

Formulation of Multifactorial Regression Model of Life Expectancy based on Correlation and Regression Analysis

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Abstract: The paper considers the formulation of a multifactorial regression model to study life expectancy dependence on 11 factors (GDP per capita according to purchasing power parity (PPP) in US dollars; prevalence of any type of tobacco use among women over 15 (cigarettes, pipes, hookah, etc.) as percentage of the total female population over 15 years of age; prevalence of any type of tobacco use among men over 15 as percentage of the total male population over 15 years of age; diabetes risk as percentage for the population from 20 to 79 years old; Gini index; number of children who died under 1 year of age; air pollution with PM2.5 particles (in micrograms per cubic meter) per capita; health expenditure as percentage of GDP; health expenditure per capita; annual consumption of pure alcohol per capita for the population over 15 years of age; amount of population over 65 years of age; household consumption in US dollars for further modeling and forecasting in decision-making. The aim presented in the paper consists in obtaining information about the type and extent of relationship between life expectancy and factors, exclusion of insignificant factors and formulation of the econometric model. The authors used R-Studio language to achieve this goal.


1 INTRODUCTION


Econometric models reflect statistic patterns, the purpose of which is to use quantitative analysis and forecast of the relationships of indicators describing an economic object to prepare and make informed economic decisions. There are software products that can potentially be used to formulate a multifactorial regression model and not only, such as MS Excel, Gretl, SPSS, STATA, Eviews, R.

In recent years, many works on this topic have been published by authors from Russia, both on the assessment of the influence of many factors on life expectancy (Zvezdina and Ivanova, 2015; Merkushova, 2015; Mironova, 2020, Shibalkov and Nedospasova, 2020), and on the influence of individual factors, e.g., the impact of alcohol consumption on life expectancy in Russia (Kosova et al., 2017), on gender and regional differences in life expectancy in Russia (Rodionova and Kopnova, 2020).

There are also works that study life expectancy in different countries (Kontis V. et al., 2017), influence of factors on early mortality (Platt O. S. et al., 1994). The influence of a single factor on life expectancy in a particular country during a certain period is often investigated. For instance, P. T. Katzmarzyk & I. M. Lee (Katzmarzyk and Lee, 2012) describe the impact of sedentary lifestyle on life expectancy in the United States, and J. E. Bennett (Bennett J. E. et al., 2019) estimates the impact of air pollution with solid particles on life expectancy in the United States.

In this paper, it is proposed to illustrate the formulation of a multifactorial regression model of life expectancy dependence based on the use of correlation and regression analysis apparatus, applying a correlation matrix based on the correlation coefficients between all available factors and exclusion of variables that are not significant for the model.

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2 METHODOLOGY

Correlation analysis is a method of mathematical statistics used to study and investigate the relationship between (general) economic indicators based on their observed statistic (sample) analogs.

Paired correlation analysis is the study of the relationship between two economic indicators that describe the properties of the same type of objects from a certain population. Consequently, the paired correlation analysis is one of the tools for selecting factors for the regression model and selecting the form of relationship between the dependent variable and determining factors. Using this method, you can check the relevance of some selected factors for this model. In order to draw conclusions about the mutual dependence of the selected indicators, it is necessary to build correlation fields and analyze the locations of points on them. If the points are located approximately on the same straight line and have slight deviations in both directions from it, then we can talk about the presence of linear relationship. The type of relationship can be determined by the straight line slope. It can be direct or reverse. If it is possible to trace a certain curvature of the band of points on the graph, then we can talk about the presence of nonlinear dependence between the indicators. And when the points are scattered over a certain area, it can be stated that there is no correlation. Based on the graphical analysis, it is necessary to make a visual assessment and present the data in the form of the correlation field.

The relationship graphical description consists in constructing an empirical regression line that connects the points on the correlation field, the abscissae of which are the values of the attribute-factor (individual or group), and the ordinates are the average values of the resulting attribute.

As researches demonstrate (Salnikova, 2020), the relationship strength is assessed using the correlation indicators:

1. K. Pearson linear coefficient of paired correlation (r_{xy}) evaluates the strength and direction of only linear relationship between two attributes calculated by formulas (1), (2) and (3):

$$r_{xy} = \frac{\sum \left(\frac{x-\bar{x}}{\sigma_x} \right) \cdot \left(\frac{y-\bar{y}}{\sigma_y} \right)}{n} \quad (1)$$

$$r_{xy} = \frac{\overline{xy} - \bar{x}\bar{y}}{\sigma_x \sigma_y} \quad (2)$$

$$r_{xy} = \frac{n \sum xy - \sum x \sum y}{\sqrt{(n \sum x^2 - (\sum x)^2) \cdot (n \sum y^2 - (\sum y)^2)}} \quad (3)$$

where σ_x and σ_y - standard deviations of factorial and resultant attributes;

x and y - values of the attributes.

The value of the linear correlation coefficient is within (formula (4):

$$-1 \leq r_{xy} \leq 1 \quad (4)$$

1) if $r_{xy} = \pm 1$, the relationship is functional, i.e. ratio $y=a+bx$ is fulfilled for all observations:

- if $r_{xy} = 1$, the relationship is direct ($b > 0$), which means functional relationship between x and y ;

- if $r_{xy} = -1$, the relationship is reverse ($b < 0$);

2) if $r = 0$, there is no linear relationship, but there may be a nonlinear one;

3) if $0 < r \leq 1$, the relationship is direct (y increases with x increase);

4) if $-1 \leq r < 0$, the relationship is reverse (y decreases with x increase).

The closer the absolute value of $|r_{xy}|$ is to 1, the stronger the linear relationship is and the better the linear relationship agrees with the observation data. Any intermediate value of r from 0 to 1 characterizes the degree of approximation of the correlation relationship between x and y to the functional one.

To determine the relationship strength between the indicators, it is necessary to build a matrix of interfactor correlation coefficients.

The aim of the work is to study the dependence of life expectancy based on the use of the apparatus of correlation-regression analysis. Next, we will build a regression model of the dependence of life expectancy in all countries of the world on the value of the value of various factors in 2015. and see how much it has changed in 2020. Since life expectancy depends on various factors: income level, lifestyle, education, heredity, ecology, nutritional quality, development of the health care system and others. Then, to build a predictive econometric model, we take 11 factors (World Health Organization):

- gdppc - GDP per capita according to PPP in US dollars;
- smokingfemale - prevalence of any type of tobacco use among women over 15 (cigarettes, pipes, hookah, snus, etc.) as percentage of the total female population over 15 years of age;
- smokingmale - prevalence of any type of tobacco use among men over 15 as percentage of the total male population over 15 years of age;
- diabetes - diabetes risk as percentage for the population from 20 to 79 years old;
- gini - Gini index;
- childdeath - number of children who died under 1 year of age;

- airpollution - air pollution with PM2.5 particles (in micrograms per cubic meter) per capita;
- healthexpenditure - health expenditure as percentage of GDP;
- healthexpenditure1 - health expenditure per capita;
- alcohol - annual consumption of pure alcohol per capita for the population over 15 years of age;
- undersixtyfive - amount of population over 65 years of age;
- hconsump - household consumption in US dollars.

Let us find a set of factors that should be included in the regression model, and also identify those that are insignificant (do not affect the dependent variable value). We also find the type of dependence between the dependent variable and predictors.

The purpose of correlation analysis is to obtain information about the type and extent of relationship between life expectancy and factors.

Determination of the type and extent of correlation relationship between life expectancy and factors.

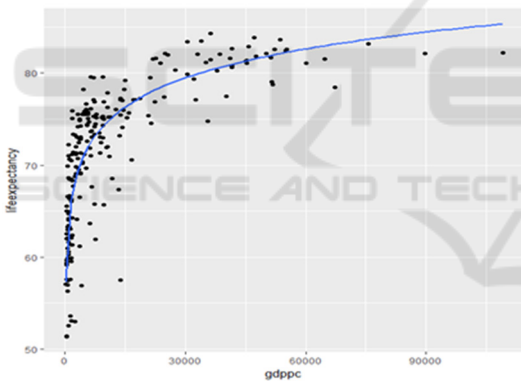


Figure 1: Correlation field of life expectancy & gdppc.

Below are the graphs of the correlation field for all factors.

In Figure 1, the graph demonstrates direct linear correlation dependence between GDP level per capita in prices of 2015. This dependence is also called Preston Curve and has been around for about 100 years, so it is often mentioned in medical researches.

The dependence shows that a person born in a rich country can, on average, according to forecasts live longer than a person born in a poor country. As income increases, life expectancy goes up, but after 60,000 US dollars per person life expectancy growth slows down.

Countries can be located above or below Preston Curve. For instance, SAR is far below the curve,

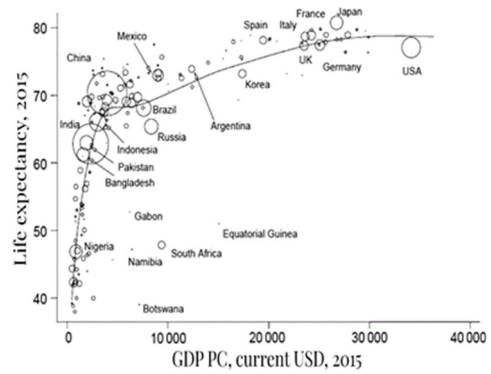


Figure 2: Correlation field of life expectancy & GDP PC/2.

meaning that life expectancy is much lower than the potential one at the current income level. At the same time, Mexico is above the curve, which means that life expectancy is higher than the average for the given income level (Figure 2).

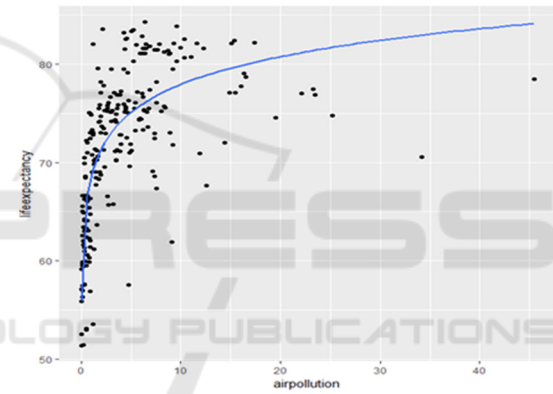


Figure 3: Correlation field of life expectancy & airpollution.

The third graph in Figure 3 demonstrates contradictory results, so we will later eliminate the air pollution factor.

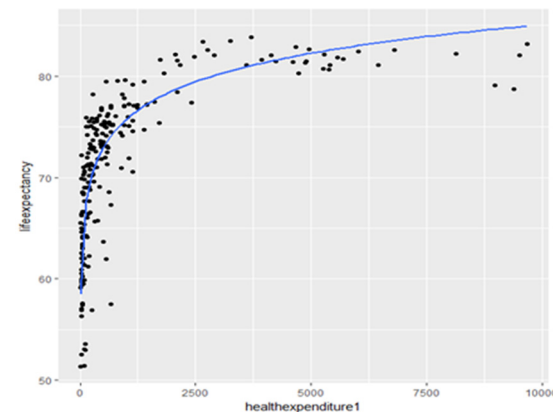


Figure 4: Correlation field of life expectancy & smokingfemale.

Mathematically, this graph in Figure 4 means that the first derivative of the life expectancy function is positive depending on the proportion of smoking women, but it decreases, and the second one is negative. In other words, the function increases and is convex upwards.

The derivative of the life expectancy function dependence on the proportion of smoking women (derivative) decreases with an increase in the proportion of smoking women, and turns to zero at the maximum life expectancy and then becomes negative, and life expectancy, having reached the maximum value, begins to decrease.

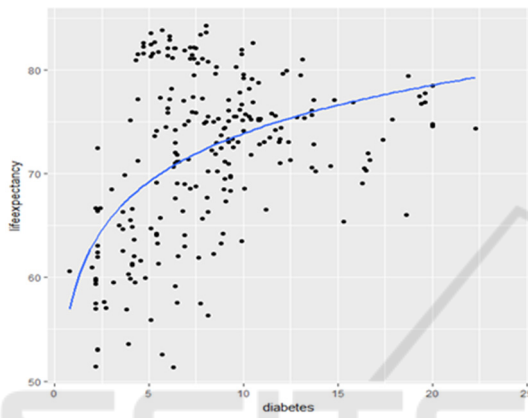


Figure 5: Correlation field of life expectancy & smokingmale.

According to the graph in Figure 5, there is negative dependence between the life expectancy and number of smoking men, but the dependence is weak.

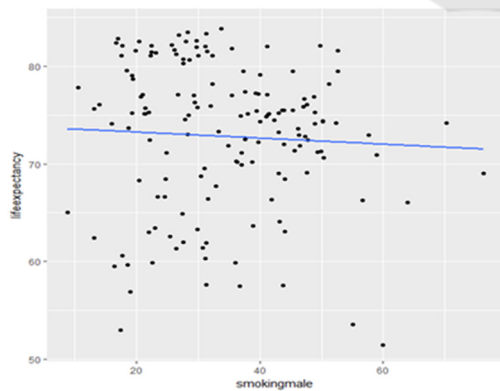


Figure 6: Correlation field of life expectancy & diabetes.

The dependence turns out to be contradictory, and it is impossible to give full interpretation (Figure 6). A possible explanation can be found in the fact that in countries where food is better, the diabetes rate is higher. However, the life expectancy is higher in the same countries.

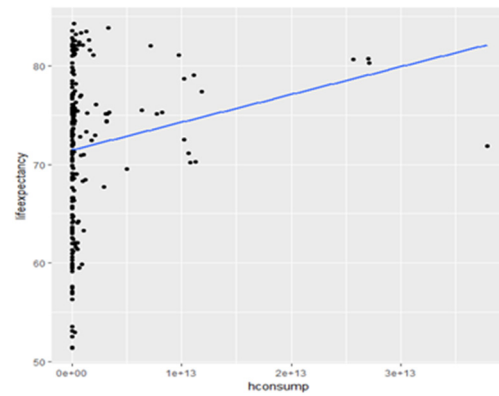


Figure 7: Correlation field of life expectancy & healthexpenditure1.

According to the World Health Organization, there is direct dependence between total health expenditure (per capita) and life expectancy.

From the graph in Figure 7 we can conclude that this statement is correct.

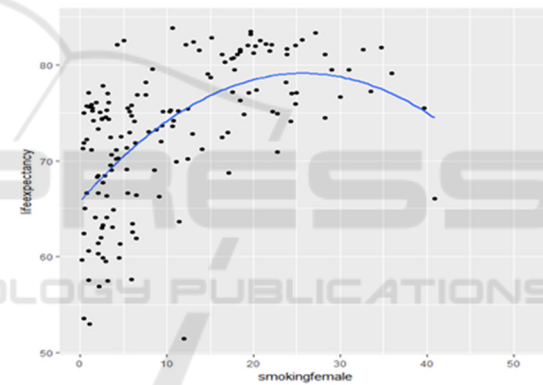


Figure 8: Correlation field of life expectancy & hconsump.

Thus, it can be seen that there is no dependence between life expectancy and household consumption (Figure 8).

Let us prepare a correlation matrix based on the correlation coefficients between all available variables (Figure 9):

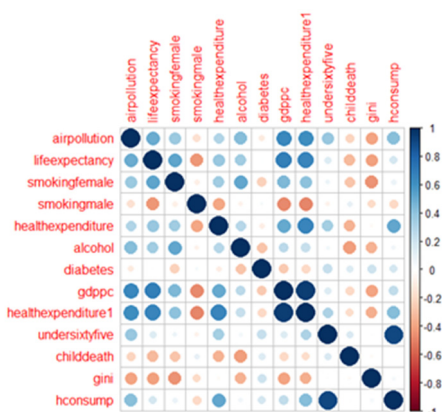


Figure 9: Correlation matrix based on the correlation coefficients between all available variables for 2015.

From the data in Figure 9 it can be seen that the correlation dependence between life expectancy and factorial variables does not exist everywhere. This table confirms the assumptions expressed on the graphical analysis basis.

Determination of the extent of mutual influence on life expectancy of all factors

$$R^2 = 0.6235$$

We can conclude that the change in life expectancy by 62.35% is due to the change in the factors included into the model.

Let us check the combined influence of the factors on life expectancy, calculated by formula (5):

$$F_{observ} = \frac{R^2(n-k-1)}{(1-R^2)k} \tag{5}$$

where R^2 – determination coefficient (square of multiple correlation coefficient), R^2 (Multiple R – Squared) = 0.6235

n – sample size, $n = 264 - 108 = 156$, 108 observations deleted due to missingness,

k – number of predictors, $k = 5$

$$F_{observ} = \frac{0.6235 \cdot (156 - 5 - 1)}{(1 - 0.6235) \cdot 5} = 49.681$$

$$F_{crit}(5; 150; 0,05) = 4.3650$$

Since $F_{observ} > F_{crit}$, consequently, with 95% probability, together the factors significantly affect the life expectancy.

In comparison with the data for 2020, the correlation dependence between the life expectancy and factorial variables does not exist everywhere, as

well as in the correlation matrix based on the correlation coefficients for 2015.

Therefore, it is proposed to exclude variables that are not significant for the model based on the data array used. After identifying significant factors using t-statistics, there were suspicions about the existence of multicollinearity between the two factors: gdppc and healthexpenditure1, as the graphical analysis contradicted the results of the t-statistics check. VIF test is used to check the factors for multicollinearity.

Results of the test conducted in R-Studio (Table 1):

Table 1: VIF test.

Gdppc	16.384332
Healthexpenditure1	18.906159
Smokingmale	1.709373
smokingfemale	1.702277
diabetes	1.212224
gini	1.909564
hconsump	2.075939
alcohol	1.722765
childdeath	1.353089

As you can see, there is high multicollinearity between gdppc and healthexpenditure1, that is, pairwise correlation dependence between the factors. This means that one of these factors should be eliminated.

Variable elimination method - used to eliminate multicollinearity. It consists in removing highly correlated explanatory variables from the regression and reevaluating it.

The selection procedure for the main factors includes the following steps:

1. The analysis of the values of the coefficients of pair correlation $r_{x_i x_j}$ between the factors x_i and x_j is carried out.

2. The identified pairwise-dependent factors are analyzed according to the closeness of the relationship between the explanatory factors and the effective variable.

If we exclude healthexpenditure1 from the model, then R^2 (Adjusted) = 0.6254. If we exclude gdppc from the model, then R^2 (Adjusted) = 0.6121. So, it makes more sense to exclude healthexpenditure1 from the model.

Now, after conducting VIF test, we can exclude the remaining insignificant factors from the model, such as: alcohol, hconsump. At the same time, R^2 (Adjusted) will increase up to 0.649.

3 RESULTS

After passing all the analysis stages, our econometric model takes the following form:

$$\text{lifeexpectancy} = 3.43 \cdot \log(\text{gdppc}) + 2.94 \cdot \log(\text{diabetes}) - 0.09719 \cdot \text{gini}$$

The factors such as the percentage of smoking women (smokingfemale), child mortality (childdeath) were excluded from the model after checking the significance of regression coefficients based on t-statistics.

According to VIF test, the model is not multicollinear (Table 2).

Table 2: VIF-test.

log(gdppc)	log(diabetes)	gini
1.232833	1.035453	1.211515

Heteroscedasticity was revealed in the model. After adjusting for heteroscedasticity, the gini factor was insignificant. However, if we exclude it from the model, R^2 (adj) will decrease from 0.8 to 0.71, which significantly affects the model quality.

Moreover, the heteroscedasticity in our model is very weakly expressed, this is evidenced by a rather high p – value = 0.01104. So, it was decided to leave the gini factor in the model.

With 95% probability, the factors that are included into the model collectively have a statistically significant effect on the change in lifeexpectancy (the regression is generally significant).

4 DISCUSSION OF RESULTS

Most of the phenomena and processes in the economy are closely related. Its identification and analysis is a primary task at the initial stage of developing a forecasting model. This allows us to discard insignificant factors, to understand the process of cause-and-effect relationships between factors. It is very convenient to investigate dependencies using correlation-regression analysis.

In light of the impact of COVID-19 on life of people, the current topic of research is the forecast of life expectancy, taking into account significant factors and possibility of further development of the necessary measures. The econometric model obtained in the course of research shows that life expectancy in a country depends on such factors as the gross

domestic product per capita, prevalence of diabetes among the population, and Gini index.

5 CONCLUSION

So, the following stages of data analysis were implemented:

- based on open statistic data, the set of 11 factors that affect life expectancy was formed;
- insignificant factors were excluded from the model: healthexpenditure1. Using the software product, VIF test was performed in R-Studio to check the factors for multicollinearity. The insignificant factors were excluded from the model, such as: alcohol, hconsump;
- econometric model was obtained and tested for heteroscedasticity.

The resulting methodology can be applied in life expectancy analysis, and can also be used by specialists in the process of forecasting various decisions in the adoption of regional policy.

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