for K+ (K−). The value m ∈ B, B ⊆ M is essential in B if (obj(B \ m), (B \ m)) is not a DT for a given set of objects.

Generally, we are interested in finding one of the maximal subsets sbmax(B) ⊂ B such that (obj(B), B) is a DT but (obj(sbmax(B)), sbmax(B)) is not a DT for a given set of positive (negative) objects. Then sbmin(B) = B \ sbmax(B) is one of minimal subsets of essential values in B.

The number of subtasks of the second kind is determined by the number of essential values in M or its subsets. Let the set Lev be equal to sbmin(M).

Proposition 1. Each essential value is included in at least one positive object description.

Proof of Proposition 1. Assume that for an object description δ(g), g ∈ G+, we have δ(g) ∩ Lev = ∅. Then δ(g) ⊆ M \ Lev. But M \ Lev is included in at least one of the negative object descriptions and, consequently, δ(g) also possesses this property. But this contradicts the fact that δ(g) is the description of a positive object.

Corollary 1 (of Proposition 1). If B ⊆ M and B ∩ Lev = ∅, then (obj(B), B) is not a test for K+.

Corollary 2 (of Proposition 1). For finding all the GMRTs contained in K+, it is sufficient to find all the GMRTs only for sub-contexts associated with essential values in Lev for M.

Definition 10. Let A ⊆ G+, assume that (A, val(A)) is not a DT for K+ (K−). The object g, g ∈ A is said to be an essential in A, if (A\g, val(A\g)) proves to be a DT for a given set of positive objects.

Generally, we are interested in finding one of the maximal subsets sbmax(A) ⊂ A such that (A, val(A)) is not a DT but (sbmax(A), val(sbmax(A))) is a DT for K+.

It is clear that if m enters into the intent of a test for K+, then its extent is in obj+(m). It is theoretically possible to find one of the maximal A* subsets of obj+(m), such that (A*, val(A*)) is a DT for K+ (K−). This operation allows to find the initial content of Sgood (Naidenova & Parkhomenko, 2020).

The quasi-minimal subset of essential values in M and quasi-minimal subset of essential objects in obj+(m), for all m∈M can be found by a simple procedure described in (Naidenova & Parkhomenko, 2020). This procedure is of linear computational complexity w.r.t. the cardinality of M.

The process of using the decomposition of formal context based on choosing essential object or value to form the projections consists in the following steps:

Choose an essential value (object) in a projection; forming the corresponding sub-projection;

Find all the GMRTs in the sub-projection (sub-context);

Delete value (object) from the parental context;

Reducing the parental context;

Determine whether the procedure of finding all the GMRTs is over.

Using the third decomposition based on selecting an essential object and an essential value simultaneously is effective when this value enters the quasi-minimal set of essential values in the description of this selected object and this essential object enters the quasi-minimal set of essential objects for this selected value.

If the essential value is the only one w.r.t. the selected object, then one can remove this object from consideration after the subtask is resolved. Similarly, if the essential object is the only one related to selected value, then one can remove this value from consideration after the subtask is over. These deletions result in a very effective reduction in the formal context considered.

Another advantage of selecting essential values and objects simultaneously is the fact that this way greatly supports the property of the complete separating the families of extents and intents of GMRTs.

It is important to formulate some unsolved and nontrivial problems related to the decomposition considered in this paper. These problems are:

How to recognize a situation that current formal classification context contains only the GMRTs already obtained (current context does not contain any new GMRTs)?

How to evaluate the number of recurrences necessary to resolve a subtask in inferring GMRTs? (if we use a recursive algorithm like DIAGARA)?

How to evaluate the perspective of a selected sub-context with respect to finding any new GMRT?

These problems are interconnected and the subject of our further research. The effectiveness of the decomposition depends on the properties of the initial classification context (initial data). Now we can propose some characteristics of data (contexts and sub-contexts) useful for choosing a projection: the number of objects, the number of attribute values, the number of the GMRTs already obtained and covered by this projection. It may be expedient to select essential object with the smallest number of entering elements of Sgood and, simultaneously, with the largest number of entering obj+(m), $m \in M$.

Our experiments show that the number of subtasks to be solved always proved to be smaller than the number of essential values.

6 CONCLUSION

In this paper, we considered one of the possible methods for decomposing classification contexts to find all GMRTs in them. We gave the definitions of three type of decomposing and, accordingly, three type of context projections and subtasks of inferring GMRTs. We revealed the role of finding essential attribute values and objects for choosing and resolving subtasks. Some ways to select the projections were given in this paper. Two methods of preprocessing the formal contexts (sub-contexts) greatly decreasing the computational complexity of inferring GMRTS are proposed: finding the number of subtasks to be solved (the number of essential values) and the initial content of the set Sgood. Some unsolved problems difficult for analytical investigations have been formulated. Currently, experimental studies of the decompositions' computational effectiveness on various data sets are conducted.

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