Reflexive Change of the Manager's Objective Function

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Keywords: Game theory, reflexive logic, economic agents, an objective function.

Abstract: Game-theoretic models of the theory of reflexive games and logical-reflexive models of algebraic methods for modelling reflexive processes are investigated. To solve the problem of changing the objective functions of economic agents in different cultural environments, the structural method of searching for the correspondence of mathematical models to the six levels of reflexivity of a manager known in the theory of the school's activity of general methodology was applied. It is shown that the change in the agent's objective function, which is not available in game-theoretic models, is possible when an economic agent gains the level of problem addressing in models of reflexive logic, as well as at higher levels of reflexivity. Game-theoretic models are a special case of logical-reflexive models and do not allow phenomenologically reliable modelling of the reflexive processes of agents. The use of logical-reflexive models gives a broader scope of research and more accurate tools for modelling reflexive processes.

1 INTRODUCTION

In game-theoretic modelling of the behavior of institutional agents, researchers traditionally proceed from the invariability of the objective functions of agents. It is assumed that the objective functions specified in the models a priori reflect the interests of these agents. These interests are considered constant and immutable. When interests change, a new model is built with a new objective function. Empirical experience often gives a different pattern: interests can change having made a decision. This is due to the fact that the real agent has an idea of all possible benefits deriving from the decision. And in the presence of certain opportunities, an agent can switch from one goal to another. From the point of view of modelling, such an objective function of the agent is multi-criteria and multifactorial. In this case, individual criteria and factors are elastic only to certain external conditions, and in the absence of these conditions, the agent is not subjected to them.

Modelling innovative development in different cultural environments this principle is even more essential. The types of culture are shown in Table. 1.

Table 1: The structure of social values in the main cultures of the world. Types of culture.

Principles of social self-identification	Equality	Inequality
Individualism	Freedom: Western Culture Alienating culture	Justice: Middle Eastern and Indian Cultures Isolating culture
Collectivism	Trust, solidarity: Eastern European (Eurasian), Latin American, African, Southeast Asian cultures Equalizing culture	Virtue (Mercy and Respect): East Asian Culture Consolidating culture

Source: Savelyev, 2015.

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Savelyev, M., Savchenko, A., Koretsky, V. and Koshcheeva, A.
Reflexive Change of the Manager's Objective Function.
DOI: 10.5220/0010693900003169
In *Proceedings of the International Scientific-Practical Conference "Ensuring the Stability and Security of Socio-Economic Systems: Overcoming the Threats of the Crisis Space" (SES 2021)*, pages 104-110
ISBN: 978-989-758-546-3

In the study of the objective attitudes of managers, the needs of a manager invariant in the studied cultural environments were revealed as power and career. But considering the constituent elements of this objective attitude, significant differences were revealed depending on the cultural environment. Researchers among the interests of managers identified income, social guarantees, psychic income (public reputation), patronage, the results of the institution's activities, the freedom to make changes and the freedom to manage the institution. When considering the types of cultural environments, the interests of managers dominating in these environments were revealed (see Table. 2).

Table 2: The structure of the manager's motives in different cultural environments and the leading elements of objective function.

Principles of social self-determination	Equality	Inequality
Individualism	Residual income	The likelihood of meeting the interests of allies (the likelihood of staying in power)
Collectivism	The likelihood of growth in the size of the organization by employment / investment / sales	The likelihood of meeting the interests of allies * the likelihood of growth

Source: Savelyev, 2020.

The objective function of a manager in different cultures is more elastic to peculiar to the corresponding culture variables (Savelyev, 2020):

$$MIB = FQ \times P_{inc} \times P_m, (1)$$

where: *MIB* is the future income of the manager; *FQ* - residual income, defined as the remainder of the organization's income without income of employees and investors; P_{inc} - the likelihood of growth in the size of the organization in terms of employment, investment or sales; P_m - the probability of satisfying the interests of allies (the probability of staying in power).

It is assumed that the favorable change of manager will first react the dominant element of the objective function, and adverse changes - the most distant from the dominant interests in a particular cultural environment element of the objective function. For example, in an isolating culture for favorable events, the manager will first of all share the additional income with allies, and in case of an unfavorable change, will reduce the size of the organization in parameters that are not in the interests of allies. In an equalizing culture, if the situation is favorable, the organization will grow to the maximum possible size, and if the situation is unfavorable, it will maintain the size of the organization even if the income of allies is reduced.

This behavior can be modelled only if ones formally describe the agent's ideas about the future before the decision is made and the logical operators for choosing the objective parameter of the objective function. And this tool is absent in traditional game-theoretic models but is being actively developed in logical-reflexive models.

In general, the combined application of the methods of game theory and reflexive logic brings closer the resolution of the epistemological gap between empirical studies of the influence of culture on the activities of economic entities and the activities of territorial authorities and the institutional description of their activities, if causally predetermined supra-constitutional rules are included in the modelled institutional systems.

These systems make it possible to design institutional provisions for the management of economic entities and territories in the conditions of completed industrialization and the transition to an innovative economy in accordance with the specifics of the culture of society, and, thereby, lay the foundation for the fundamental provision of the ideologeme of the "multipolar world" and a reasonable refusal from institutional unification under the implementation of the global project "unipolar world". For the mathematical modelling of institutional systems and the process of designing institutional transformations, it is necessary to develop a theory of reflexive processes, involving the combined application of the methods of game theory and reflexive logic.

Algebraic methods for modelling reflexive processes were developed by V. Lefebvre (Lefebvre, 1965, 1966, 1967, 1973, 1991, 1997, 2009; Lefebvre, 1982, 1992, 1998, 2008).

As noted in one of this works (Lefebvre, 2009), a significant contribution to the theoretical understanding of reflection was made by T. Taran, who built a Boolean valued model of the choice of social norms (Taran, 1998, 2001); V. Krylov, who studied the problems of axiomatics of reflexive models (Krylov, 2000); Yu. Schrader, who considered continuous-valued logics as languages of

reflection and proposed formal models of reflexive structures and (Schrader, 1999). The contribution of T. Taran Ramming to the theory of reflexive processes can, with no exaggeration, be called the second most important after the results of V. Lefebvre. Her research and research have significantly expanded the capabilities of reflexive models.

The theory of reflexive games as a new class of game-theoretic models describing the interaction of subjects (agents) making decisions based on the hierarchy of ideas about parameters and ideas about representations was built in the works of D. Novikova, A. Chkhartishvili, M. Gubko (Novikov, 2003, 2004, 2013; Chkhartishvili, 2004; Gubko, 2002).

Reflexive logic, despite the active calls of V. Lefebvre, is not used in economic and mathematical methods. In our opinion, this situation has several reasons.

First, it will call into question the paradigm of the mainstream of Western economics, in particular the assumption of the rationality of agent behavior and of methodological individualism. In the case of applying reflexive logic, rationality will be broken up into many logics of different cultures, and the rejection of methodological individualism will cause the “specter of communism” and the assumption of the existence of many ideal economic systems corresponding to different cultures of the world. In the mainstream, these innovations are ideologically taboo, because the mainstream is used as an ideological means in hybrid warfare, so is the means of determining the rules of international economic policy.

Secondly, the basis for the development of the theory of reflexive games was the hierarchical representation of the players' information reflection. Such a representation set the information structure as a kind of infinite tree, which makes the algorithmic problem of optimizing the choice of strategy in a reflexive game practically impracticable. In this regard, the creation of a universal algebraic apparatus that corresponds to the processes of reflection actually presented in different cultures and allows one to model these reflexive processes, including in institutional design, is significant for the development of scientific knowledge and relevant for addressing practical management tasks. For example, it is known that the solution of the problem of modelling the emergence of organizations in the form of a game to develop the rules of the game stumbles upon the problem of “bad infinity” (“reflexive candle”), which requires its solution by logical-algebraic methods on the basis of actually existing reflexive processes studied in cognitive psychology.

Thirdly, on the one hand, there are no empirical studies of reflexive processes in economic and political agents that confirm the models of the theory of reflexive games, and on the other hand, the concept of self-reflection (self-reflection or reflection of the first kind (Novikov, 2013)) lies outside the scope of research. As indicated in this work: “If the only reflexive subject is an economic agent that seeks to maximize its objective function by choosing one of the ethically acceptable actions, then natural reality is included in the objective function as a parameter, and the results of reflection (ideas about ideas, etc.) objective function arguments are not. Then self-reflection is “unnecessary” because it does not change the action chosen by the agent."

There is a bulk of references dedicated to the general methodology (system-thought-activity methodology) for the level of development of the subject's reflexive abilities to be classified. The theory of activity classifies group formation according to their 6 levels of cohesion (Anisimov, 2001):

1. sympathy (antipathy) or in life, - lack of reflection,
2. coordination of the results of reflection of behavior and about the opposition, - situational reflection with a minimum of problematization and random regulation,
3. reflection of the process of implementing norms (addressing problems), - full-fledged "three-sided" reflection with problematization and renormalization of unrealizable norms,
4. the grounds for organizing reflection, - the use of concepts (for example, science) in reflection,
5. the way of developing the foundations for organizing reflection, - using values as the means of problematizing a situation and concepts (for example, a particular methodology of science),
6. search for a way to develop the foundations of the organization of reflection - problematization not only of the situation and concepts but also of the values themselves (for example, a general methodology).

The penultimate level assumes that the result of reflection is influenced by the subject's relationship to the cultural environment, while the latter assumes a conscious choice of conformity to culture.

The purpose of this study is to determine the level of development of the agent's reflexive abilities when one is able to change the objective function and what logical-reflexive model can describe this process. Research hypothesis: the logical-reflexive models

developed by previous researchers allow describing the process of changing the agent's objective function.

- determine within which logical-reflexive models it is possible to change the objective function of the agent;
- to determine the algorithm for changing the investigated objective function of the manager.

2 MATERIALS AND METHODS

The research is mostly based on the structural method (Gretsky, 1983). It is planned to be applied in relation to existing logical-reflexive models in order to find the correspondence of these models to 6 levels of reflexivity.

In the works (Lefebvre, 2007, 2008; Lefebvre, 2008), a model is consistently introduced into consideration, which allows one to build hypotheses about the individual choices of group members that influence each other, and it is explained how this model can be used in practice. It is noted (Lefebvre, 2007) that the constructed model is based on the construction described by V.Lefebvre (Lefebvre, 1982, 2007), as well as on the work performed by Taran (Taran, 2001), in which the reflexive model was extended to Boolean algebra with many elements.

The work (Taran, 2001) considers the basic principles of constructing reflexive models of behavior in a situation of choice. First, based on the reflexive model of bipolar choice, V.A. Lefebvre (Lefebvre, 2003) naturally builds a logical model of the subject's reflexive behavior, formalized in Boolean logic $B_1 = <\{0,1\}, \&, \vee, \neg >$, gives an ethical interpretation of the reflexive choice and indicates that, depending on the domain of definition variables, various models of reflexive choice can be built. Further, we propose a generalization of V. Lefebvre's model in the form of a vector Boolean model, in which the choice-making activity of the subject under certain conditions is regulated by certain norms, models of choice are built on multivalued Boolean scales, correlating with a set of actions characterizing the behavior of the subject in society, regulated by a system of certain norms. In this study, we rely on logical-reflexive models proposed by T. Taran (Taran, 1998, 2001, 2004; Taran, 1998, 2001).

The basic model of reflexive choice developed by V. Lefebvre, represents the behavior of the agent in the form of a hierarchical reflexive structure of the factors (Taran, 1998: 51; Taran, 2004):

$$\text{Behavior} = \text{feeling}^{\text{knowledge}_3{}^{\text{desire}}}.$$

The decision-making by the agent is considered as a sequence of binary choices (elementary decision-making acts). Each elementary choice is made on a binary scale with two options: positive and negative. The agent is under pressure from the outside world to choose one of the options and acts as an operator: at each moment in time, an agent can choose one of the options (perform some action: positive (modelled by a Boolean value 1) or negative (modelled by a Boolean value 0)). This pressure of the outside world is not realized by the subject, but subconsciously makes them do a choice. These perceptions bring to the mind of the agent opinion about the pressure of the external world - subjective knowledge, which does not always coincide with reality (it is included in the "self-image" that the subject has). This "self-image" also includes the subject's ideas about themself: a conscious image of themself, which forms the subject's intentions to choose one or another option, which can be considered as personal desires. Under the influence of the pressure of the outside world, the subject's idea of this pressure and intentions the agent goes amenable to making a choice, accessible to an external observer. The images available to the subject is visible only for an external observer. For the subject, they seem to be some kind of reality. The apparent behavior of the subject is reality for an external observer. The self-image is a reality for the subject. The self-image that the self-image has is the subject's idea of reality.

The reflexive structure is described by a function of the form:

$$f(x_1,x_2,x_3) = x_1{}^{x_2{}^{x_3}} \quad (2),$$

where x_1 describes the pressure of the outside world towards one of the options; x_2 - the subject's idea of the pressure of the outside world; x_3 — subject's intentions. The function $f(x_1,x_2,x_3)$ describes the state of the subject at the moment of choice.

The expression a^b is a logical implication: $b \to a$, so function (2) can also be represented by the logical formula

$$X_1 = (x_3 \to x_2) \to x_1, \quad (3)$$

where x_1 is the perception of the pressure of the external world (unconscious perception by a person of impulses from the external environment); x_2 is assessment of the subject's psychological attitude (the expected pressure of the outside world, which is formed on the basis of a person's past knowledge and psychological attitude); variable x_3 is an assessment of intentions (plans and desires that reflect a person's

perceptions of a complicated situation and about himself in this situation), the implication $x_3 \to x_2$ - the subject's self-assessment ("self-image").

The scope of these variables and functions is a set of type ***2={0,1}*** (a Boolean lattice of two elements (0 and 1)) with the operations of disjunction, conjunction, negation and implication ($b \to a = \neg b \vee a$). Therefore X_1 is a Boolean function $f: 2^3 \to 2$ (Kuznetsov, 2008).

This model can be generalized as a vector Boolean model, in which (3) is a function $f: 2^n \to 2$,, where 2^n n is a Boolean lattice of binary vectors of length n (this generalization was proposed by T. Taran).

Suppose the subject has no plans. Then the simplest model of the subject, when they readiness for action to choose only under the pressure of the external world and psychological attitude, has the form:

$$X_1 = f\,(x_1, x_2) = x_2 \to x_1 \quad (4)$$

This expression is 0 only if x_2 = 1, x_1 = 0. This means that having no plans, the subject chooses the negative option in the only case: if the pressure of the outside world inclines one to choose the negative option, and the subject imagines this pressure as positive so the expectations regarding the situation are overestimated. A subject with no intention is called a primal subject, and the choice made at the second stage of reflection is called a primal choice. The primitive subject is not aware of their own ideas about the world, does not have a model of the world that generates intentions and changes behavior.

3 RESULTS AND DISCUSSION

Within this work, the variable x_3 from (2) and (3) correlates with the needs and interests of the manager described in formula (1). And the formula (1) itself is the formula of intentions.

Within the considered model of choice, the objective function of a manager in different cultural environments corresponds to the variable x_3 in formula (2). Note that the formulae of the remaining agents and limited resources are constraints (environmental pressure), and our ideas about the norm correspond to the variable x_2.

That is, at the first level of reflexivity, the variable x_2 corresponds to the assigning stereotypes of the subject's behavior (at this level, other subjects and objects do not differ but are perceived as objects of consumption). At the second level of reflexivity, the variable x_2 corresponds to the expectation from interaction with other agents (here already other subjects and objects differ in the sense that the subject agrees with them or enters into opposition).

At the first and second levels, any agent does not yet have intentions, since at the first level there is still no normative activity, there are only consumer stereotypes, and at the second level we also have ideas about other subjects and interaction with them. Thus, the agent's readiness for action is described by formula (4).

Starting from the third level, the agent's readiness for action is described by formula (3), which includes the variable x_3. Here the objective function of the manager already arises, which is intention. At this level of reflexivity, the knowledge variable includes only the agent's subjective idea of the previous norm, which agent changes during the reflexive process. This means that the variable x_2 is an argument x_3, in other words, there is a functional dependence of x_3 on x_2, because the agent uses the relationship between the new objective function and the norm of activity.

At the fourth level, the agent has an idea not only of the previous unrealizable norm but also of an abstract description of the situation, that is, a concept that includes scientific concepts (theories). In this case, the agent has the opportunity to choose the objective function, albeit so far without criteria.

At the fifth level, the subject introduces into the element of subjective knowledge not only ideas about an executable norm and an abstract description of the situation but also value grounds, which, in addition to an abstract description of the situation, are applicable to the entire class of being of a given agent, they are universal, describe wants and certainly motivate agents to take action. Thus, it is only at this level that the agent's knowledge includes abstract idea of own desires. This element becomes a criterion for choosing possible norms, and not random but conscious. In other words, thanks to this element, the choice from possible conceptual norms is carried out consciously, and not accidentally.

At the sixth level, the subject is able to think of their own wants (at the abstract level) and develops technologies for changing the existing reality. Hence the agents can assess their moral standards, treat them critically, and for this, they need to generate a new logic.

4 CONCLUSIONS

The inclusion of game-theoretic models in reflexive logic made it possible to come to the conclusion that a change in the objective function of an agent is

possible starting from the third level of development of reflexive abilities - the level of the problem. This process can be described within the framework of logical-reflexive models considered in the works of V. Lefebvre and T. Taran.

Thus, models of reflexive logic provide a broader scope of research and more accurate tools for modelling reflexive processes. This confirms the statement of V. Lefebvre, that a reflexive player can "outplay" a player using the classical maximin, and game-theoretic models are a special case of logical-reflexive models in which the objective functions are unchanged.

ACKNOWLEDGEMENTS

The reported study was funded by RFBR, project number 20-010-00869.

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