Lattice Boltzmann Simulation of the Three-dimensional Natural Convection with a Regularly Heated Cavity Slab

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Abstract: In this paper, the Lattice Boltzmann method (LBM) associated with the Multi-Relaxation Times (MRT) is performed to investigate the three-dimensional (3D) natural convection of the air in a cubical cavity for various values of the Rayleigh number ranged between 10^3 <Ra<10^5. The main 3D-LBM-MRT code was compared and validated with the experimental and numerical work for different cases. The main objective of this work is to analyze the heat transfer between the two cold vertical walls and the hot cavity base by using the D3Q7 model for thermal field, and the evolution of the velocity field employing the D3Q19 model. Furthermore, the effects of the conjugate heat transfer also take into consideration to investigate the heat transfer rate by using different values of the thermal conductivity.

1 INTRODUCTION

The natural convection in a cubical cavity within a heat generation source is commonly used to investigate the fluid flow and heat transfer enhancement for the various fields. This prototype system is considered one of the main subjects of many researchers because of its privileges in many industrial and engineering applications related to the thermal insulation of buildings, cooling the electronic chips, etc. (Admi, 2020; Li, 2019). The study of natural convection heat transfer using a three-dimensional for complex configuration presents a great challenge to understand the mechanism and the behavior of the flow of fluid and heat transfer from numerical simulations related to real physical problems. Time and cost for experimental studies are some of the main problems which oriented most researchers to find alternative solutions to reduce the time and lack of the experimental equipment. The numerical simulation enables to solve concrete problems masterly and efficiently, where it offers valuable solutions by providing a clear view of complex systems.

In this context, the lattice Boltzmann method (LBM), as a numerical simulation, allows to describing the fluid behavior and the heat transfer using different models. Moussaoui et al. (Moussaoui, 2019) analyzed the effect of the Rayleigh number of two-dimensional heated obstacle inside a square enclosure by using the double MRT-LBM, they conclude that the increase of obstacle dimension enhances the heat transfer by increasing the surface exchange. The recirculation zone loses its symmetric when the position of the heated obstacle varied, and the isotherm field also disturbs because of this variation in the local heating. Chavez-Modena et al. (Chávez-Modena, 2020) realized the under-resolved turbulent flow simulation using the D3Q19 model with multi-relaxation time with the central moment. They interpret that the model used to obtain a satisfactory result compared with the results BGK and MRT-CM model for the under-resolved systems. Wang et al. (Wang, 2017) performed a numerical investigation of a three-dimensional heated cubical cavity for a high Rayleigh number. This study focuses on the evolution of the heat transfer and fluid behavior inside the differential heated cavity, where

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the obtained numerical results conclude that the thermal and velocity contour formed close to the adiabatic walls, and the thickness of the contour layers narrowness near these walls for high values of Rayleigh number. Liu et al. (Liu, 2019) also investigate 3D convection heat transfer in porous media at the REV scale. They found that the model employed for studying the porous channel is suitable to investigate the heat transfer and the fluid flow.

The present work was performed to analyze the heated cavity slab with different values of thermal conductivity which is related to the nature and thickness of the heating source.

2 NUMERICAL STATEMENT DETAILS

2.1 Configuration System

The configuration system studied is a cubical cavity consist of two vertical cold walls and a heated floor as shown in Figure 1. The cavity base undergoes a dimensionless temperature T_h=1 according to different values of the thermal conductivity (*Ks*). While the cold surfaces kept fixed cold dimensionless temperatures *Tc*=0. The front, rear, and top cavity surfaces are insulated. The thickness of the cavity base characterized by a different thermal conductivity (*ks*) compared to the thermal conductivity of the airflow (*kf*). The continuity of the temperature and heat flux is taken into account for the interface solid/fluid (See Figure 1).

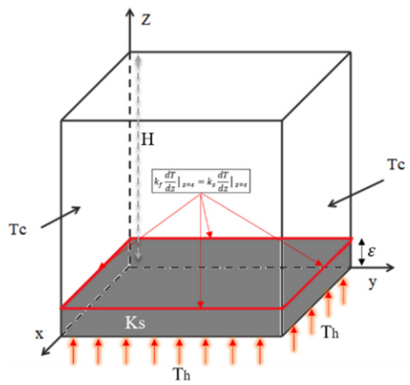

Figure 1: 3D Geometry system of a cubical cavity.

2.2 Discrete Numerical Model

In this study, The Natural convection of the air in a cubical cavity was performed using the three-dimensional lattice Boltzmann method including the D3Q19 and D3Q7 model to define the velocity and the thermal field (Liu, 2019; Wang, 2017), respectively as illustrate in Figure 2.

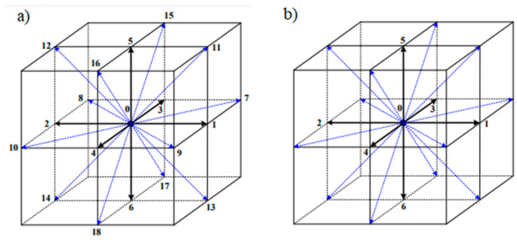

Figure 2: a) Discrete Velocities for D3Q19 model b) discrete temperature for D3Q7 model.

The above figure describes the nineteen discrete speeds for the D3Q19 model $\langle e_i | i = 0, \ldots, 18\rangle$ such as the speed coordinate was proposed as below:

$$e_i = c\begin{bmatrix} 0\,1\,1\,0\,0\,0\,0\,1\,1\,1\,1\,1\,1\,1\,1\,0\,0\,0\,0 \\ 0\,0\,0\,1\,1\,0\,0\,1\,1\,1\,1\,0\,0\,0\,0\,1\,1\,1\,1 \\ 0\,0\,0\,0\,0\,1\,1\,0\,0\,0\,0\,1\,1\,1\,1\,1\,1\,1\,1 \end{bmatrix} \quad (1)$$

The quantity **c** considers the lattice speed related to the speed of sound where:

$$c^2 = 3 \times c_s^2 \quad (2)$$

Furthermore, the seven discrete speed was proposed for this investigation for the D3Q7 scheme where the temperature direction is correspondent to seven speed direction of the D3Q19 scheme (Liu, 2019).

2.3 Numerical Approach

Recently, the lattice Boltzmann method associated with multi-relaxation times (LBM-MRT) consider one of the powerful numerical estimation to describe the physics phenomena, the precision, and the simplicity of the implementation in the code allows to define of most physics and engineering problem for various complex geometries. This approach makes it possible to found satisfactory results with a reduced computation time and for weak grids compared with the other numerical approaches.

The lattice Boltzmann equation (LBE) defines two main parts: the propagation and the collision process of the fluid popularity, such that the study of complete fluid is done in such a way that the fluid discretized into a finite set of particles, namely the distribution function or the probability of finding a group of particles in a position $\boldsymbol{x_i}$ with velocity $\mathbf{e_i}$ at time $\boldsymbol{t}$ (Lahmer, 2019; Admi, 2020; Li, 2019; Moussaoui, 2017).

2.3.1 Lattice Boltzmann Equation for the Flow Field

The LBM-MRT equation related to the D3Q19 model for the velocity field is expressed as follows:

$$f(x+ei\times\Delta t,t)-f(x,t)=M^{-1}\times S\times\left(m(x,t)-m^{eq}(x,t)\right)+F \quad (3)$$

Where M and S denote the 19×19 transformation matrix and the relaxation matrix, respectively, which is related to the moment $\boldsymbol{m}$ and the equilibrium moment m^{eq}. The quantity F is the body force related to the gravity aspect.

The numerical resolution of this equation makes it possible to describe the fluid flow behavior from the macroscopic quantities (Lahmer, 2019).

2.3.2 Lattice Boltzmann Equation for the Thermal Field

The thermal LBE-MRT equation is adopted to simulate the temperature field using the D3Q7 model as shown in the following expression:

$$g(x+ei\times\Delta t,t)-g(x,t)=N^{-1}\times Q\times\left(n(x,t)-n^{eq}(x,t)\right) \quad (4)$$

The quantities N and Q define the 7×7 transformation matrix and the relaxation matrix, respectively. During the numerical computation, the data obtained from the thermal Boltzmann equation allow modelling the temperature field at each point of the computation domain. This process was performed by the transformation of obtained data to macroscopic quantities which is the temperature.

All the parameters used for this study have been presented in detail in the reference (Liu, 2019; Wang, 2017).

2.4 Boundary Conditions

2.4.1 Velocity Boundary Condition

The boundary conditions used in the numerical calculation procedure correspond to the Boltzmann equation considered as conditions easy to implement in the calculation code.

Regarding the fluid flow, the "bounce-back" scheme was employed to describe the cavity walls as shown in the expression below:

$$f_i(x,y,z)=f_{\tilde{\imath}}(x,y,z) \;\; i=0,\ldots,18 \quad (5)$$

Where $\boldsymbol{i}$ denotes the direction of the discrete velocity, and $\tilde{\imath}$ defines the opposite speed direction. The positions x, y, and z describe the position of each cubical cavity walls.

2.4.2 Thermal Boundary Condition

For the thermal problem, the boundary conditions adopted to define the cold and hot walls which are expressed by the equation related to the macroscopic quantity:

$$T=\sum_{i=0}^{6} g_i \quad (6)$$

From the equation above, the boundary conditions can be expressed for the hot bottom wall and the cold vertical walls by the following expressions:

$$\begin{cases} z=0 \rightarrow g_5(x,y)=T_{hot}-\sum_{\substack{i=0\\ i\neq 5}}^{6} g_i \\ y=0 \rightarrow g_1(x,z)=T_{cold}-\sum_{\substack{i=0\\ i\neq 1}}^{6} g_i \\ \qquad\quad g_2(x,z)=T_{cold}-\sum_{\substack{i=0\\ i\neq 2}}^{6} g_i \end{cases} \quad (7)$$

Concerning the adiabatic walls, Neumann Boundary Condition was executed by the following expressions:

$$\begin{cases} z=H \rightarrow g_6(x,y)=g_5(x,y) \\ x=0 \rightarrow g_4(y,z)=g_3(y,z) \\ x=H \rightarrow g_3(y,z)=g_4(y,z) \end{cases} \quad (8)$$

3 VALIDATION AND NUMERICAL RESULTS

To cognize the consistency and accuracy of our simulation code, we executed Intensive studies in order to know that the current code used is more suitable for studying the fluid flow and convective heat transfer in various geometries.

3.1 Validation Code for Double MRT-LBM

In this context, the developed code is compared to the literature results (Corvaro, 2008). The first comparison was made with the experimental results of the isotherm and velocity contour obtained by Corvaro and Paroncini. The results showed a good agreement with the reference (Corvaro, 2008) for Rayleigh number equal Ra=2.02×10^5 and δ=0.5. (See Figures 3 and 4). The interest of studying the Ra number is to characterize the heat transfer within a fluid during its flow.

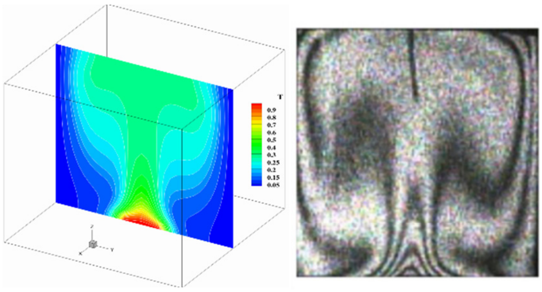

Figure 3: Comparison between the 3D and 2D projection of LBM-MRT simulation result, and the experimental measurement using the double-exposure interferogram (Corvaro, 2008) For Ra=2.02×10^5, δ=0.5.

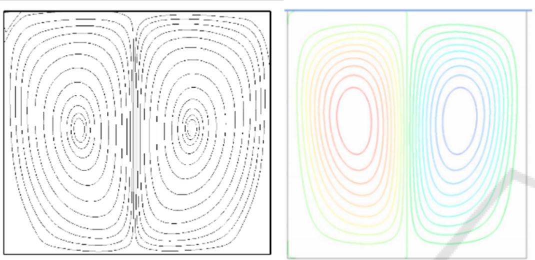

Figure 4: Comparison study between the 3D projection of LBM-MRT simulation results, and the numerical simulation of the streamline For Ra=2×10^5, δ=0.5.

To reinforce the reliability of our simulation code, the Nusselt number is carried out for different values of the Rayleigh number where δ=0.5. Note that the role of the Nusselt number is to characterize the nature of thermal transfer involved between the hot cavity base and the two vertical cold walls as shown in the expression below:

$$Nu = L_c \times \frac{dT}{dz}\Big|_{z=0} \tag{9}$$

Where Lc denotes the characteristic length correspond to the heated part of the cubical cavity.
The experimental and numerical results obtained by the literature (Corvaro, 2008) are converged towards the results obtained by the numerical simulation carried out by the double MRT-LBM as illustrate in Table 1.

Table 1: Comparison of the average Nusselt number between the experimental (Corvaro,2008) and numerical results for δ=0.5.

Ra	*Nu (exp)*	*Nu (num)*	|Δ|*[%]*
1.71E+05	6.3	6.306	0.095
1.98E+05	6.45	6.488	0.59
2.32E+05	6.65	6.697	0.70
2.50E+05	6.81	6.8	0.15

Furthermore, the maximum deviation of each average Nusselt number results under 1%. Therefore, the code used in the current study able to make numerical simulations that correspond to real physical problems of the fluid flow and thermal transfer.

3.2 Results and Discussion

Isotherm contours, velocity fields, and average Nusselt number for different values of the thermal conductivity ratio ($Kr=K_s/Kf$) and Rayleigh number are investigated in this section. The objective proposed in this study is the analysis of the heat transfer evolution by the air cooling of the electronic equipment inside a cubical box.

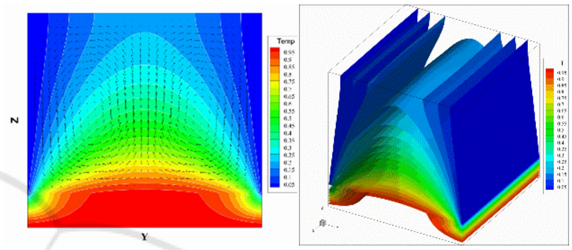

Figure 5: 2D projection for isotherms, velocity field and the 3D isosurfaces temperature for Ra=3.10^3, Kr=10.

Figures 5 and 6 present the evolution of the fluid flow and heat transfer by natural convection for two different values of Rayleigh number with fixed thermal conductivity ratio. The results show that the energy transport increase promptly upwards of the cavity such that the fluid flows symmetrically relative to the cavity mid-length. Two recirculation zone appear clockwise and anticlockwise. Furthermore, the isotherms field evolves symmetrically where the heat transfer enhances in the left and right of the cold cavity walls. This explains that the fluid flow and convective heat transfer behavior refer to the thermal exchange between the cold walls and the heated slab, where the magnitude of heat transfer more enhanced at the two corners of the cavity slab.

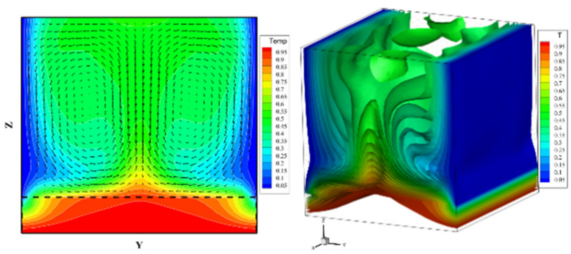

Figure 6: 2D projection of isotherm and velocity pattern, and 3D isosurface of the thermal field for Ra=5.10^5, Kr=10.

The evaluation of the thermal transfer and fluid flow was also investigated for two various values of thermal conductivity ratio (*Kr*=10 and *Kr*=6600) with fixed *Ra*. As shown in Figures 6 and 7, the slab became almost isothermal as the thermal conductivity ratio increases. In addition, the energy transport of the heated air near the slab propagates rapidly upward of the cavity. The presence of the cold walls in the vicinity of the heated base provokes an efficient cooling compared with the center of the heated slab.

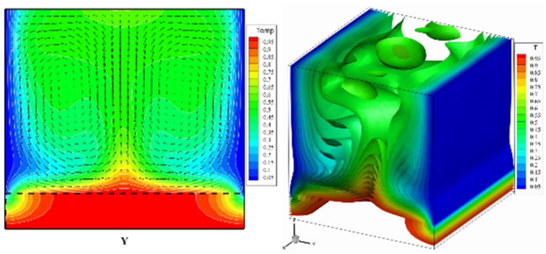

Figure 7: 2D projection of isotherm and velocity pattern, and 3D isosurface of thermal field for Ra=5.10^5, Kr=6600.

To understand substantially the mechanisms of heat transfer involved in this work, the local Nusselt number evolution was realized for several values of Ra and Kr for the heated cavity base (see Figure 8). The thermal exchange increases as Ra increases from 3.10^3 to 5.10^5. While the heat exchange reduced from 5.10^5 to 6.10^5. This diminishes of the Nu refers to the fact that the fluid receives more heat through convective heat transfer, and the buoyancy forces increase, and in turn, thermal boundary layer thickness near the hot wall decreases. In order to enhance the heat transfer in this case, for a high thermal conductivity ratio, the heat exchange improves for $Ra=5\times10^5$, Kr=6600.

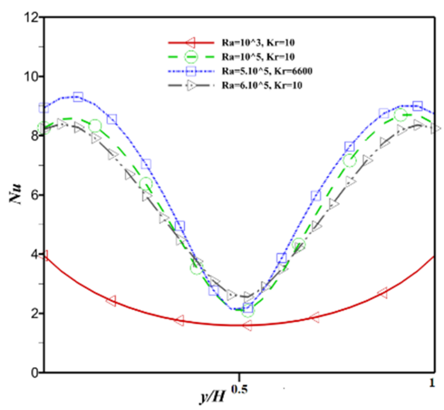

Figure 8: Local Nusselt number evolution.

4 CONCLUSION

In this work, the examination of the three-dimensional natural convection heat transfer with a regularly heats cubical cavity slab is illustrated. The results obtained using the double MRT-LBM method had a good agreement with the experimental investigation which gives credibility to our simulation code. To achieve the perfect cooling of electronic components whatever its nature in a cubical enclosure, one can reconsider that the effects of introducing a large Rayleigh number provoke a decrease in heat exchange if it exceeds the critical point which is equal to 6.10^5. On the other hand, the increase in thermal conductivity considers a strong point to reinforce the heat exchange for the case where the high Rayleigh number.

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