Optimization of Large-scale Transport Network as a Factor of Sustainable Development

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Abstract: The paper investigates the problem of optimal planning of passenger and freight transportation routes in a large-scale transport network. Optimizing the structure of the transport network and analyzing the spatial relationships of the functioning of the infrastructure are key ways to ensure the sustainability of regional development. It is proposed to use fractal graphs and their limited counterpart, prefractal graphs, which are graphs with fractal properties as a model of a large-scale transport network. The mathematical formulation of the problem is presented as a multicriteria discrete optimization problem, where the criteria are the most significant requirements for the system. In this formulation, the problem under study becomes a multicriteria problem of covering a prefractal graph with simple intersecting paths. The solution of the multicriteria discrete optimization problem is constructed using special algorithms, the quality of which is estimated by computational complexity. We have built one of the effective algorithms for optimizing the problem according to one of the presented criteria that allows us to select the maximum paths. The common problem of discrete multicriteria problems is to find many alternatives, but in this paper, attention is paid to finding at least one optimal solution from many alternatives and evaluating it according to other criteria. The advantage of using this approach using prefractal graphs is justified by a reduction in the computational complexity of the algorithms.

1 INTRODUCTION

The task of optimal planning of passenger and freight transportation routes is a key problem in ensuring the efficiency of transport infrastructure (Comtois, 2013). Infrastructure is sustainable if it brings social, economic and environmental benefits throughout its life cycle. When solving such a problem, it is necessary to take into account various optimization requirements (criteria). For example, when finding optimal routes, it is necessary to take into account not only economic requirements, i.e., optimization of transportation costs but also social or environmental requirements. As a rule, in such problems, a solution that optimizes one of the criteria is not optimal according to other criteria, then these tasks are multi-criteria problems. The solution to the multicriteria problem is not one single solution, but a set of alternatives (Cochrane, 1973). Currently, the problem of finding a set of alternatives is poorly studied, including for the multicriteria discrete problem in modelling transport routes (Emelichev, 1991).

The article studies the mathematical model of the problem of planning transport routes in a large-scale transport network in a multi-criteria environment. It is proposed to use prefractal graphs (Kochkarov, 1999; Skums, 2019; Kochkarov, 2004) with the property of a «small-world» as a model of a large-scale transport network. Prefractal graphs are used to model the structure of large-scale complex systems, such as the global Internet, electric networks, and large-scale clustering of matter in the Universe (Kochkarov, 2004; Perepelitsa 1999; Kochkarov, 2015).

In the study of any multicriteria problem, three stages can be distinguished, each of which is a

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separate task. The first step is the construction of a set of feasible solutions. The second stage consists in isolating from the set of feasible solutions the Pare-to optimal so-called Pareto set (Cochrane, 1973; Emelichev, 1991). The solution is Pareto-optimal if the value of any of the criteria can be improved only due to the deterioration of the values of other criteria. At the third stage, from the Pareto set, it is necessary to choose a solution that will be implemented taking into account the essence of the problem (Emelichev, 1991; Kochkarov 1998).

In this paper, attention is paid to finding at least one optimal solution from a variety of alternatives. The concept of asymptotic time complexity is used – the behaviour of computational complexity as a function of input size in the limit with increasing size of the problem (Garey, 1979). For this, a polynomial algorithm (Garey, 1979) is constructed that allows one to single out an effective solution with an estimate according to given criteria.

2 METHODS

2.1 Basic Concepts in Fractal and Prefractal Graphs

Prefractal and fractal graphs are a model of structures growing in discrete time according to the same rules from each of its vertices. The formal reflection of these rules is the operation of replacing a vertex by seed, which underlies the definition of prefractal graphs. The term *seed* is any connected graph $H = (W, Q)$. The essence of the operation vertex replacement by seed (VRS) is as follows. In the given graph $G = (V, E)$, the vertex $\tilde{v} \in V$ chosen for replacement is distinguished by the set of $\tilde{V} = \{\tilde{v}_j\} \subseteq V$, $j = 1,2, \dots, |\tilde{V}|$ adjacent vertices. Further, this vertex $\tilde{v}$ and all its incident edges are removed from the graph G. Then each vertex $\tilde{v}_j \subseteq V$, $j = 1,2, \dots, |\tilde{V}|$ is connected by an edge to one of the vertices of the seed $H = (W, Q)$. The vertices are joined arbitrarily (randomly) or according to a certain rule if necessary.

Denote the prefractal graph by $G_L = (V_L, E_L)$, where V_L is the set of vertices of the graph, and E_L is the set of its edges. We define it recurrently, gradually replacing each vertex in the graph G_l constructed at the previous stage $l = 1,2, \dots, L-1$ each its vertex with the seed $H = (W, Q)$. At the stage $l = 1$, the prefractal graph corresponds to the seed $G_1 = H$. The process of generating a prefractal graph G_L is the process of constructing a sequence of prefractal graphs $G_1, G_2, \dots G_l, \dots, G_L$, called a trajectory (see Figure 1). The fractal graph G generated by the seed H is determined by an infinite trajectory.

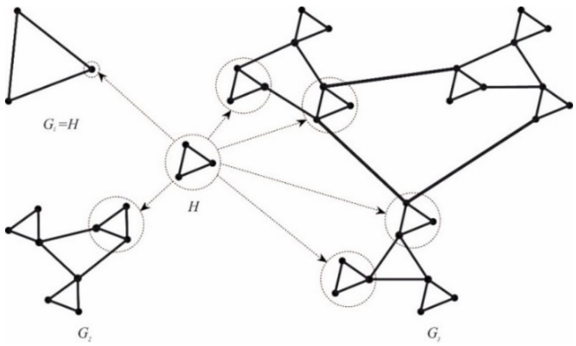

Figure 1: The trajectory G_1, G_2, G_3 of the prefractal graph G_3 generated by the seed-triangle where the adjacency of the old edges is chosen arbitrarily.

For a prefractal graph G_L, edges that appear at the l-th, $l \in \{1,2, \dots, L\}$ generation stage will be called *edges of rank l.* The new edges of the prefractal graph G_L are the edges of rank L, and all the other edges are called the *old edges*.

If we remove all edges of ranks $l = 1,2, \dots, L-r$ from the prefractal graph G_L, we obtain the set $\{B_{L,i}^{(r)}\}$, $r \in \{1,2, \dots, L-1\}$ *blocks* of the r-th rank, where $i = 1,2, \dots, n^{L-r}$ is the block ordinal number. We call block $B_{l,s}^{(1)}$, $s = \overline{1, n^{l-1}}$, of the first rank of prefractal graph G_l, $l = \overline{1, L}$ from the trajectory as *seed subgraph* $z_s^{(l)}$.

Prefractal graph $G_L = (V_L, E_L)$ is called *weighted* if for each edge $e^{(l)} \in E_L$ there is a real number $w(e^{(l)}) \in (\theta^{l-1}a, \theta^{l-1}b)$, where $l = \overline{1, L}$ is the rank of the edge, $a > 0$, and $\theta < \frac{a}{b}$.

A prefractal graph generated by one or a set of seed multigraph (Harary, 1979) is called a prefractal *multigraph*.

2.2 Discrete Multi-criteria Problem Statement

Let weighted prefractal graph $G_L = (V_L, E_L)$ generated by seed $H = (W, Q)$ be given. On feasible solution set (FSS) $X = X(G_L) = \{x\}$, $x = (V, E_x)$, $E_x \subseteq E_L$ consisting of all kinds of coverings of weighted prefractal graph G_L by simple intersecting paths, a vector-valued objective function (VVOF) is defined as follows:

$$\mathrm{F}(X) = \{F_1(x), F_2(x), F_3(x),$$

$$F_4(x), F_5(x), x \in X\} \quad (1)$$

$$F_1(x) = \sum_{e \in E_x} w(e) \to \min \quad (2)$$

where $\sum_{e \in E_x} w(e)$ is sum of all edges included in covering x;

$$F_2(x) = \min_{k=\overline{1,K}} w(C_k) \to \max \quad (3)$$

where $w(C_k)$ is length of the maximal path from covering $x \in \{C_1, C_2, \dots, C_k, \dots, C_K\}$.

$$F_3(x) = N(x) \to \min \quad (4)$$

where $N(x)$ is number of all maximal paths in covering x;

$$F_4(x) = i \to \min \quad (5)$$

for any mixed path C^i from covering x.

$$F_5(x) = \left|\rho_x(u,v) - \rho_{G_L}(u,v)\right| \to \min \quad (6)$$

where $\rho_x(u,v)$ is the distance (between any vertices $u, v \in V_L$) passing through the edges belonging to covering x, while $\rho_{G_L}(u,v)$ is the distance between any vertices $u, v \in V_L$ in graph G_L.

In terms of transport systems, the above criteria of the VVOF (1)-(6) have a certain meaningful interpretation (Comtois, 2013). The weights of the edges of the prefractal graph G_L may correspond to certain costs and restrictions when moving vehicles along the nodes of the transport network. Criterion (2) factors in the costs incurred by passengers and the authorities that are managing the transport system. During operation, costs should be minimal. Optimization by criterion (3) allows you to find routes containing the largest number of nodes in your path. Optimal for this criterion is a coating containing maximum paths. To get to the desired node of the transport network with the least number of transfers, it is necessary to reduce the total number of routes in the system; for this purpose, criterion (4) is used. Important features of the transport system are the locality and differentiation of its routes. Intra-regional (city, intra-district) should be transport routes of shorter length and less weight, thereby ensuring locality. This simplifies the process of administering the transport system at a certain level (district, city, etc.). Interregional routes are longer and with more weight. Differentiation refers to the separation of routes according to their functions into inter-regional and intra-regional. At the intersection of intra-regionality and inter-regionality, a violation of differentiation may occur, i.e., deterioration in the functionality of the route. Criterion (5) is responsible for preventing such situations in the operation of the transport system in the VVOF (1)-(6). Mixed path C_k is a route model combining both functions – intra-regional and inter-regional – since its old edges connect the blocks and seed subgraphs of prefractal graph G_L, which correspond to the maps of the roads of districts, cities, etc. When operating a transport system, it is often required that the final destination will be reached with the least number of stops. Criterion (6) reflects these requirements on construction of such routes.

3 RESULTS

3.1 The Algorithm for Finding the Largest Maximum Paths

The β_2 algorithm finds covering $x_2 = J = (V_L, E_J) = \{C_1, C_2, \dots, C_k, \dots, C_K\} \in X$ on prefractal graph, where all $C_k = \{v_k, u_k\}$ paths are simple, $k = \overline{1,K}$. β_2 is based on the largest maximal path finding algorithm (LMPF algorithm) on an arbitrary graph. Using the LMPF algorithm as a procedure, the β_2 algorithm finds subgraph $J_s^{(l)} = (V_s^{(l)}, E_{J_s^{(l)}}) = \{C_1, C_2, \dots, C_k, \dots, C_{K_{J_s^{(l)}}}\}$ on each seed subgraph of set $Z(G_L) \in z_s^{(l)}, l = \overline{1,L}, s = \overline{1,n^{l-1}}$ of prefractal graph G_L such that all paths $C_k = \{u,v\}$ are maximal (i.e. $|C_k| = \min$), $k = \overline{1,K_J}$, among all paths between vertices $u, v \in V_s^{(l)}$ of seed subgraph $z_s^{(l)}$ and the largest. Set of coverings $\{J_s^{(l)}\}, l = \overline{1,L}, s = \overline{1,n^{l-1}}$, selected on seed subgraphs of prefractal graph G_L, forms covering $x_2 = J = (V_L, E_J)$.

3.1.1 LMPF Algorithm

INPUT: graph $G = (V, E)$.

OUTPUT: spanning subgraph $J = (V, E_J) = \{C_1, C_2, \dots, C_k, \dots, C_{K_J}\}$.

STEP 1. Find set $\{C'_{i_1}, C'_{i_2}, \dots, C'_{i_k}, \dots, C'_{i_{K'}}\}$ of all shortest paths between each pair of vertices $u, v \in V$ of graph G. From $\{C'_{i_1}, C'_{i_2}, \dots, C'_{i_k}, \dots, C'_{i_{K'}}\}$ remove all those paths that are completely contained in others. Combine the remaining ones into set $\{C_{i_1}, C_{i_2}, \dots, C_{i_k}, \dots, C_{i_{K^*}}\}$, assigning them indices such that the length of the i_{k+1}-th path is not greater than the length of the i_k-th path, $k = \overline{1,K^*}$. Consider paths $\{u,v\}$ and $\{v,u\}$ for any pair of vertices $u, v \in V$ as identical and include only one of them in set $\{C_{i_1}, C_{i_2}, \dots, C_{i_k}, \dots, C_{i_{K^*}}\}$. Set $\{C_{i_1}, C_{i_2}, \dots, C_{i_k}, \dots, C_{i_{K^*}}\}$ is a set of maximal paths of graph G, where C_{i_1} is the diametral path.

STEP 2. Cover the vertices and edges of graph G with paths from set $\{C_{i_1}, C_{i_2}, \dots, C_{i_k}, \dots, C_{i_{K^*}}\}$ sequentially, starting with the i_1-th. For the covering of graph G with path C_{i_k} we will have in mind selection of vertices and edges forming the C_{i_k}. path on graph G. We use only paths that satisfy the condition that each new path selects at least one other vertex of graph G that is not covered by previous paths.

STEP 3. Assign numbers (in the order they are used) to all paths from set $\{C_{i_1}, C_{i_2}, \dots, C_{i_k}, \dots, C_{i_{K^*}}\}$ used to cover vertices and edges of graph G. Cover graph until there are no unselected vertices left.

STEP 4. Set of paths $\{C_1, C_2, \dots, C_k, \dots, C_{K_J}\} \subseteq \subseteq \{C_{i_1}, C_{i_2}, \dots, C_{i_k}, \dots, C_{i_{K^*}}\}$, used to cover graph G, form the desired covering $J = (V, E_J) = \{C_1, C_2, \dots, C_{K_J}\}$, consisting of the largest maximal paths $C_k = \{v_k, u_k\}$ $k = \overline{1, K_J}$.

Theorem 1. *The computational complexity of the LMPF algorithm that finds covering $J = (V, E_J)$ on graph $G = (V, E), |V| = n$, is $O(n^5)$.*

PROOF. Finding the shortest distance between any two vertices of graph G will take no more than n^2 simple operations. In its first step, the LMPF algorithm finds all the shortest paths of graph G, and they are equal to $n(n-1)/2 < n^2$ in number. Next, the algorithm selects (by comparing the paths) some part of these paths. Since all the vertices and edges that make up the paths are known, comparing the paths will take n^2 operations. In total, the computational complexity of the LMPF algorithm is $O(n^2 n^2 + n^2) < O(n^5)$.

3.1.2 The β_2 Algorithm

INPUT: prefractal graph $G_L = (V_L, E_L)$.
OUTPUT: connected spanning subgraph $J = (V_L, E_J) = \{C_1, C_2, \dots, C_k, \dots, C_K\}$.

STEP 1. Construct a set of seed subgraph $Z(G_L) = \{z_s^{(l)}\}$, $l = \overline{1, L}$, $s = \overline{1, n^{l-1}}$ for prefractal graph G_L. In accordance with constructed set $Z(G_L)$, number all the edges of prefractal graph G_L.

STEP 2. One at a time, in a decreasing order of rank $l = L, L-1, \dots, 2, 1$ find spanning subgraphs $J_s^{(l)} = (V_s^{(l)}, E_{J_s^{(l)}}) = \{C_1, C_2, \dots, C_{K_{J_s^{(l)}}}\}$ on all seed subgraphs $z_s^{(l)}$, $s = \overline{1, n^{l-1}}$, from set $Z(G_L)$, using the LMPF algorithm. After finding $\{J_s^{(L)} = (V_s^{(L)}, E_{J_s^{(L)}})\}$, $s = \overline{1, n^{l-1}}$, create set of paths $\{C_1^*\} = \{C_{1,1}, C_{1,2}, \dots, C_{1,k}, \dots, C_{1,K_1}\} = \{J_s^{(L)} = (V_s^{(L)}, E_{J_s^{(L)}})\}$.

Further, each time after path set $\{C_{L-l+1}^*\} = \{C_{L-l+1,1}, C_{L-l+1,2}, \dots, C_{L-l+1,k}, \dots, C_{L-l+1,K_{L-l+1}}\}$, $l = L, L-1, \dots, 2, 1$, is created, connect each of its $C_{L-l+1,k}$ paths to the edges of the paths of seed subgraphs $\{z_s^{(l-1)}\}$ and combine them into a new path set $\{C_{L-l+2}^*\}$ as follows.

STEP 3. Attach any edge $\in C_k = \{v_k, u_k\}$, $k = \overline{1, K_{J_s^{(l)}}}$, of path $C_k \in J_s^{(l-1)}$ of seed subgraph $z_s^{(l-1)}$, $s = \overline{1, n^{l-1}}$, to that path from set $\{C_{L-l+1}^*\}$, to which it is incident at the end. The path formed in this way is introduced into a new path set $\{C_{L-l+2}^*\}$.

If edge e is incident to one of its ends by several paths from$\{C_{L-l+1}^*\}$, then all the paths formed in this case are introduced into set $\{C_{L-l+2}^*\}$. If both vertices v_k, u_k of edge e are incident to the ends of two different paths C_{L-l+1,k_1} and C_{L-l+1,k_2} respectively, then a path formed by paths $C_{L-l+1,k_1}, C_{L-l+1,k_2}$ and edge e is added to set $\{C_{L-l+2}^*\}$ only if the ends of paths C_{L-l+1,k_1} and C_{L-l+1,k_2} that are not incident to edge $e$are also not incident to the ends of other paths from $\{C_{L-l+1}^*\}$. Otherwise, add the paths formed by several paths from $\{C_{L-l+1}^*\}$ and several edges of the paths of seed subgraphs of $(l-1)$-th rank to set $\{C_{L-l+2}^*\}$.

If edge e is not incident to any paths of $\{C_{L-l+1}^*\}$, then insert it into set $\{C_{L-l+2}^*\}$ as a separate path.

STEP 4. At the input of the previous step, after the paths of all seed subgraphs have been processed, a set of paths $\{C_L^*\} = \{C_{L,1}, C_{L,2}, \dots, C_{L,k}, \dots, C_{L,K_L}\}$will be obtained. Set of paths $\{C_1, C_2, \dots, C_k, \dots, C_K\}$ obtained from $\{C_L^*\}$ by changing the numbering defines the required spanning subgraph $J = (V_L, E_J)$.

Theorem 2. The β_2 algorithm finds connected spanning subgraph $J = (V_L, E_J) = \{C_1, C_2, \dots, C_k, \dots, C_K\}$, where C_k are simple paths, on prefractal graph $G_L = (V_L, E_L)$, generated by seed $H = (W, Q)$, $|W| = n$.

The proof of the theorem is based on the design features of the construction of prefractal graphs and the operation of the algorithm β_2.

Theorem 3. The computational complexity of the β_2 algorithm that selects covering $J = (V_L, E_J)$ on prefractal graph $G_L = (V_L, E_L)$, generated by seed $H = (W, Q)$, where $|W| = n$, $|V_L| = N = n^L$, is equal to $O(Nn^5)$.

Proof. The β_2 algorithm is essentially a multiple execution of step 2. Step 2, in turn, is a multiple invocation to the LMPF algorithm, whose computational complexity is equal to $O(n^5)$. Since the β_2 algorithm invokes the SPS (shortest path selection) algorithm $k = \frac{n^L-1}{n-1}$ times, it will perform no more than $k \cdot O(n^5)$ operations.

Then, $O(k \cdot n^5) = O(\frac{n^L-1}{n-1} \cdot n^5) = O(n^L \cdot n^5) = O(Nn^5)$.

Hence, the computational complexity of the β_2 algorithm is equal to $O(Nn^5)$.

Theorem 4. The β_2 algorithm selects covering $x_2 = J = (V_L, E_J) = \{C_1, C_2, \dots, C_k, \dots, C_K\} \in X$, where C_k are the shortest paths of same rank, on prefractal graph $G_L = (V_L, E_L)$ generated by seed $H = (W, Q)$, $|W| = n$, $|Q| = q$, estimated by the first criterion:

$$F_1(x_2) \in \left[a(n-1)\frac{(na/b)^L-1}{\frac{na}{b}-1}; qb\frac{(na/b)^L-1}{\frac{na}{b}-1}\right].$$

Proof. Covering $x_2 = J = (V_L, E_J) = \{C_1, C_2, \dots, C_k, \dots, C_K\}$ selected by the β_2 algorithm on prefractal graph G_L generated by seed, belongs to feasible solution set X of vector-valued objective function (1)-(6).

We first establish the upper bound of the estimate. The $F_1(x)$ criterion is weighted and its value is equal to the sum of the weights assigned to the edges of covering $x \in X$. Obviously, the covering from the feasible solution set consisting of all edges of prefractal graph G_L will have the greatest weight, i.e., when $x = G_L$. Using the prefractal graph weighting rule, we give an estimate of the total weight $w(G_L)$ of prefractal graph G_L. We denote the total weight of seed subgraph $z_s^{(l)} \in Z(G_L)$ of rank l, $l = \overline{1, L}$ under serial number s, $s = \overline{1, n^{l-1}}$ as $w(z_s^{(l)})$, then $w(G_L) = \sum_l \sum_s w(z_s^{(l)})$. The weight of a single seed of rank l, $l = \overline{1, L}$ is estimated as $w(z_s^{(l)}) < q(a/b)^{l-1}b$, where $|Q| = q$ is the number of edges in seed H. Accordingly, the sum of the weights of all same-rank seed subgraphs of prefractal graph G_L is limited by inequality $\sum_s w(z_s^{(l)}) < qb(a/b)^{l-1}n^{l-1}$. As a result, the weight of the prefractal graph is limited as $w(G_L) < \sum_l qb(a/b)^{l-1}n^{l-1} < qb\frac{(na/b)^L-1}{na/b-1}$.

We now establish the lower bound of the estimate. The smallest (by weight) covering from the feasible solution set should be some spanning tree of prefractal graph G_L. To get the lower bound of the estimate by the first criterion, we only need to estimate the weight of the minimum spanning tree (Swamy, 1983) $T = (V_L, E_T)$ selected on prefractal graph G_L. Each edge of seed subgraph $z_s^{(l)}$ of rank l, according to the prefractal graph weighting rule, cannot be less than $(a/b)^{l-1}a$. Then $w(T_s^{(l)}) > (n-1)(a/b)^{l-1}a$, where $(n-1)$ is the number of edges of any spanning tree, while the total weight of the minimum spanning tree of the seed subgraph of same rank is $\sum_s w(T_s^{(l)}) > a(n-1)(a/b)^{l-1}n^{l-1}$, $l = \overline{1, L}$. For the weight of minimum spanning tree T, the following inequality holds: $w(T) > \sum_l a(n-1)(a/b)^{l-1}n^{l-1} > a(n-1)\frac{(na/b)^L-1}{na/b-1}$.

Thus, the value of function $F_1(x_2)$, from the covering constructed by the β_2 algorithm falls within the interval

$$F_1(x_2) \in [a(n-1)\frac{(na/b)^L-1}{na/b-1}; qb\frac{(na/b)^L-1}{na/b-1}].$$

Theorem 5. The β_2 algorithm selects connected spanning subgraph $J = (V_L, E_J) = \{C_1, C_2, \dots, C_k, \dots, C_K\}$, where C_k are the shortest paths, on prefractal graph $G_L = (V_L, E_L)$ generated by seed $H = (W, Q)$, $|W| = n$, $|Q| = q$, for which the adjacency of its old edges is not violated.

Proof. We define the operation of "gluing" together two arbitrary graphs $G' = (V', E')$ and $G'' = (V'', E'')$. Two vertices – $v' \in V'$ and $v'' \in V''$ – are selected for merging. Graph $\tilde{G} = (\tilde{V}, E' \cup E'')$, derived from graphs G' and G'' by merging vertices v' and v'' into some vertex $\tilde{v} \in \tilde{V}$ such that all edges incident to vertices v' and v'' become incident to vertex $\tilde{v}$, is called glued from graphs G' and G''.

Prefractal graph $G_L = (V_L, E_L)$, generated by seed $H = (W, Q)$, such that the adjacency of its old edges in the generation process is not violated. Then, prefractal graph G_L can be obtained by gluing together all $\frac{n^L-1}{n-1}$ seed subgraphs $Z(G_L) = \{z_s^{(l)}\}$, $l = \overline{1, L}$, $s = \overline{1, n^{l-1}}$ (Kochkarov, 1998). First, first-rank seed subgraph $z_1^{(1)}$ is glued at each of its vertices with second-rank seed subgraph $z_s^{(2)}$, $s = \overline{1, n}$. Further, each prefractal graph G_l generated in this way at the l-th step, $l = \overline{1, L-1}$, is glued at each of its vertices with seed subgraphs $z_s^{(l+1)}$, $s = \overline{1, n^{l-1}}$. As a result, we obtain prefractal graph G_L at the L-th step of which the adjacency of its old edges is not violated.

If connected spanning subgraphs $D_s^{(l)}$, $l = \overline{1, L}$, $s = \overline{1, n^{l-1}}$ are selected on all seed subgraphs $\{z_s^{(l)}\}$ of prefractal graph G_L, then graph D obtained by gluing together graphs $\{D_s^{(l)}\}$, similarly to generation of graph G_L described above, will become the spanning subgraph of graph G_L. This will happen due to the mutual correspondence of the edge numbers of seed subgraphs $\{z_s^{(l)}\}$, participating in the generation of graphs D and G_L.

The β_2 algorithm selects a spanning subgraph $J_s^{(l)} = (V_s^{(l)}, E_{J_s^{(l)}})$, consisting of a set of simple shortest paths $\{C_1, C_2, \dots, C_k, \dots, C_{K_{J_s^{(l)}}}\}$, on each seed subgraph $z_s^{(l)} \in Z(G_L)$, $l = \overline{1, L}$, $s = \overline{1, n^{l-1}}$ of prefractal graph $G_L = (V_L, E_L)$.

Recall that all paths $\{C_1, C_2, \dots, C_k, \dots, C_K\}$ forming covering $J = (V_L, E_J)$ are either paths of coverings $J_s^{(l)} = (V_s^{(l)}, E_{J_s^{(l)}})$, $l = \overline{1, L}$, $s = \overline{1, n^{l-1}}$, or consist of paths of these coverings. In both cases, all the paths are the shortest. In the first case, the paths are shortest thanks to the LMPF algorithm, and in the second, this is a consequence of the special way of defining prefractal graph G_L (the adjacency of its old edges is not violated).

Thus, covering $J = (V_L, E_J)$ consists of many simple paths $\{C_1, C_2, \dots, C_k, \dots, C_K\}$, and each path $C_k = \{v_k, u_k\}$ is the shortest $k = \overline{1, K}$, among all possible paths between vertices $v_k, u_k \in V_L$ of prefractal graph G_L.

4 DISCUSSION

The proposed mathematical model of a large-scale transport network is based on the apparatus of the theory of fractal graphs (Kochkarov, 1998). Let L be the rank of the simulated system, which can correspond to a certain level of the hierarchical structure of the administrative-territorial administration of the region (Comtois, 2013). The mathematical model of the road map is constructed in the form of the trajectory of a prefractal graph generated by a seed set $H = \{H_1, H_2, \dots, H_t, \dots, H_T\}$. Consider the process of building a transport network using the example of Russian roads. Geographically, Russia consists of 8 federal districts (see Figure 2). At the first stage, the $G_1 = H_1$ seed is a multigraph in which the federal roads correspond to the edges and the federal districts correspond to the vertices (see Figure 3). Further, in G_1, each vertex is replaced by a seed corresponding to the regions within the federal district. The structure obtained in the second stage corresponds to the prefractal graph G_2 (see Figure 4). In the next step, each vertex is replaced by a set of seeds, corresponding to regional or municipal districts. This process continues until the necessary level of hierarchy of the system under study is reached.

Figure 2: An example of a two-level hierarchy of the territorial division of the Russian Federation is presented. Eight federal districts of the Russian Federation designated 1-8, which consist of 85 constituent entities of the Russian Federation.

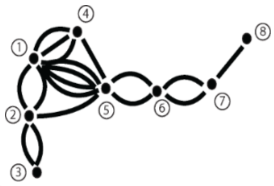

Figure 3: The G_1 multigraph is presented, where the federal districts correspond to the vertices, and the federal roads correspond to the edges.

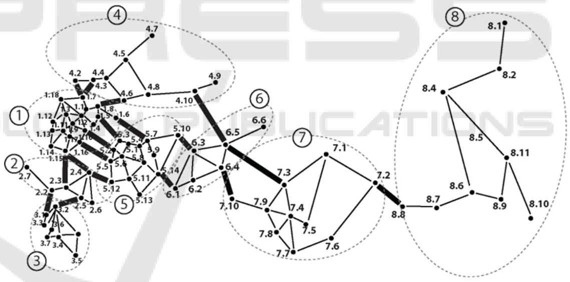

Figure 4: The prefractal graph G_2 of rank L=2 is obtained from the multigraph G_1, in which each vertex is replaced by seeds corresponding to the structure of roads between the federal subjects belonging to the federal district. Bold edges are edges of rank $L = 1$ (federal highways), the remaining edges belong to rank $L = 2$ (roads of federal subjects).

As the whole system of transport routes, we took the coverage of the prefractal graph consisting of paths corresponding to some routes. All necessary requirements and restrictions imposed on routes are expressed as a vector-valued objective function.

The use of prefractal graphs as a model of a large-scale transport network can significantly reduce the computational complexity of algorithms for finding optimal solutions. Comparing the computational complexity of the LMPF and β_2 algorithms on prefractal graph G_L, we obtain $O(N^5) < O(Nn^5)$.

Therefore, the computational complexity of β_2 is n^{L-5} times less than the computational complexity of the LMPF algorithm.

It is worth noting that it is convenient to construct parallel algorithms on prefractal graphs.

5 CONCLUSIONS

In the article, an approximate algorithm was used, which is called the algorithm with estimates. The search for efficient and accurate methods for many NP-hard or intractable problems has no practical sense. In this situation, we are forced either to proceed to the study of more particular problems and to search for low-laborious algorithms for them, or to build approximate algorithms. This gives rise to an approach to algorithmic problems, which is called "algorithms with estimates". We are talking about a vector assessment of the quality of algorithms. Criteria, i.e. the components of this vector function (i.e., estimates) are computational complexity, accuracy, memory size, size of the region within which the desired solution (many alternatives) is almost always obtained at the output of the algorithm, etc.

The constructed model and the algorithm for allocating maximum routes in terms of inclusion makes it possible to effectively solve the problem of route planning in large-scale transport networks.

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