

Logical Duality in Reactions of Amoeba Proteus

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Abstract: We consider some emergent properties in the motility of *Amoeba Proteus* in its reactions on attractants and repellents. In these reactions, we cannot define a logical composition $\Psi(x_1, \dots, x_n)$ as an n -place logical function Ψ over x_1, \dots, x_n , where each x_i is an atomic proposition or its negation. Each x_i should occur only without negation. Nevertheless, we face there a self-organised process with different reaction under stress or safety conditions.

1 INTRODUCTION

The swarm motion can be quite complicated. In order to simulate it, there were proposed many algorithms (Cuevas et al., 2013; Dorigo and Stutzle, 2004; Karaboga, 2005; Kennedy and Eberhart, 2001; Passino, 2002; Rajabioun, 1987). In the *Particle Swarm Optimization* (PSO) (Kennedy and Eberhart, 2001; Kennedy and Eberhart, 1995) it is assumed that the particles (agents) know (i) their best position ‘local best’ (lb) and (ii) their neighbourhood’s best position ‘global best’ (gb). The next position is determined by velocity. Let $x_i(t)$ denote the position of particle i in the search space at time step t , where t is discrete. Then the position x_i is changed by adding a velocity to the current position:

$$x_i(t+1) = x_i(t) + v_i(t+1),$$

where $v_i(t+1) = v_i(t) + c_1 r_1 (lb(t) - x_i(t)) + c_2 r_2 (gb(t) - x_i(t))$ and i is the particle index, c_1, c_2 are acceleration coefficients, such that $0 \leq c_1, c_2 \leq 2$, r_1, r_2 are random values (such that $0 \leq r_1, r_2 \leq 1$) regenerated every velocity update.

One of the possible PSO algorithms can be exemplified by the *bird flocking* (Reynolds, 1987; Reynolds, 1994). In flocks ‘local best’ and ‘global best’ of birds are defined by the following three rules: (i) collision avoidance (birds fly away before they crash into one another); (ii) velocity matching (birds fly about the same speed as their neighbours in the

flock); and (iii) flock centering (birds fly toward the center of the flock as they perceive it). So, the position of a bird i at time t is given by its placement x_i at time $t-1$ shifted by its current velocity v_i . This v_i is determined by the rules (i) – (iii).

Another type of algorithms was developed for explicating the motility of multinucleated giant amoebae *Physarum polycephalum* (Schumann, 2019; Tsuda et al., 2004; Tsuda et al., 2012). Here the position $x_i(t)$ of particle i at time step t changes due to biologically active matters: (i) attractants (pheromone and other good conditions) which attract the particles and (ii) repellents (strong light and other bad conditions) which repel the amoeboid particles. Some collisions (merging the particles) which are avoided in PSO are always possible for multiagent reactions of *Physarum polycephalum*. From this it follows that the plasmodium of *Physarum polycephalum* can respond to a contradictory situation (consisting of a mixture of an attractant and a repellent), in which there is no single optimal solution, differently. In other words, the plasmodia showed diverse responses that could not be explained by a simple model of the stimulus-response system (Shirakawa et al., 2020). In this paper, we consider an abstract model of *Amoeba Proteus* motility. This model also is based on reactions of amoeboid particles on external stimuli: attractants and repellents.

In this approach, we can implement some logical functions: negation as repelling the particles, conjunction as attracting both particles simultaneously, disjunction as attracting one or another particle. We can assume that in this way we can always define a logical composition $\Psi(x_1, \dots, x_n)$ as an n -place log-

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ical function Ψ over x_1, \dots, x_n , where each x_i is an atomic proposition or its negation. But it is impossible. The point is that in any logical composition $\Psi(x_1, \dots, x_n)$ each x_i should occur only without negation. Thereby, we are going to show that some emergent properties in the amoeboid motility appear due to the principal impossibility of composing negations for atomic propositions into a complex logical function (see Section 2). It is a more general abstract result than (Shirakawa et al., 2020). Nevertheless, we can define the logical duality for this motility, i.e. reactions on external stimuli under stress or safety conditions (Section 3). It means that we observe a proto-psychic structure in reactions with distinguishing stress reactions from safety reactions.

Our model of attraction is based on nutrition in amoebae. For them, nutrition is holozoic. It means that experiments can be performed with different organisms such as algae, rotifers or other protozoans, which are eaten by amoebae. These organisms can be captured by pseudopodia of amoebae to form a food cup and to be completely surrounded by cytoplasm. In our model, we test reactions of amoebae on several small organisms (two and more) located near.

2 REACTIONS OF AMOEBA PROTEUS ON ATTRACTANTS AND REPELLENTS

Let us consider basic amoeboid reactions of *Amoeba Proteus*. These reactions are swarm-like, because the membrane of this organism is very elastic and can be readily deformed at different places in reactions to different external signals. These deformations are caused by assembling and dissembling actin filament networks which are represented by different bunches or trees of F-actin proteins growing either towards attractants or in the opposite direction of repellents (Carlier, 1989; Carlier, 1991; Etienne-Manneville, 2004; Hill, 1981; Maly and Borisys, 2001; Mayne and Adamatzky, 2015; Mogilner and Oster, 1996; Moore et al., 1970; Mooseker and Tilney, 1975; Pollard and Cooper, 2009). So, an external signal influences the grow of actin filaments in one of the following two directions: either toward the signal (if it is coming from an attractant), see Fig.1, or counter to the direction of the signal (if it is coming from a repellent), see Fig.2. Hence, actin filament wavefronts are presenting some sensitive and reacting agents of *Amoeba Proteus*. In other words, active zones of assembling actin filaments which are responsible for changing the cell shape are considered agents. They appear

and disappear under different external conditions to change the membrane of amoeba.

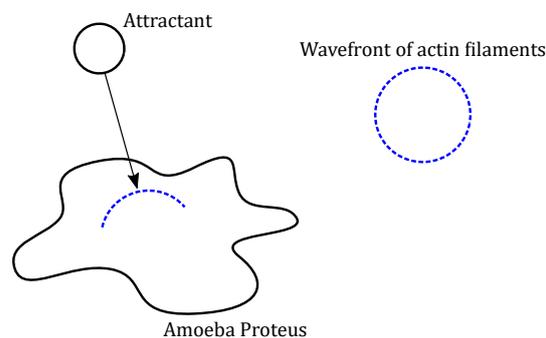


Figure 1: If we locate an attractant before the amoeba, then it causes assembling a wavefront of actin filaments which begins to make a pseudopodium – a deformation of membrane towards this attractant.

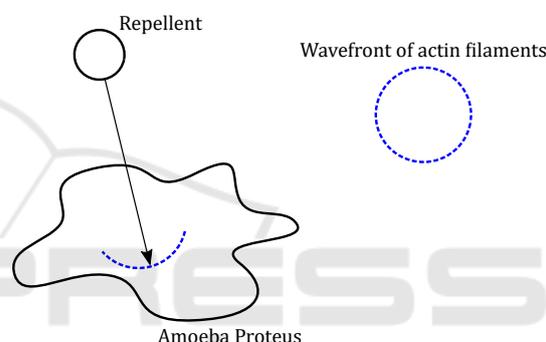


Figure 2: If we locate a repellent before the amoeba, then it causes assembling a wavefront of actin filaments which begins to make a pseudopodium in the opposite direction – a deformation of membrane counter to the direction of this repellent.

Let $(A_1, \mathbf{r}_1^A), \dots, (A_n, \mathbf{r}_n^A)$ be different active zones of an amoeba with their corresponding positions, where A_i is an agent (active zone) and \mathbf{r}_i^A is its position. So, each (A_i, \mathbf{r}_i^A) can be identified with a zone of growing actin filaments. They start to grow faster and to interconnect into bunches and trees through the adhesion due to external signals. Even if the signal is the same, it can be detected by (A_i, \mathbf{r}_i^A) differently because of their different location.

Similarly, let $(X_1, \mathbf{r}_1^X), \dots, (X_m, \mathbf{r}_m^X)$ denote the family of external signals, where X_i can be either an attractant At_i or a repellent Rp_i . Suppose that each signal X_i carries its own interaction coefficient $\chi_i \in [-1, 1]$. While $\chi_i > 0$ for attractive At_i and $\chi_i < 0$ for repulsive Rp_i , the precise value of an interaction coefficient depends on the particular signal. Hence, we propose that zone dynamics due to external signals can be described by a potential $V(\chi_j, \mathbf{r}_i^A, \mathbf{r}_j^X)$ and, by an abuse of notation, it holds

$$V(\chi_j, \mathbf{r}_i^A, \mathbf{r}_j^X) \equiv \chi_j V(|\mathbf{r}_i^A - \mathbf{r}_j^X|). \quad (1)$$

This gives rise to forces $\mathbf{F}_{ij}^A = -\chi_j \nabla V(|\mathbf{r}_i^A - \mathbf{r}_j^X|)$, therefore each zone (A_i, \mathbf{r}_i^A) is subjected to an external, resultant force

$$\mathbf{F}_i^A = \sum_{j=1}^m \mathbf{F}_{ij}^A = - \sum_{j=1}^m \chi_j \nabla V(|\mathbf{r}_i^A - \mathbf{r}_j^X|). \quad (2)$$

Let us sketch a model for the physical mechanism of an interaction between the system of amoeba and some external signals, driving the desired behaviour eventually. We have already stated that, in principle, the external signals are classified into those of attractive and repulsive character. In these reactions to external forces there is a small memory effect – a short Δt , when the amoeba continues its motoring stage even under new conditions. If discretized, the model would therefore satisfy the so-called Markov chain property, i.e. the future state depends on both the present and previous state.

Assume that we have two attractants A_{t_1} and A_{t_2} before the amoeba and these attractants have different power of pheromone (intensity) χ_1 and χ_2 , respectively, see Fig.3. As a result, we have two appropriate forces \mathbf{F}_{11}^A and \mathbf{F}_{22}^A made on active zones A_1 and A_2 , respectively. Both attractants are placed close together and if we add their two force vectors $\mathbf{F}_{11}^A + \mathbf{F}_{22}^A$, then the sum of these vectors is obtained as a quite long new vector. We know experimentally that under this location of two attractants the amoeba will try to occupy both of them simultaneously. This kind of behaviour corresponds to the conjunction of both attractants. So, we can introduce the following rule: (i) the longer the sum of two force vectors $\mathbf{F}_{11}^A + \mathbf{F}_{22}^A$ is, the more appropriate to conjunction the behaviour is; (ii) the shorter the sum of two force vectors $\mathbf{F}_{11}^A + \mathbf{F}_{22}^A$ is, the more appropriate to disjunction the behaviour is.

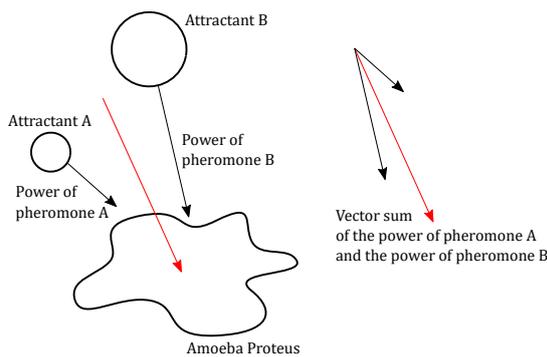


Figure 3: The addition of two vectors denoting the power of intensity of two pheromone pieces A and B.

Suppose, (A_1, \mathbf{r}_1^A) and (A_2, \mathbf{r}_2^A) denote two actin filament wavefronts of amoeba and X_1 and X_2 are two external signals with two appropriate forces \mathbf{F}_{11}^A and \mathbf{F}_{22}^A on A_1 and A_2 , respectively. Define the

fuzzy membership function $\mu_{(A_i, \mathbf{r}_i^A)}(x) = \alpha \in [0, 1]$ for $i = 1, 2$ with the following meaning: an actin filament x with the location \mathbf{r}^x belongs to the actin filament wavefront A_i with a degree of membership (probability) α that corresponds to the force \mathbf{F}_{ij}^A of perceived external signal X_j at the zone A_i :

$$\mu_{(A_i, \mathbf{r}_i^A)}(x) = \frac{-\chi_j \nabla V(|\mathbf{r}^x - \mathbf{r}_j^X|)}{-\chi_j \nabla V(|\mathbf{r}_i^A - \mathbf{r}_j^X|)} = \frac{\mathbf{F}_{ij}^x}{\mathbf{F}_{ij}^A}, \text{ see (2), where } -\chi_j \nabla V(|\mathbf{r}_i^A - \mathbf{r}_j^X|) \neq 0, \text{ i.e. we suppose that there is a force } \mathbf{F}_{ij}^A \text{ on } A_i \text{ indeed.}$$

Let us define a fuzzy set $\tilde{A}_i = \{x: \mu_{(A_i, \mathbf{r}_i^A)}(x) > 0\}$. Its complement $\neg \tilde{A}_i$ is as follows: $\neg \tilde{A}_i = \{x: \mu_{(A_i, \mathbf{r}_i^A)}(\neg x) = 1 - \mu_{(A_i, \mathbf{r}_i^A)}(x)\}$. Now, we can define intersection and union for \tilde{A}_1 and \tilde{A}_2 :

$$\text{intersection: } \tilde{A}_1 \cap \tilde{A}_2 = \tilde{A}_3 = \{z: \mu_{(A_3, \mathbf{r}_3^A)}(z) = \mu_{(A_3, \mathbf{r}_3^A)}(x \wedge y) = \left(\frac{\mathbf{F}_{11}^x}{\mathbf{F}_{11}^A + \mathbf{F}_{22}^A} + \frac{\mathbf{F}_{22}^y}{\mathbf{F}_{11}^A + \mathbf{F}_{22}^A} - \frac{\mathbf{F}_{11}^x}{\mathbf{F}_{11}^A} \cdot \frac{\mathbf{F}_{22}^y}{\mathbf{F}_{22}^A} \right) > 0, x \in \tilde{A}_1, y \in \tilde{A}_2\}.$$

$$\text{union: } \tilde{A}_1 \cup \tilde{A}_2 = \tilde{A}_3 = \{z: \mu_{(A_3, \mathbf{r}_3^A)}(z) = \mu_{(A_3, \mathbf{r}_3^A)}(x \vee y) = \frac{\max(\mathbf{F}_{11}^x, \mathbf{F}_{22}^y)}{\max(\mathbf{F}_{11}^A, \mathbf{F}_{22}^A)} > 0, x \in \tilde{A}_1, y \in \tilde{A}_2\}.$$

After that we define the order as follows: $\tilde{A}_i \subseteq \tilde{A}_j$ if and only if for all $x \in \tilde{A}_i$ its membership function $\mu_{(A_i, \mathbf{r}_i^A)}(x) \leq \mu_{(A_j, \mathbf{r}_j^A)}(x)$. According to this definition, $(\tilde{A}_1 \cap \tilde{A}_2) \subseteq (\tilde{A}_1 \cup \tilde{A}_2)$.

3 LOGICAL DUALITY

Suppose that f is an n -place logical composition of fuzzy sets $\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n$. Another n -place two-valued logical composition f' is said to be *dual* (or *logically dual*) to f if and only if either

$$f'(\tilde{A}_1, \dots, \tilde{A}_n) \subseteq f(\tilde{A}_n, \dots, \tilde{A}_1)$$

or

$$f(\tilde{A}_1, \dots, \tilde{A}_n) \subseteq f'(\tilde{A}_n, \dots, \tilde{A}_1).$$

According to this definition, if f' is dual to f , then f is dual to f' . So, the duality is always mutual.

Let us notice that intersection and union defined above are dual to each other:

$$(\tilde{A}_1 \cap \tilde{A}_2) \subseteq (\tilde{A}_1 \cup \tilde{A}_2).$$

Now, let us introduce a standard propositional logical language consisting of propositional variables p, q, r, \dots and logical connectives: \neg (negation), \wedge (conjunction), \vee (disjunction). The semantics of this language is as follows.

atomic proposition: let p be an atomic proposition and $m_{\chi_i,t}^{A_i}$ be its truth evaluation, then $m_{\chi_i,t}^{A_i}(p) = \top$ if and only if the set \tilde{A}_i for the active zone A_i and interactive coefficient χ_i at the time step t is not empty; otherwise $m_{\chi_i,t}^{A_i}(p) = \perp$;

negation: let p be a formula and $m_{\chi_i,t}^{A_i}$ be its truth evaluation, then $m_{\chi_i,t}^{A_i}(\neg p) = \top$ if and only if the set \tilde{A}_i for the active zone A_i and interactive coefficient χ_i at the time step t is not empty and its force F_i^A is negative; otherwise $m_{\chi_i,t}^{A_i}(\neg p) = \perp$;

conjunction: let p, q be two formulas and $m_{\chi_i,\chi_j,t}^{A_i,A_j}$ be their truth evaluation, then $m_{\chi_i,\chi_j,t}^{A_i,A_j}(p \wedge q) = \top$ if and only if the set $\tilde{A}_i \cap \tilde{A}_j$ for active zones A_i, A_j and interactive coefficients χ_i, χ_j at the time step t is not empty; otherwise $m_{\chi_i,\chi_j,t}^{A_i,A_j}(p \wedge q) = \perp$;

disjunction: let p, q be two formulas and $m_{\chi_i,\chi_j,t}^{A_i,A_j}$ be their truth evaluation, then $m_{\chi_i,\chi_j,t}^{A_i,A_j}(p \vee q) = \top$ if and only if the set $\tilde{A}_i \cup \tilde{A}_j$ for active zones A_i, A_j and interactive coefficients χ_i, χ_j at the time step t is not empty; otherwise $m_{\chi_i,\chi_j,t}^{A_i,A_j}(p \vee q) = \perp$;

On the basis of logical duality, we can define contrary, subcontrary, subaltern, and contradictory logical functions:

contrary: two functions h and h' are *contrary* if and only if $(h \wedge h') \equiv \perp$, but not always $(h \vee h') \equiv \top$;

subcontrary: two functions h and h' are *subcontrary* if and only if $(h \vee h') \equiv \top$, but not always $(h \wedge h') \equiv \perp$;

subaltern: a function h is *subaltern* to h' if and only if $(h' \Rightarrow h) \equiv \top$;

contradictory: two functions h and h' are *contradictory* if and only if $(h \vee h') \equiv \top$ and $(h \wedge h') \equiv \perp$.

Now, let us show that $(p \wedge q \wedge \dots \wedge r)$ can be interpreted as ‘stress from p, q, r ’ and $(p \vee q \vee \dots \vee r)$ as ‘safety from p, q, r ’. Then we can construct a square of opposition, see Fig.4, 5, where

contrary: $(p \wedge q \wedge \dots \wedge r)$ and $\neg(p \vee q \vee \dots \vee r)$ are *contrary*;

subcontrary: $\neg(p \wedge q \wedge \dots \wedge r)$ and $(p \vee q \vee \dots \vee r)$ are *subcontrary*;

subaltern: $(p \vee q \vee \dots \vee r)$ is *subaltern* to $(p \wedge q \wedge \dots \wedge r)$ as well as $\neg(p \wedge q \wedge \dots \wedge r)$ is *subaltern* to $\neg(p \vee q \vee \dots \vee r)$;

contradictory: $(p \wedge q \wedge \dots \wedge r)$ and $\neg(p \wedge q \wedge \dots \wedge r)$ are *contradictory* as well as $(p \vee q \vee \dots \vee r)$ and $\neg(p \vee q \vee \dots \vee r)$.

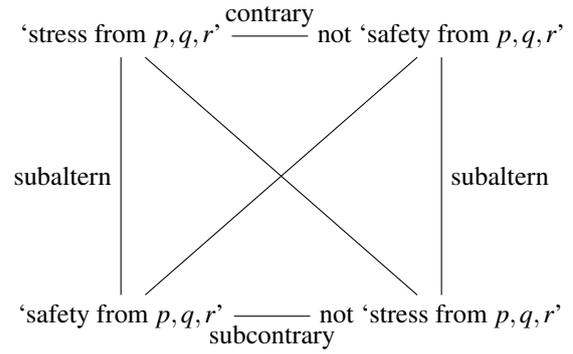


Figure 4: The square of opposition for the expressions ‘safety from p, q, r ’ and ‘stress from p, q, r ’.

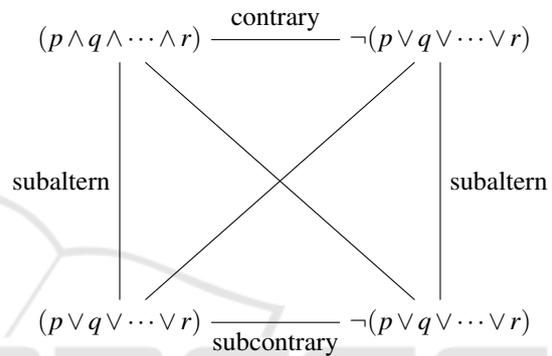


Figure 5: The square of opposition for the expressions $(p \cup q \cup \dots \cup r)$ and $(p \cap q \cap \dots \cap r)$.

In this square of opposition, the predicates ‘stress from p, q, r ’ and ‘safety from p, q, r ’ are considered dual: if ‘stress from p, q, r ’ holds true, then ‘safety from p, q, r ’ holds true. It is shown (Schumann, 2019) that in each swarm networking, including even networks of actin filaments in one cell, there are two basic reactions to outer stimuli: *lateral activation* (a reaction under safety) and *lateral inhibition* (a reaction under stress). The *lateral activation* is a reaction of swarm particles (such as active zones of *Amoeba Proteus*) to outer stimuli, according to which different particles are not concentrated on the same stimuli. As a result, we observe a decreasing of the intensity of the external signals and the contrast of the signals is made less visible. The *lateral inhibition* is a reaction of swarm particles (such as active zones of *Amoeba Proteus*) to external stimuli, according to which different particles are concentrated on the same stimuli. This has led us to an increasing of the intensity of the outer signals and the contrast of the signals is made more visible. The amoebae of *Amoeba Proteus* follow the lateral activation if they detect normal attractants and they follow the lateral inhibition if they face standard repellents (Schumann, 2019).

4 CONCLUSION

To sum up, we see that the amoebae of *Amoeba Proteus* realize a kind of logical duality in their reactions towards outer stimuli p, q, r , since either they behave under lateral activation and realize ‘safety from p, q, r ’ or they can behave under lateral inhibition and realize ‘stress from p, q, r ’, see Fig.4–5. In the meanwhile, the transmission between stress and safety is smooth and it depends upon force vectors.

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