Non-linear Motorcycle Dynamic Model for Stability and Handling Analysis with Roll Motion and Longitudinal Speed Regulation

Vincenzo Maria Arricale[a], Renato Brancati[b], Francesco Carputo[c], Antonio Maiorano[d] and Guido Napolitano Dell'Annunziata[e]
Department of Industrial Engineering, University of Naples "Federico II", Via Claudio 21, Naples, Italy

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Abstract: The use of computer simulations in motorcycle engineering makes it possible both to reduce designing time and costs and to avoid the risks and dangers associated with experiments and tests. The multi-body model for computer simulations can be built either by developing a mathematical model of the vehicle or by using commercial software for vehicle system dynamics. Even though the first method is more difficult and time-consuming than the second, maximum flexibility in the description of the features of the model can be obtained only by using an analytical model. Moreover, mathematical modelling has a high computation efficiency, whereas multi-body software requires a lot of time to carry out simulations. For the reasons above, the aim of this work was to develop a mathematical model of a motorcycle.

1 INTRODUCTION

The goal of many inventors over the past six centuries was to discover a device for fast and easy road transport. The invention of motorcycles began after the development of bicycles and engines. In fact, the first motorcycles were merely bicycles with small engines thrust into the frame. Nowadays, as one of the world's most popular means of transport, the motorcycle is not the early period monster that was made of metal and solid wood; it is rich in variety, advanced technology and well-made. Compared with other methods of transport, it has unparalleled advantages: it is economical, convenient and fast way to travel far away (Sharp et al., 2001; Herlihy, 2004)

During nearly 130 years of development, pioneering builders have exhausted their own intelligence and have created numerous milestone achievements, leaving their name in the history books. From the twenties of the 20th century to the present, improvements have become the main theme of the development of motorcycles (Limebeer et al., 2002). The modeling and control of a motorcycle are different from the process for a bicycle for three main reasons. First of all, the weight of a motorcycle is much larger than that of a bicycle; the difference is about ten times (Limebeer et al., 2002; Sharp, 1971). Secondly, due to the disparate weight, the rider has a different role to play during the model building process between a motorcycle and a bicycle. Thirdly, the speed is also hugely different. In fact, usually, the speed of a bicycle can be around 20 km/h whereas a modern motorcycle can achieve a top speed of about 230 km/h. For some sport motorcycles, the speed is able to reach even 300 km/h. Under this speed, the modeling process should not only consider the normal dynamics of the bicycle but should also consider the aerodynamics force analysis and the relevant thermal phenomena arising during the vehicle motion (Farroni et al., 2019). Over the years the theory of motorcycle dynamics has been perfected gradually, and some scholars have even done experimental research based on their own experience (Escalona et al., 2018; Sharp et al., 1980; Spierings, 1981).

The kinematic study of motorcycles is important, especially in relation to the effects on their dynamic behavior. Therefore, in this paper, in addition to the

[a] https://orcid.org/0000-0002-1292-1566
[b] https://orcid.org/0000-0003-2718-5706
[c] https://orcid.org/0000-0001-8436-2516
[d] https://orcid.org/0000-0002-9321-626X
[e] https://orcid.org/0000-0002-7293-4975

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kinematic study, some simple examples of the dynamic behavior of motorcycles are reported in order to show how kinematic peculiarities influence the directional stability and maneuverability of motorcycles (Bruni et al., 2020; Cossalter, 2014; Cossalter et al., 2002). Finally, the aim of this work is the development of a simple and effective motorcycle model easily implementable in control logics on board.

2 KINEMATICS OF MOTORCYCLES

Motorcycles are composed of a great variety of mechanical parts, including some complex ones. From a strictly kinematic point of view, by considering the suspensions to be rigid, a motorcycle can be defined as simply a spatial mechanism composed of four rigid bodies (Genta, 1997; Gillespie, 1996):

- The rear assembly (frame, saddle, tank and motor-transmission drivetrain group);
- The front assembly (fork, steering head and handlebars);
- The front wheel;
- The rear wheel.

These rigid bodies are connected by three revolute joints (the steering axis and the two-wheel axles) and are in contact with the ground at two wheels/ground contact points as shown in Fig. 1.Each revolute joint inhibits five degrees of freedom (DoF) in the spatial mechanism, whereas each wheel-ground contact point leaves three DoF free (Koenen, 1983; Schwab et al., 2004).

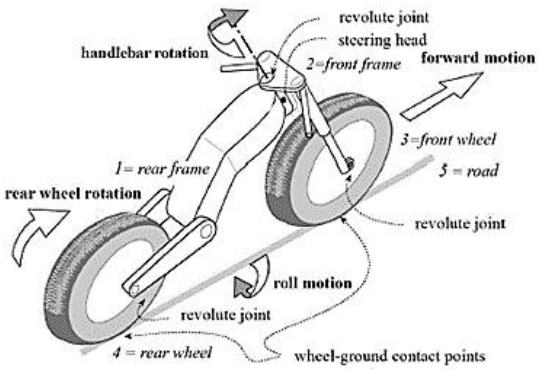

Figure 1: Kinematics representation of a motorcycle (Cossalter, 2014).

Considering the hypothesis of the pure rolling of tires on the road to be valid, it is easy to ascertain that each wheel, with respect to the fixed road, can only rotate around (Farroni et al., 2019; Kooijman et al., 2011):

- The contact point on the wheel plane (forward motion);
- The intersection axis of the motorcycle and road planes (roll motion);
- The axis passing through the contact point and the centre of the wheel (spin).

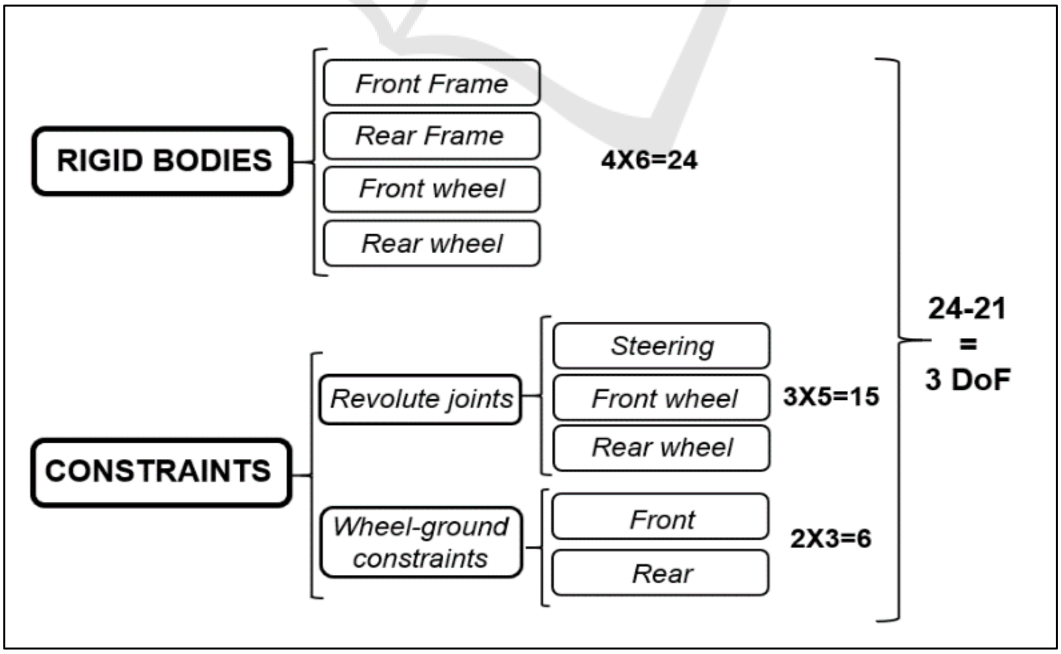

Figure 2: Degrees of Freedom of the schematized motorcycle model.

Table 1: List of symbols.

Symbol	***Description***	***Symbol***	***Description***
B	Rotation matrix between body-fixed reference frame and Euler-axis reference frame	R_{nz}	Translation of the centre of mass along the z-axis
C_x	Longitudinal stiffness of tires	T	Kinetic energy
C_α	Lateral stiffness of tires	T_{di}	Driving torque input
F_x	Longitudinal force of tires	V	Potential energy
F_y	Lateral force of tires	V_F	Vehicle forward velocity
F_z	Vertical force of tires	V_n	Centre of mass velocity
g	Gravity acceleration	v_r	Reference velocity
h_G	Centre of mass height	v	Acquired velocity
I	Mass moment of inertia tensor	x_G	Centre of mass coordinate along the x-axis
k_d	Coefficient for derivative term	α	Tire sideslip angle
k_i	Coefficient for integral term	γ_e	Rotational coordinates provided by the Euler Angles
k_p	Coefficient for proportional term	ε_x	Longitudinal slip ratio of tires
m	Motorcycle mass	θ	Rotation of the centre of mass around the y-axis
Q	Non-conservative force acting on the system	μ_{max}	Maximum friction coefficient
q_j	Generalized coordinate	φ	Rotation of the centre of mass around the z-axis
R_c	Turn radius	ψ	Rotation of the centre of mass around the x-axis
R_{nx}	Translation of the centre of mass along the x-axis	Ω	Angular yaw rate
R_{ny}	Translation of the centre of mass along the y-axis	ω_e	Angular velocity in the Euler axis-frame

In conclusion, a motorcycle's number of degrees of freedom is equal to three, given that the fifteen DoF inhibited by the three revolute joints and the six degrees of freedom eliminated by the two wheel-ground contact points must be subtracted from the four rigid bodies' twenty-four DoF, as summarized in Fig. 2 (Escalona et al., 2012; Lowell et al., 1982).

These three degrees of freedom may be associated with three principal motions (Schwab et al., 2004; Yi et al., 2009):

- Forward motion of the motorcycle (represented by the rear wheel rotation);
- Roll motion around the straight line which joins the tire contact points on the road plane;
- Steering rotation.

The rider manages all the three major movements, according to his personal style and skill: the resulting movement of the motorcycle and the corresponding trajectory (e.g. a curve) depend on a combination, in the time domain, of the three motions related to the three degrees of freedom. This generates one manoeuvre, among the thousands possible, which represents the personal style of the driver. These considerations have been formulated assuming that the tires move without slippage. However, in reality, the tire movement is not just a rolling process. The generation of longitudinal forces (driving and braking forces) and lateral forces requires some degree of slippage in both directions, longitudinally and laterally, depending on the road conditions. The number of degrees of freedom is therefore seven (Dugoff et al., 1969; Pacejka, 2006; Rajput et al., 2007; Seffen et al., 2001):

- Forward motion of the motorcycle;
- Rolling motion;
- Handlebar rotation;
- Longitudinal slippage of the front wheel (braking);
- Longitudinal slippage of the rear wheel (thrust or braking);

- Lateral slippage of the front wheel;
- Lateral slippage of the rear wheel.

This kinematic study refers to a rigid motorcycle, without suspensions and with the wheels fitted to non-deformable tires, schematized as two toroidal solid bodies with circular sections (Leonelli et al., 2015; Pacejka et al., 1991).

3 MOTORCYCLE DYNAMIC MODEL

The dynamic model of the motorcycle has been derived with the specific goal of a model simple but able to capture all the dynamics relevant of two-wheeled vehicles. For these purposes, this work presents a four degrees of freedom model that considers rear-wheel driving and the front wheel steering; three of those DoF refer to in-plane longitudinal, lateral and yaw vehicle body motions whereas the last DoF refers to out-plane roll body motion. Moreover, in this paper, a velocity tracking and stability control for agile manoeuvres using steering rotation and rear thrust as control inputs is presented (Sakai, 1990).

The analytical equations of motion are given by the Lagrangian approach: the result is a non-linear second-order ordinary differential equation (ODE) system in four unknowns: roll and yaw angles and the centre of mass coordinates in the plane road. The model considers both longitudinal and lateral forces exerted by the tires and has as inputs the steering torque and the rear wheel torque.

The motorcycle model's assumptions are:

$$\{q_j\} = \{R_{nx} \quad R_{ny} \quad R_{nz} \quad \psi \quad \theta \quad \varphi\} \tag{1}$$

The translational coordinates are the translation of the centre of mass measured parallel to the axes of the ground reference frame, whereas the rotational coordinates are provided by the Euler angles:

$$\{\gamma_e\} = \begin{Bmatrix} \psi \\ \theta \\ \varphi \end{Bmatrix} \tag{2}$$

The angular velocity in the Euler-axis frame as already stated is simply the time derivative of the Euler angles:

$$\{\omega_e\} = \frac{d}{dt}\{\gamma_e\} \tag{3}$$

Using the Lagrange equation, the motion equation can be obtained as:

$$\frac{d}{dt}\frac{\partial L(q_j,\dot{q}_j)}{\partial \dot{q}_j} - \frac{\partial L(q_j,\dot{q}_j)}{\partial q_j} = Q_{q_j} \tag{4}$$

where $L(q_j,\dot{q}_j) = T(q_j,\dot{q}_j) - V(q_j)$ is the Lagrangian function.

$T=T(q_j,\dot{q}_j)$ is the kinetic energy expressed in terms of generalized coordinates $\dot{q}_j$ and it is given by:

$$T=\frac{1}{2}\{V_n\}^T m\{V_n\} + \frac{1}{2}\{\omega_e\}^T[B]^T[I]\{\omega_e\}[B] \tag{5}$$

Applying Lagrange's equation, the equation of motion of the model are:

$$\begin{gathered} m[\ddot{x} + h_G \cos\theta \sin\psi\, \ddot{\theta} - (x_G \sin\psi \\ - h_G \sin\theta \cos\psi)\ddot{\psi} - (x_G \cos\psi \\ + h_G \sin\theta \sin\psi)\, \dot{\psi}^2 + h_G \sin\theta \cos\psi\, \dot{\theta}^2 \\ + 2h_G \cos\theta \cos\psi\, \dot{\theta}\dot{\psi}] = Q_x \end{gathered} \tag{6}$$

$$\begin{gathered} m[\ddot{y} - h_G \cos\theta \sin\psi\, \ddot{\theta} + (x_G \cos\psi \\ + h_G \sin\theta \sin\psi)\ddot{\psi} - (x_G \sin\psi \\ - h_G \sin\theta \cos\psi)\dot{\psi}^2 + h_G \sin\theta \cos\psi\, \dot{\theta}^2 \\ + 2h_G \cos\theta \sin\psi\, \dot{\theta}\dot{\psi}] = Q_y \end{gathered} \tag{7}$$

$$\begin{gathered} \ddot{\psi}[\left(I_{yy} + mh_G^2\right) \sin^2\theta + mx_G^2 \\ -(I_{yy} - I_{zz}) \cos^2\theta] + (I_{xz} \cos\theta \\ - h_G m x_G \cos\theta)\ddot{\theta} + m(x_G \cos\psi \\ h_G \sin\theta \sin\psi)\ddot{y} - m(x_G \sin\psi \\ h_G m \sin\theta \cos\psi)\ddot{x} + (h_G m x_G - I_{xz}) \sin\theta\, \dot{\theta}^2 \\ +[h_G^2 m + \left(I_{yy} - I_{zz}\right)] \sin 2\theta\, \dot{\theta}\dot{\psi} = Q_\psi \end{gathered} \tag{8}$$

$$\begin{gathered} \left(I_{xx} + mh_G^2\right)\ddot{\theta} + (I_{xz} \cos\theta - h_G m x_G \cos\theta)\ddot{\psi} \\ + mh_G \cos\theta\, (\sin\psi\, \ddot{x} - \cos\psi\, \ddot{y}) \\ - \frac{\dot{\psi}^2 \sin 2\theta}{2}\left(I_{yy} - I_{zz} + h_G^2 m\right) - h_G mg \sin\theta = Q_\theta \end{gathered} \tag{9}$$

The mathematical model presented is a non-linear Ordinary Differential Equation system which depends on the front and rear lateral forces and on the longitudinal.

To model the tire behaviour, it is possible to use physical models, divided into:

- physical-analytical model: which are physical models based on measurable physical quantities that have a closed-form solution, as described in (Romano et al., 2019);
- physical-numerical model: which are physical models that have not a closed-form solution. In this case, equations that regulate the phenomenon are very complex and cannot be solved analytically so that the problem is solved by using a computer and therefore the equation resolution is numerical.

Otherwise, it is possible to use interpolative empirical models, based on experimentation but which do not have a clear physical connotation. This model approximates a series of data obtained experimentally and generates a functional that interpolates and fits these data. A well-known empirical model is the Magic Formula (Dugoff et al., 1969; Hans B. Pacejka, 2006). This model provides an excellent fit for tire effort curves, which makes it more suitable for vehicle motion simulations. But at the same time, it provides a poor insight into tire behaviour. On one hand, empirical models rely on experimental measures to make the simulation more accurate, and on the other hand, physical models rely on physics to give more insight about tire behaviour. In regards to the calculation of the forces at the tire/road interface, complex multiscale friction models (Genovese et al., 2019) are based on the knowledge of the road surface and of the viscoelastic properties of tire tread (Genovese et al., 2020). Here, the Dugoff physical model to describe the friction forces between the tire/road interface is adopted. With a simple form, the Dugoff tire model can calculate the longitudinal and lateral tire force under pure longitudinal slip, pure side slip and combinate longitudinal-side slip situation. Dugoff developed an analytic model based on the classical analysis of Fiala (Sakai, 1990). Dugoff in his model assumed a constant friction coefficient and a constant vertical load distribution. These assumptions give:

$$F_x = C_x \frac{\varepsilon_x}{1+\varepsilon_x} f(\lambda) \tag{10}$$

$$F_y = C_\alpha \frac{\tan\alpha}{1+\varepsilon_x} f(\lambda) \tag{11}$$

In particular, λ and the function $f(\lambda)$ are described as:

$$\lambda = \frac{\mu_{max}\, F_z(1+\varepsilon_x)}{2\sqrt{(C_x\varepsilon_x)^2+(C_\alpha \tan\alpha)^2}} \tag{12}$$

$$f(\lambda) = \begin{cases} (2-\lambda)\lambda & \lambda<1 \\ 1 & \lambda\geq 1 \end{cases} \tag{13}$$

4 PROPORTIONAL INTEGRAL DERIVATIVE LONGITUDINAL CONTROLLER FOR SPEED REGULATION

The input variables of the motorcycle model are the steering angle and the driving torque at the rear wheel. The steering angle assigned during the simulation process, derives from experimental acquisitions, whereas the driving torque input to assign at the rear wheel has been evaluated through the implementation of a Proportional-Integral-Derivative (PID) controller, on the longitudinal velocity.

The PID controller is a closed-loop control system. It requires a sensor that is able to measure the controlled variable and sends the corresponding information to the controller. The controller receives as input the error made on the controlled variable, i.e. between the velocity signal and the target velocity; based on that error and using a proper control law evaluates the control signal to be sent to the actuator that applies the control force on the system in such a way the controlled variable follows the reference.

The PID controller generates an output that is given by the summation of three different contribute that are respectively proportional to the error between the reference signal and the output signal, to its derivative and to its integral over time.

Therefore, the driving torque input assumes the following form:

$$T_{di} = k_p\big(v_r(t)-v(t)\big) + k_d\big(\dot{v}_r(t)-\dot{v}(t)\big) + k_i\int_0^t \big(v_r(t)-v(t)\big)dt \tag{14}$$

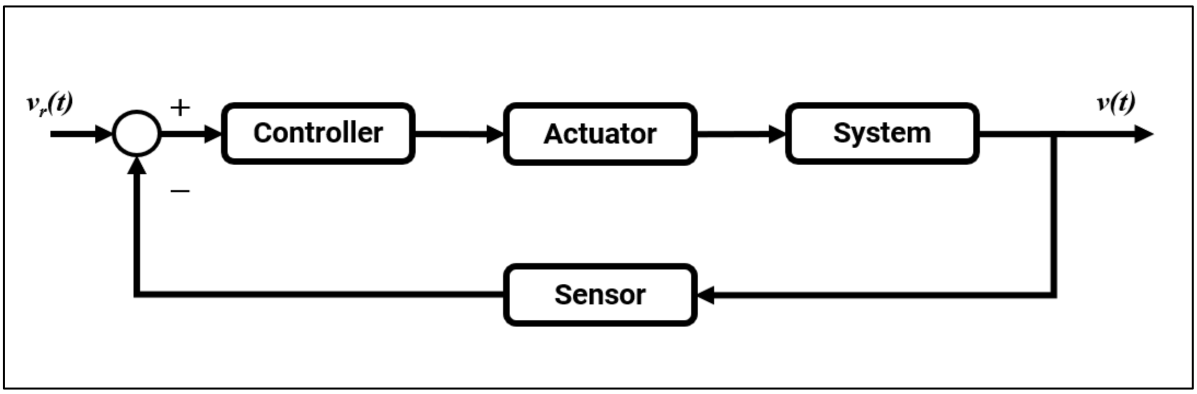

Figure 3: Closed loop control system - block diagram.

5 ROLL ANGLE STABILIZATION

Motorcycle is an unstable system that is kept in the stable zone by its rider that acts as a feedback controller acting on throttle, brake, steer, roll and by moving himself on the seat influencing the centre of gravity position, so the forces and the torques acting. Reproducing the driver's behaviour with a math model has always been very tricky since there are many variables that must be taken into consideration. Furthermore, there is no optimal strategy to adopt due to the fact that it depends on parameters related to the riding style of a specific riders.

Defining an optimal strategy for the roll motion control is a hard and complex thing. The usual approach is to balance the bike and to put it in the stable zone at the beginning; then, once it is stable, the body movement is controlled to exploit the best grip and speed. Many difficulties figure out on that strategy, for example, the body cannot be considered as a point mass but should at least be schematized like a solid with homogeneously distributed weight. Since this control part requires a lot of time to be tuned and to be simulated, the approach used here starts from the ideal roll angle.

The motorcycle, in steady turning, is subject to both a restoring moment, generated by the centrifugal force that tends to return the motorcycle to a vertical position, and to a tilting moment, generated by the weight force, that tends to increase the motorcycle's inclination or roll angle.

The following simplifying hypotheses have been introduced:

- the motorcycle runs along a turn of constant radius at constant velocity (steady-state conditions);
- the gyroscopic effect is negligible.

Considering the cross-section thickness of the tires to be zero, the moments equilibrium allows to derive the roll angle φ in terms of the forward velocity V_F and the radius of the turn R_c (measured from the centre of gravity to the turning axis):

$$\varphi = \tan^{-1}\left(\frac{R_c\Omega^2}{g}\right) = \tan^{-1}\left(\frac{V_F^{\,2}}{gR_c}\right) \qquad (15)$$

Where Ω indicates the angular yaw rate, while $V = \Omega R_c$ indicates the vehicle forward velocity.

In equilibrium conditions, the resultant of centrifugal and weight forces passes through the line joining the contact points of the tires on the road plane. This line lies in the motorcycle plane if the wheels have zero thickness and the steering angle is very small.

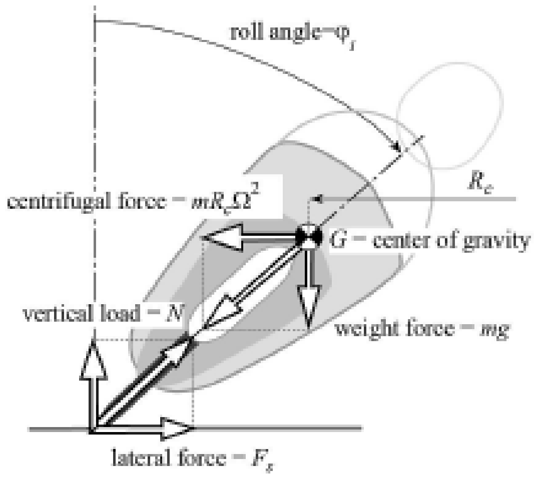

Figure 4: Steady turning: roll angle equipped with zero thickness tires (Cossalter, 2014).

In this work, therefore, to implement a control system that allowed the roll angle stabilization for the roll angle, the ideal roll angle is used as an input of the model to ensure stability. The rider presence is neglected, so no movement outside the plane of the motorcycle is considered, but it has been assumed that the rider rigidly attached to the saddle and always remains in the plane of symmetry of the motorcycle. Because of this assumption, the roll angle acquired by experimental measures and the one derived by imposing the steady turning conditions differ a bit from each other.

6 RESULTS

The data obtained from the model have been compared with those obtained from the experimental acquisitions given by a high-performance motorcycle manufacturing company; the industrial partner also provide all the information necessary to parametrize the model properly. The results have been then normalized for reasons of confidentiality.

The model inputs are the steer angle and the rear torque that were evaluated by the PID controller that, on the error between the target velocity and the measured velocity, applied a driving torque to the rear wheel of the motorcycle. For this reason, the profile velocity that is obtained is quite close to the velocity acquired experimentally, as illustrated in Fig. 5.

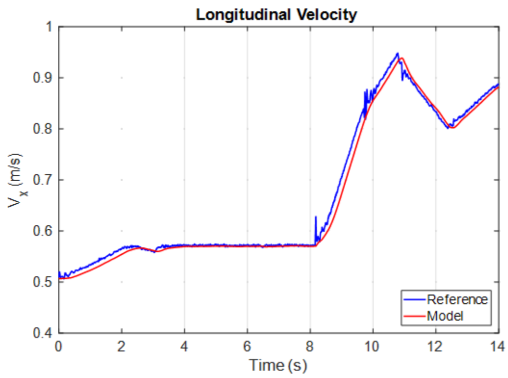

Figure 5: Velocity profile comparison.

It can be noticed that the roll angle of the model is globally greater than that acquired experimentally as expected since the rider is considered to be rigidly attached to the saddle and can't move his entire bodies to the interior of the turn to reduce the roll angle of the motorcycle.

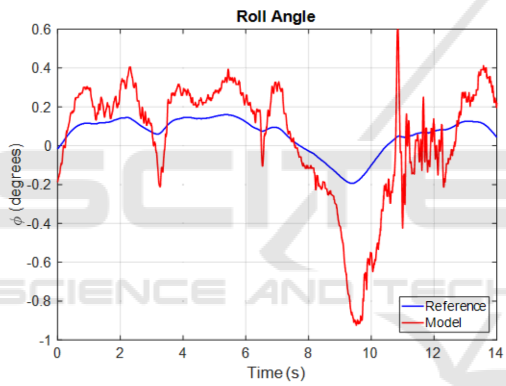

Figure 6: Roll angle comparison.

Moreover, there is a good match on the longitudinal force at the rear wheel and on the longitudinal acceleration between the model and the acquired data as shown in Fig. 7 and Fig. 8.

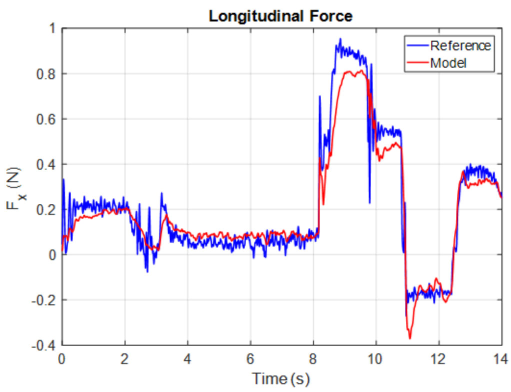

Figure 7: Longitudinal force comparison.

As it is possible to see in those figures, the model replies longitudinal force and acceleration correctly in all the dynamic conditions, except for the high acceleration zones; this is mainly due to the simplification introduced in the motorcycle schematization and parametrization.

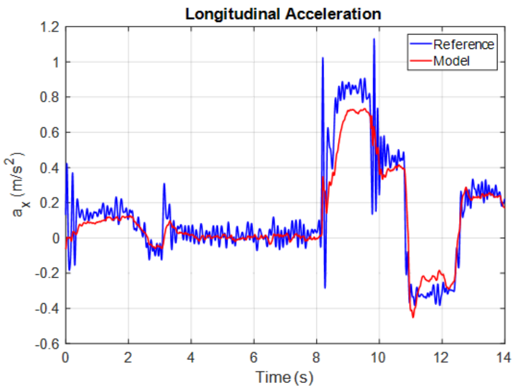

Figure 8: Longitudinal acceleration comparison.

7 CONCLUSIONS

In the present work, the mathematical model of a motorcycle with four degrees of freedom has been presented. The study has been carried out under the hypotheses of considering the front wheel steering and the rear wheel driving and braking. Any motion of the rider has been neglected, therefore the roll angle assigned during the simulation is equal to the ideal roll angle evaluated for the steady-state conditions. To simulate the behaviour of a driver who tries to reach a certain velocity has been implemented a proportional-integral-derivative controller PID which according to the error between the target velocity and the measured velocity apply a driving torque to the rear wheel of the motorcycle. For tire modelling, a physical model to describe the friction forces between the tire/road interface has been adopted, which is the Dugoff tire model.

The comparison has shown good reliability of the proposed model especially for what concerns the longitudinal dynamics, although have been found some differences between the lateral forces due to the basic hypotheses for the model of considering the ideal roll angle and to neglect any dynamic due to the rider behaviour.

The availability of non-linear equations represents an advantage with respect to the classical Jacobian linearization approach commonly used in literature. The model can be employed with an advanced non-linear model-based control system design and it can also be easily implemented on board thanks to its simplicity and robustness.

Further developments may consist in the realization of a more complex motorcycle model, considering it as a multi-body system of four bodies: the front and rear wheels, the rear assembly (including frame, engine and fuel tank), the front assembly (including steering column, handle-bar, and front fork). Moreover, a suspension system at the front and at the rear could also be considered; in this way, it could be analyzed the degrees of freedom of the motorcycle in the longitudinal plane such as the pitch motion and the vertical displacement.

The model developed could also be completed with a rider leaning model for the roll stabilization. In particular, it could be implemented a rider control in which the rider tries to stabilize the motorcycle by inclining left and right his upper body in such a way to not consider the roll angle fixed to its steady-state value, in order to have a better description of transverse dynamics.

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