Synergy Conformal Prediction for Regression

Niharika Gauraha and Ola Spjuth
Department of Pharmaceutical Biosciences, Uppsala University, Uppsala, Sweden

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Abstract: Large and distributed data sets pose many challenges for machine learning, including requirements on computational resources and training time. One approach is to train multiple models in parallel on subsets of data and aggregate the resulting predictions. Large data sets can then be partitioned into smaller chunks, and for distributed data sets the need for pooling can be avoided. Combining results from conformal predictors using synergy rules has been shown to have advantageous properties for classification problems. In this paper we extend the methodology to regression problems, and we show that it produces valid and efficient predictors compared to inductive conformal predictors and cross-conformal predictors for 10 different data sets from the UCI machine learning repository using three different machine learning methods. The approach offers a straightforward and compelling alternative to pooling data, such as when working in distributed environments.

1 INTRODUCTION

Data has become one of the key assets of many organizations, and the amount of data continues to increase in virtually all domains; sometimes referred to as Big Data (Hashem et al., 2015; Stephens et al., 2015; Gandomi and Haider, 2015). Training statistical (machine) learning models on large data sets can be challenging from different perspectives (Elish and danah boyd, 2018). For example, large data sets require substantial computational hardware, and in some cases large data can be difficult to move into a single computer (Zhou et al., 2017). A multitude of approaches have been developed for machine learning on large data sets (Zhou et al., 2017; Peteiro-Barral and Guijarro-Berdiñas, 2013). The most popular methodology is probably to train a global model in distributed environments that preserves data locality, using frameworks such as Apache Spark (Meng et al., 2016). Another methodology is to distribute (partition) data into smaller units and train multiple models in parallel and aggregate predictions (Vapnik and Izmailov, 2016).

Conformal Prediction is a relatively recent methodology where conformal predictors are built on top of standard machine learning algorithms and complement the predictions with valid measures of confidence (Vovk et al., 2005). The two main approaches are Transductive Conformal Prediction (TCP) (Vovk, 2013) and Inductive Conformal Prediction (ICP) (Papadopoulos, 2008) and they can be used for both classification and regression problems. The main drawback of using TCP is that it is computationally demanding; for every test example a re-training of the model is required. ICP was developed to overcome this issue; it has little computational overhead to the underlying algorithm but there is some loss in terms of informational efficiency due to a subset of training examples are set aside for calibration. To address this problem of information efficiency, ensembles of conformal predictors were introduced such as Cross Conformal Prediction (CCP) (Vovk, 2015) and Aggregated Conformal Prediction (ACP) (Carlsson et al., 2014a). The validity of TCP and ICP is proven in that they produce $1-\varepsilon$ expectation tolerance regions, where ε is the selected significance level (Vovk et al., 2005). However, the validity of ensembles of CPs has not been theoretically proven and has been discussed in (Carlsson et al., 2014a) and (Linusson et al., 2017).

Synergy Conformal Prediction (SCP) was recently proposed to address the validity problem of ensembles of conformal predictors that combines monotonic conformity scores instead of p-values (Gauraha and Spjuth, 2018). Its applicability has been shown in two scenarios; where data is partitioned in order to reduce the total model training time, and where an ensemble of different machine learning methods is used to improve the overall efficiency of predictions.

In this paper, we extend the SCP methodology for regression problems and explore its usefulness when partitioning large data and aggregating results, and also for working with distributed data without pooling into a single dataset. The paper is organized in the following way. In section 2, we outline the background

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concepts and notations used throughout the paper. In Section 3 we introduce synergy conformal prediction for regression and discuss its properties. In Section 4 we perform numerical analysis on a set of real data sets. In Section 5 we summarize our results, and in Section 6 we conclude and discuss implications and future outlook.

2 BACKGROUND

In this paper we mainly focus on regression problems and assume exchangeability of observations. The object space is denoted by $\mathcal{X} \subset \mathbb{R}^p$, where p is the number of features, and label space is denoted by $\mathcal{Y} \subset \mathbb{R}$. We assume that each example consists of an object and its label, and its space is given as $\mathcal{Z} := \mathcal{X} \times \mathcal{Y}$. In a classical regression setting, given ℓ data points $Z = \{z_1, ..., z_\ell\}$ where each example $z_i = (x_i, y_i)$ is labeled, we want to predict the label of a new object x_{new}.

In the conformal prediction setting, a nonconformity measure is a way of measuring the strangeness of an example in relation to the previous examples (Vovk et al., 2005). In our experiments, we use the standard regression nonconformity measure

$$\alpha_i = |y_i - \hat{y}_i|, \tag{1}$$

where $\hat{y}_i$ is the estimated output for the object x_i using regression algorithms such as Support Vector Regression (SVR) or Random Forests (RF).

Definition 1. Inductive Conformal Prediction (ICP) for Regression (Papadopoulos et al., 2002)
Given a training set of ℓ examples, $Z = \{z_1, ..., z_\ell\}$, drawn from an exchangeable distribution P, the training data is first divided into a proper training set $\{Z_T\}$ and a calibration set $\{Z_C\}$, where (T, C) is a partition of $\{1, ..., \ell\}$. The regression algorithm (e.g. SVR or RF) is applied to the proper training set, and using the decision rule a strangeness measure is associated with every example in the calibration set. In particular, we use the standard regression nonconformity measure

$$\alpha_i = |y_i - f(x_i)| = |y_i - \hat{y}_i|, i \in C \tag{2}$$

where $f : X \to Y$, is a prediction rule of the model trained on the proper training set Z_T, and $\hat{y}$ is the estimated output. To denote the dependency on the proper training set we write the decision rule as $f(Z_T, .)$. Let us denote by $\alpha_{(1)}, ..., \alpha_{(|C|)}$ the sequence of all α_i corresponding to the calibration set sorted in the ascending order. Let x_{new} (following the same distribution P) be the new object we want to predict, and let $\hat{y}_{new}$ be its estimated label using the same function $f(Z_T, .)$. The Prediction Interval (PI) for the new object x_{new} is then computed as

$$(\hat{y}_{new} - \alpha_{(s)},\ \hat{y}_{new} + \alpha_{(s)}), \tag{3}$$

where $s = \lfloor \varepsilon(|C| + 1) \rfloor$, and $\varepsilon \in (0, 1)$ is a chosen significance level, and $(1 - \varepsilon)$ is the confidence level. We denote an ICP by a tuple $(Z_C, f(Z_T, .))$, which consists of the calibration Z_C set and the decision rule given by its training set, Z_T.

As ICP uses only part of the training examples for training its underlying algorithm, and part of the examples for calculating the α-scores, it may result in lower informational efficiency. The Cross Conformal Predictor (CCP) for regression was introduced in (Papadopoulos, 2015) and is based on a cross-validation approach which helps overcome the informational loss of ICP; instead of only using a subset of the data as a calibration set, data is divided into K disjoint folds where each fold is treated as the calibration set, and the others as the proper training set. Other ensembles of conformal predictors for classification discussed in (Gauraha and Spjuth, 2018) have also been effectively applied to regression problems. Most of these ensembles methods for regression problems aim to get more informational efficient conformal predictors by combining p-values (or by combining prediction intervals). Since the combined p-values need not be uniformly distributed, as a result the final models are not guaranteed to be valid, see (Linusson et al., 2017) for more details. Another related method is Ensemble Cross Conformal Prediction (ECCP) (Beganovic and Smirnov, 2018) for classification settings, that has been shown to be computationally efficient. The synergy conformal prediction for classification was introduced in (Gauraha and Spjuth, 2018) which is an ensemble method that combines monotonic conformity scores, and is capable of producing valid prediction interval.

3 REGRESSION SYNERGY CONFORMAL PREDICTION

The objective of this paper is to extend SCP to regression problems, which was originally inspired by "Synergy of Monotonic Rules" proposed in (Vapnik and Izmailov, 2016). For classification problems, it has been shown that, by combining monotonic conformity scores of the calibration set and the test examples, SCP is capable of producing valid prediction regions. As with SCP for classification, for regression problems also one can suggest to combine nonconformity scores of the calibration set and test examples computed from multiple trained models on the

Algorithm 1: Synergy Conformal Predictor for Regression with data partitioning.

Input: training dataset:Z, object to predict:x_{new}, a regression algorithm:A, number of partitions: M
Output: Prediction Interval (PI)
Step1: Split the training set into two smaller sets, $\{Z_T\}$ and the calibration set $\{Z_C\}$. (T,C) is a partition of $\{1,...,|Z|\}$.
Step2: Split the set Z_T into M proper training sets, $\{Z_{T_m}, m = 1,\ldots,M\}$. $(T_1,...,T_M)$ is a partition of T.
Step3: For each part Z_{T_m}, train and construct the rule to generate nonconformity scores.
Step4: Compute the combined nonconformity scores across M models for each example in the calibration set, α_j, for $j \in C$, using eq. 5.
Step5: Compute the averaged estimated value across M regression models for the new object x_{new}, which results in $\hat{y}_{new}$.
Step6: Compute prediction interval, PI using eq 3.
return PI

individual partitions of the proper training set. However, for regression settings it is difficult to compute nonconformity score for a test example as the postulated output can be a real number. Therefore, we propose to combine predicted outputs for test examples for constructing the prediction intervals using Equation 3. Synergy conformal prediction for regression is described in the following.

Consider ℓ examples, $Z = \{z_1,...,z_\ell\}$, drawn from an exchangeable distribution P. Similar to ICP, the training data is first divided into the proper training set $\{Z_T\}$ and the calibration set $\{Z_C\}$, where (T,C) is a partition of $\{1,...,\ell\}$. Then the proper training data is further divided into M non-empty disjoint subsets and each subset $Z_{T_m}, m = 1,\ldots,M$ is then used for training. Here $(T_1,...,T_M)$ is a partition of T. The M predictive models trained on the individual partitions are then used to compute the nonconformity scores for the calibration set denoted by α_{mj}, for $j \in C$ and $m = 1,\ldots,M$. For example,

$$\alpha_{mj} = |y_i - f_m(x_i)|, \tag{4}$$

where $f_m(x)$ is the prediction rule defined by the predictive model trained on the m^{th} part of the training set. The aggregated nonconformity scores across models are then defined as

$$\alpha_j = \frac{1}{M}\sum_m \alpha_{mj}. \tag{5}$$

Let x_{new} (follows the same distribution P) be the object we want to predict, and let $\hat{y}$ be the averaged estimated value across models. The synergy conformal predictor corresponding to the tuple $(Z_C, f_1(Z_{T_1},.),...,f_m(Z_{T_M},.))$ is defined as a prediction interval (PI) as given by eq. (3).

The SCP method differs from most of the other ensemble methods discussed previously in that SCP combines nonconformity scores rather than combining the conformal p-values or prediction intervals. In the following, we discuss the validity property of Regression SCP.

Proposition 1. The synergy conformal predictor for regression is valid

Proof. For unpartitioned data, when the proper training set as a whole is used for training, in that case SCP is exactly ICP and hence valid (Papadopoulos et al., 2002). For partitioned training data, SCP can be viewed as a single ICP (hence valid), when the ensemble of M regression methods is considered as one function producing the (combined) nonconformity scores. To illustrate this, let us consider partitioning of the set Z_T, into M subsets, $Z_{T_1},\ldots,Z_{T_M}$, and let their corresponding decision rules be $f_1(Z_{T_1},.),\ldots,f_m(Z_{T_M},.)$ respectively. Define a new decision rule $f(Z_T,.)$ which combines the estimated values of an example z, $f(Z_T,z) = \frac{1}{M}\sum_{m=1}^{M} f_m(Z_{T_m},z)$. Then the nonconformity scores can be computed using the combined estimated values. The pair $\{Z_C, f(Z_T,.)\}$ forms an ICP corresponding to the new decision rule $f(Z_T,.)$, hence valid. □

4 EXPERIMENTS

We evaluate Regression SCP on ten regression datasets from UCI machine learning repository (Lichman et al., 2013), namely Boston Housing Data (Housing)Wine Quality (Wine), Parkinson Speech Dataset with Multiple Types of Sound Recordings (PD), Combined Cycle Power Plant (PowerPlant), Energy Efficiency (Energy), Concrete Compressive Strength (Concrete), Electrical Grid Stability Simulated Data (GridStability), Superconductivty Data (SuperConduct), Condition Based Maintenance of Naval Propulsion Plants (CBM) and SkillCraft1 Master Table Dataset (Game). Since ICP and CCP, running on whole datasets, are computationally expensive, we select randomly 2000 data points from larger datasets. In all our experiments, the data is scaled for target (or label) and for all features to be within

Table 1: Datasets from UCI repository that are used in the evaluation. Train refers to the number of examples in the training set, Calib refers to the number of examples in the calibration set, Test refers to the number of objects in the test set, Features refer to the number of features in the dataset.

Dataset	Train	Calib	Test	Features
Housing	282	122	102	13
Wine	1120	480	400	10
PD	582	250	208	26
PowerPlant	1120	480	400	4
Energy	429	185	154	8
Concrete	576	248	206	8
GridStability	1120	480	400	12
SuperConduct	1120	480	400	81
CBM	1120	480	400	16
Game	1120	480	400	18

the [0,1] range, as feature scaling improves the convergence of the underlying optimization method and target is scaled for better comparisons.

Four experiments were carried out (the experiments will be explained in greater detail further down), where each dataset was randomly divided into a training and test subset of sizes using a $80:20$ ratio. The training set was further (randomly) divided into proper training set and calibration set of sizes using a $70:30$ ratio. The specific breakdowns for training, calibration and test sets are given in Table 1. Then the proper training set was randomly partitioned into M equal parts, and a model was trained on each individual part. We used three machine learning algorithms: Support Vector Regression using linear kernel (SVR), Support Vector Regression using RBF kernel (RBF-SVR) (implemented in Python with LIBSVM) and Random Forest (RF) (implemented in Python). The corresponding hyper-parameters were learned using 5-fold cross-validation and the best value was used to train the final model on the entire dataset. The estimated target values $\hat{y}$ were computed for each example in calibration set and test set, and were averaged across models. Then the predictions interval (PI) was computed using the averaged predicted values according to Equation 3. The whole process was repeated 10 times.

To assess a conformal predictor we consider validity and efficiency(Vovk et al., 2016; Svensson et al., 2018). In our experiments, validity is empirically assessed in terms of calibration plots; the plot of the percentage of errors against $\varepsilon \in (0,1)$ being close to the bisector of the first quadrant with some statistical fluctuations. We use size of the prediction interval as the measure of efficiency; lower width implies better informational efficiency and hence indicates a 'better' model.

4.1 Experiment 1: Synergy of Conformal Prediction using the Same Machine Learning Algorithm on Partitioned Data

The objective of this experiment is to compare efficiency of SCP with ICP on partitioned and unpartitioned data, using linear (linear SVR) and non-linear (SVR with RBF kernel) regression methods. Inspired by the experiments in (Vapnik and Izmailov, 2016) we partition the proper training set into three equal partitions in the SCP method, and results for the test sets are shown in Table 2. To illustrate the quantitative difference between ICP_p, ICP and SCP, box plots for dispersion of efficiencies are presented in Figures 1a and 1b for SuperConduct dataset (similar plots are shown in the Appendix for the remaining datasets). We also perform this experiment with more partitions (5,7,9,11,15 and 20) for a couple of datasets selected randomly, and the results are reported in Table 3.

From Table 2 we observe that ICP on unpartioned data using non-linear regression method (RBF-SVR) performs best for each individual measures as expected, whereas both ICP_p linear and non-linear have somewhat larger intervals (lower efficiency) as compared to the corresponding SCPs for most of the cases, which is more prominent for the non-linear case. This pattern is repeated for the case when we have more partitions which is evident from Table 3.

4.2 Experiment 2: Synergy of Conformal Prediction using Different Machine Learning Algorithms on Partitioned Data

The objective of Experiment 2 is similar to Experiment 1, with the difference that multiple machine learning methods were used for the different partitions. We use the same setup as in the previous experiment with three equal partitions of the proper training set, but with three different machine learning algorithms: linear SVR, Random Forest (RF) and SVR using RBF kernel (RBF-SVR) for each partition. The results are reported in Table 4. To illustrate the quantitative difference between ICP_p and SCP, box plots are presented in Figure 1c for Super Conduct dataset. From Table 4 we observe that all ICP_p have lower efficiency as compared to the SCP for most of the datasets.

Table 2: Results from Experiment 1, Synergy of Conformal Prediction using the Same Machine Learning Algorithm on Partitioned Data. Efficiency is calculated at confidence level 90% for linear SVR and non-linear SVR with RBF kernel is used as the underlying machine learning algorithm. ICP_p corresponds to the partition with the best efficiency, ICP to the efficiency where the whole dataset is used, and SCP to the synergy conformal prediction with regression method.

	Linear			NonLinear		
Dataset	ICP_p	ICP	SCP	ICP_p	ICP	SCP
Housing	0.316	0.312	0.309	0.282	0.219	0.249
Wine	0.203	0.201	0.204	0.207	0.195	0.199
PD	1.605	1.519	1.465	1.603	1.545	1.441
PowerPlant	0.207	0.208	0.209	0.191	0.19	0.186
Energy	0.265	0.28	0.276	0.255	0.177	0.245
Concrete	0.441	0.446	0.443	0.356	0.275	0.325
GridStability	0.430	0.426	0.422	0.235	0.177	0.209
SuperConduct	0.483	0.466	0.478	0.400	0.351	0.377
CBM	0.380	0.321	0.367	0.022	0.018	0.021
Game	0.534	0.533	0.538	0.535	0.528	0.535

Table 3: Results from Experiment 1, using different partitions where non-linear SVR with RBF kernel is used as the underlying machine learning algorithm.

Housing	# Partitions	ICP_p	ICP	SCP
	5	0.322	0.219	0.278
	7	0.336	0.219	0.286
	9	0.382	0.219	0.312
	11	0.397	0.219	0.317
	15	0.426	0.219	0.336
	20	0.428	0.219	0.351
SuperConduct	# Partitions	ICP_p	ICP	SCP
	5	0.430	0.351	0.380
	7	0.450	0.351	0.397
	9	0.450	0.351	0.410
	11	0.488	0.351	0.419
	15	0.506	0.351	0.434
	20	0.539	0.351	0.452

4.3 Experiment 3: Synergy of Conformal Prediction using Different Machine Learning Algorithms on Unpartitioned Data

The objective of this study is to compare SCP with ICP and CCP trained on the whole training set, in order to show that using different machine learning algorithms for training the same data (proper training set) in SCP also improves the informational efficiency. In this experiment we use three different machine learning algorithms: linear SVR, RF and RBF-SVR; the results are reported in Table 5. The results of the individual ICPs and SCP are given in columns 2-4 and column 5 respectively. Column 6-8 shows

Table 4: Results from Experiment 2, Synergy of Conformal Prediction using Different Machine Learning Algorithms on partitioned Data. Efficiency is calculated at confidence level 90% for ICP_p and SCP with synergy of three different machine learning algorithms: linear SVR, RF and RBF-SVR. The SVR-ICP_p corresponds to the ICP on first partition with linear SVR as underlying algorithm, RF-ICP_p to the ICP on second partition with RF as underlying algorithm, RBF-SVR-ICP_p to the third partition with SVR with RBF as underlying algorithm, and SCP to the Synergy Conformal Prediction with Regression method.

Dataset	SVR-ICP_p	RF-ICP_p	RBF-SVR-ICP_p	SCP
Housing	0.316	0.294	0.317	**0.256**
Wine	0.212	0.299	0.216	**0.212**
PD	1.605	1.600	1.603	**1.393**
PowerPlant	0.210	0.205	0.191	**0.187**
Energy	0.276	**0.193**	0.278	0.220
Concrete	0.441	**0.309**	0.356	0.317
GridStability	0.432	0.394	0.238	**0.299**
SuperConduct	0.485	0.424	0.400	**0.382**
CBM	0.381	0.412	**0.022**	0.224
Game	0.534	0.567	0.537	**0.525**

the average informational efficiency of Cross Conformal Prediction (CCP) applied on the whole training set with three fold cross conformal prediction using linear SVR, RF and RBF-SVR as an underlying machine learning algorithm respectively. The prediction intervals for CCP were combined by taking the medians of the lower and upper bounds as suggested in (Park and Budescu, 2015). It is important to note that in this experiment the methods ICP (70:30), SCP (70:30) and CCP (2/3:1/3, due to three folds) get almost the same amount of training and calibration data,

and that SCP and ICP combines results from three different methods, and three folds respectively. To illustrate the quantitative difference between the ICP, SCP and CCP, box plots are presented in Figure 1d for Super Conduct dataset.

From Table 5 it is observed that efficiency of SCP on unpartioned data is higher than ICP and CCP for more than half of the datasets.

4.4 Experiment 4: Validity of Conformal Predictors

In this section we study the validity of ICP, SCP and CCP. We use the same setup as in Experiment 1, using Random Forest (RF) with 10 trees as the underlying machine learning algorithm. We also train three fold CCP using RF with 10 trees. Calibration plots are available in Figure 2, indicating that all models show little deviation from validity.

5 DISCUSSIONS

The aim of this paper was to develop SCP for regression and to explore its performance in different settings. In Experiment 1, we considered partitioned proper-training data with the same machine learning algorithm applied on each partition. As with SCP for classification, SCP for regression also succeeded in combining the models (see Table 2) and for the linear case achieved an efficiency that was on par with the individual best ICP_p, whereas in the non-linear case using SVR with RBF kernel SCP showed improved efficiency (lower median prediction interval width) for all datasets as compared with ICP_p. The difference between ICP trained on the entire dataset and the SCP regression methodology is not large in absolute numbers for the NonLinear case (Table 2), but we observe a decrease the efficiency of SCP with a larger number of partitions which is expected since the individual sub-models are then trained on less data. Nevertheless, SCP outperformed the best individual partition ICP_p in all cases. In Experiment 2, we considered partitioned proper training data with different machine learning algorithms applied on each partition. Table 4 shows results from the evaluation, and we note that in 7 out of 10 datasets, SCP has the lowest (best) efficiency compared to the other methods trained on individual partitions. We here argue that using SCP for integrating predictions using different machine learning models in most cases is superior than relying on an individual prediction methodology. This would resemble a real-world use case when different organizations have implemented different modeling methods, are not able to share data, but are able to share nonconfomity measures to make up an aggregated prediction. In Experiment 3, we considered unpartitioned proper training data with different machine learning algorithms, hence all individual parties have access to the entire dataset but use different underlying machine learning methods. Table 5 shows that for 6 out of 10 datasets, SCP has improved efficiency compared to the individual methods using ICP. Further, the results comparing SCP with CCP show that SCP integrating different machine learning methods is in all cases superior than using a three-fold CCP with a single machine learning method. In Experiment 4, results show that all method indicate valid models (close to the bisector of the first quadrant), with a small deviation observed for CCP. The attractive property for ICP to produce valid models while aggregating results over multiple models constitutes a clear improvement over previous aggregation methods for conformal predictors, such as reported in (Linusson et al., 2017; Carlsson et al., 2014b).

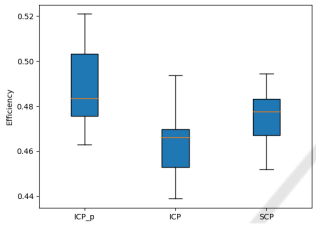
The experiments show in general that SCP has attractive properties for aggregating prediction intervals, evaluated on a set of benchmark datasets using different forms of data partitioning. While SCP do not live up to producing as efficient models as for pooling all data in one location and training an ICP, in almost all cases it is advantageous compared to the best individual model trained on partitioned data. This implicates that SCP has good applicability in distributed or federated settings, where data is located in different locations and cannot be pooled due to privacy, regulatory, or practical reasons. We also envision that SCP could be a foundation upon which implementations can train individual partition in parallel, such as in locality-aware Big Data frameworks. The main drawback of SCP is that it requires a calibration set that is shared between individual partitions. This is likely not a big concern when partitioning data, but it does constitute an additional step when working with data sources that are distributed.

6 CONCLUSIONS AND FUTURE DIRECTIONS

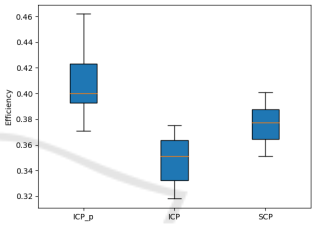
We introduced Synergy Conformal Prediction for regression and evaluated its validity and informational efficiency using various underlying machine learning algorithms. The key outcome is that SCP offers an alternative to pooling distributed data when using inductive conformal predictors, with reduced training time as models can be trained in parallel and predictions aggregated, while still preserving validity.

Table 5: Results from Experiment 3, Synergy of Conformal Prediction using Different Machine Learning Algorithms on Unpartitioned Data. Efficiency is calculated at confidence level 90% for ICP, SCP and CCP on unpartitioned data using three different machine learning algorithms (linear SVR, RF and RBF-SVR). Columns 2-4 shows results from ICP, the fifth column shows results for SCP for an ensemble of different machine learning algorithms , and columns 6-8 shows results for three fold CCPs.

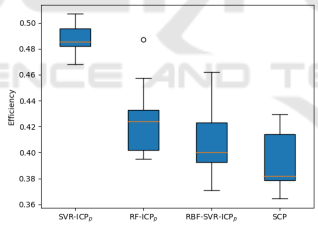
Dataset	SVR-ICP	RF-ICP	RBF-ICP	SCP	SVR-CCP	RF-CCP	RBF-CCP
Housing	0.312	0.233	0.219	**0.216**	0.316	0.241	0.224
Wine	0.201	0.258	0.195	**0.194**	0.209	0.256	0.199
PD	1.519	**1.400**	1.545	1.436	1.522	1.461	1.582
PowerPlant	0.208	0.196	0.190	**0.180**	0.205	0.195	0.186
Energy	0.28	0.190	0.177	**0.168**	0.294	0.188	0.184
Concrete	0.446	0.238	0.275	0.267	0.446	**0.233**	0.271
GridStability	0.426	0.315	**0.177**	0.251	0.419	0.324	0.180
SuperConduct	0.466	**0.340**	0.351	**0.340**	0.463	0.344	0.353
CBM	0.321	0.224	**0.018**	0.153	0.321	0.228	0.019
Game	0.533	0.533	0.528	**0.516**	0.534	0.533	0.531



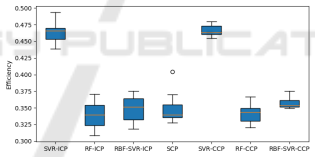
(a) Linear SVR, partitioned data



(b) RBF-SVR, partitioned data

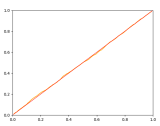


(c) Different ML Models

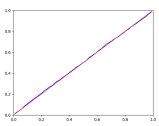


(d) Unpartitioned data

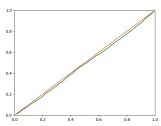
Figure 1: Results from Experiment 1, 2 and 3 comparing the efficiency between different methods for the SuperConduct dataset. (a and b): Experiment 1, comparison of individual ICPs trained on data partitions, ICP on the entire dataset, and SCP where (a) linear SVR is used, and (b) RBF SVR is used as the underlying machine learning algorithm; (c): Experiment 2, comparing individual ICPs on partitions and SCP where three different algorithms (linear SVR, RF and RBF-SVR) are used on partitioned training data; (d): Experiment 3 comparing ICP, SCP and CCP where three methods (linear SVR, RF and RBF-SVR) are used on unpartitioned training data.



(a) ICP



(b) SCP



(c) CCP

Figure 2: Illustration of validity of ICP, SCP and CCP for Super Conduct dataset using random forest (with 10 trees) for training all the underlying models.

The efficiency of SCP does not reach the level of ICP trained on pooled data, but efficiency is lower (lower median prediction interval width) compared to the predictions made on the best individual partition, indicating attractive properties in distributed and federated settings as a valid confidence predictor. Future directions when working on partitioned data include (i) studying the effect of the number and size of data partitions as well as overlapping partitions (ii) evaluating the effect of different nonconformity scores and different underlying machine learning algorithms with individual partitions.

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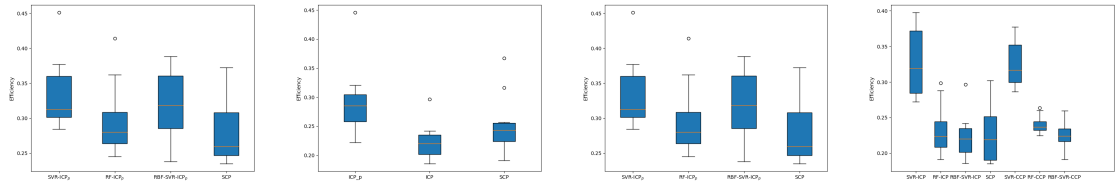
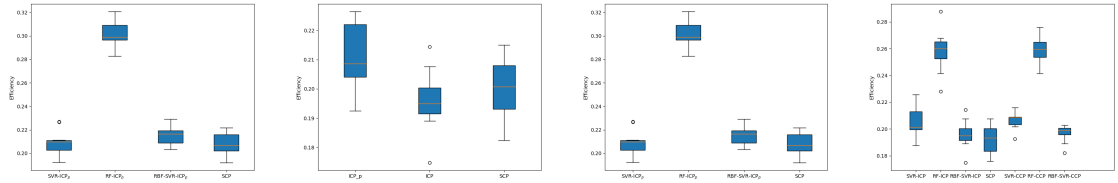
APPENDIX



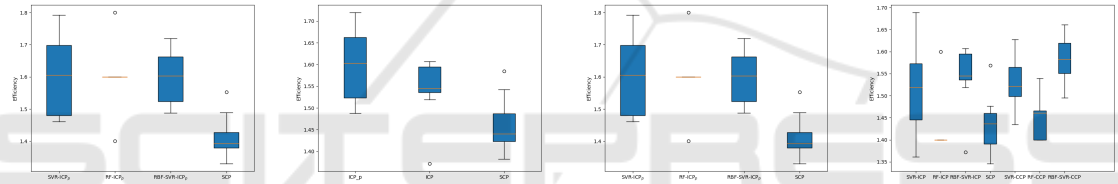
(a) Linear SVR (b) RBF-SVR (c) Different ML Models (d) Unpartitioned data

Figure 3: Results from Experiment 1, 2 and 3 comparing the efficiency between different methods for Housing dataset.



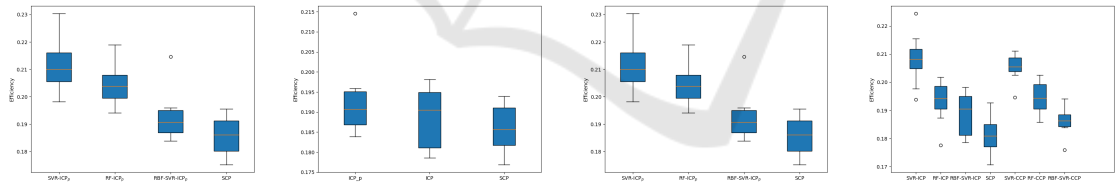
(a) Linear SVR (b) RBF-SVR (c) Different ML Models (d) Unpartitioned data

Figure 4: Results from Experiment 1, 2 and 3 comparing the efficiency between different methods for Wine dataset.



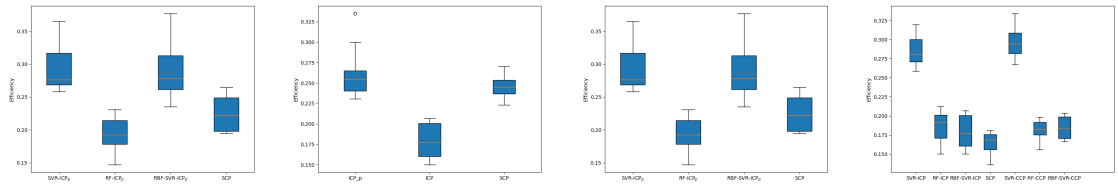
(a) Linear SVR (b) RBF-SVR (c) Different ML Models (d) Unpartitioned data

Figure 5: Results from Experiment 1, 2 and 3 comparing the efficiency between different methods for PD dataset.



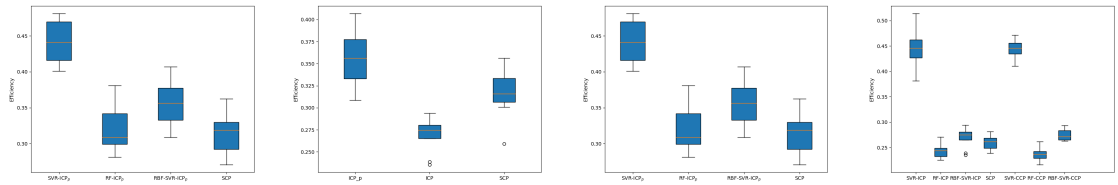
(a) Linear SVR (b) RBF-SVR (c) Different ML Models (d) Unpartitioned data

Figure 6: Results from Experiment 1, 2 and 3 comparing the efficiency between different methods for PowerPlant dataset.



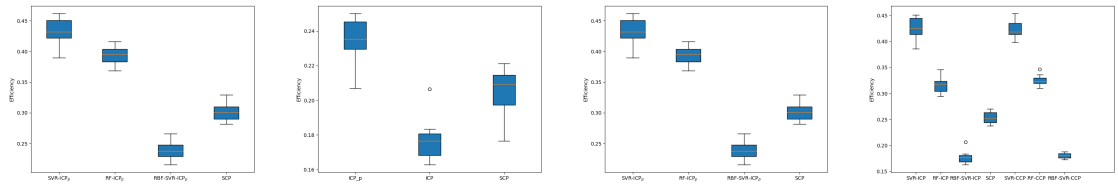
(a) Linear SVR (b) RBF-SVR (c) Different ML Models (d) Unpartitioned data

Figure 7: Results from Experiment 1, 2 and 3 comparing the efficiency between different methods for Energy dataset.



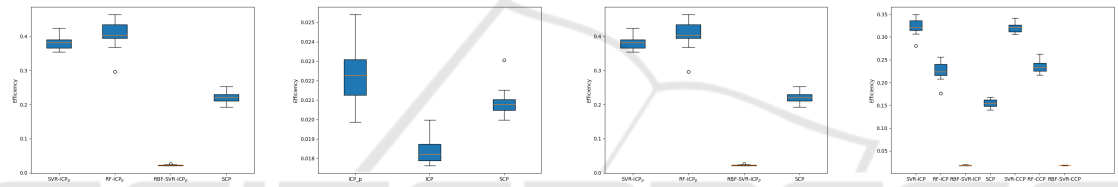
(a) Linear SVR (b) RBF-SVR (c) Different ML Models (d) Unpartitioned data

Figure 8: Results from Experiment 1, 2 and 3 comparing the efficiency between different methods for Concrete dataset.



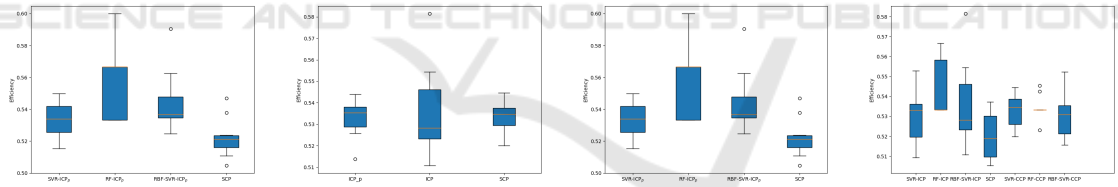
(a) Linear SVR (b) RBF-SVR (c) Different ML Models (d) Unpartitioned data

Figure 9: Results from Experiment 1, 2 and 3 comparing the efficiency between different methods for GridStability dataset.



(a) Linear SVR (b) RBF-SVR (c) Different ML Models (d) Unpartitioned data

Figure 10: Results from Experiment 1, 2 and 3 comparing the efficiency between different methods for CBM dataset.



(a) Linear SVR (b) RBF-SVR (c) Different ML Models (d) Unpartitioned data

Figure 11: Results from Experiment 1, 2 and 3 comparing the efficiency between different methods for Game dataset.