An Investigation Into the Effect of the Learning Rate on Overestimation Bias of Connectionist Q-learning

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Abstract: In Reinforcement learning, Q-learning is the best-known algorithm but it suffers from overestimation bias, which may lead to poor performance or unstable learning. In this paper, we present a novel analysis of this problem using various control tasks. For solving these tasks, Q-learning is combined with a multilayer perceptron (MLP), experience replay, and a target network. We focus our analysis on the effect of the learning rate when training the MLP. Furthermore, we examine if decaying the learning rate over time has advantages over static ones. Experiments have been performed using various maze-solving problems involving deterministic or stochastic transition functions and 2D or 3D grids and two Open-AI gym control problems. We conducted the same experiments with Double Q-learning using two MLPs with the same parameter settings, but without target networks. The results on the maze problems show that for Q-learning combined with the MLP, the overestimation occurs when higher learning rates are used and not when lower learning rates are used. The Double Q-learning variant becomes much less stable with higher learning rates and with low learning rates the overestimation bias may still occur. Overall, decaying learning rates clearly improves the performances of both Q-learning and Double Q-learning.

1 INTRODUCTION

The core idea of Reinforcement learning (RL) is to let an agent learn to optimize its behavior by interacting with an environment. The goal of the agent is to maximize its cumulative obtained reward. To achieve this goal, the agent learns from trial and error by observing the effects of different actions in different states. This process leads to a temporal sequence: $\{ s_0, a_0, r_1, s_1, a_1, r_2, \ldots \}$ of states s_t, actions a_t and rewards r_t, which can be used by the agent to optimize its action-selection policy (Sutton and Barto, 2018). The process is stopped when the task is completed or interrupted in another way, which forms a single episode (note that continuing tasks also exist, but are less common in RL). Usually many episodes are required for learning a near-optimal policy, and therefore RL algorithms can take a long time to learn good policies in complex environments.

A well-known and often used RL algorithm is Q-learning (Watkins and Dayan, 1992). Q-learning is an off-policy algorithm and uses bootstrapping for updates. Off-policy learning means the agent uses a target policy π for updating the value function but uses a different behavior policy π_b for selecting actions (Sutton and Barto, 2018). In this approach, bootstrapping entails the update of the value of the current state-action pair is based on other action-value estimates, which are usually biased and contain errors. For solving problems involving large or continuous state spaces, Q-learning needs to be combined with function-approximation methods such as neural networks, an approach which has already been used successfully in the 1990s (Lin, 1993).

The collected data (state, action, target value) during training is non-stationary and non-iid. Therefore, fitting a (non-linear) function on this data may lead to unstable learning and bad performances. The current methodology to deal with this issue is to use experience replay and a target network (Mnih et al., 2015), which helps to improve stability, but which does not solve all problems. One problem that remains is that Q-learning tends to overestimate state-action values, referred to as the overestimation bias (Thrun and Schwartz, 1993; Van Hasselt, 2010). Sutton and Barto defined the combination of off-policy

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RL, bootstrapping and function approximation as 'the deadly triad' (Sutton and Barto, 2018) and several studies have shown that such algorithms can diverge (Boyan and Moore, 1994; Wiering, 2004). These problems sometimes make the application of RL to solve challenging control tasks very hard.

The problem of overestimation bias in Q-learning has drawn attention from researchers in RL for many years. Thrun and Schwartz pointed out that the overestimation bias is the major source of failure when function approximators are combined with Q-learning (Thrun and Schwartz, 1993). They proved that this combination could cause a systematic overestimation of utility values because of the approximation error and mentioned that this could be resolved by biasing the function approximator to learn lower predictions.

Later in (Van Hasselt, 2010), van Hasselt showed that overestimation is quite general in the application of Q-learning and also occurs when lookup tables are used to store the state-action value function. To tackle this problem, van Hasselt proposed an off-policy algorithm called Double Q-learning which does not suffer from overestimation, but which can suffer from underestimation (Van Hasselt, 2010). Later, van Hasselt extended the idea of Double Q-learning to using deep neural networks and constructed an algorithm called Double DQN, which successfully learned better policies than DQN on different Atari games (Van Hasselt et al., 2016).

A problem of Double Q-learning is that it does not solve all problems concerning learning an accurate state-action value function with function approximation, because the overestimation problem is replaced by an underestimation problem. In (D'Eramo et al., 2016), the authors proposed the Weighted Estimator to generate more accurate estimations by computing the weighted averages of the sample means. In (Lu et al., 2018), the authors identified another problem of approximate Q-learning, termed delusional bias. This delusional bias occurs when value-function updates are generated which are inconsistent with each other and cannot be jointly realized. To handle this problem, the authors propose to use information sets, which are possible function approximator parameter settings that can justify the updates. These two algorithms are however complex and require much more computation than regular Q-learning.

Contributions. In this paper we study the overestimation bias of Q-learning combined with multilayer perceptrons (MLPs) when this QMLP technique is used to solve different maze-navigation problems and some other control tasks. QMLP uses experience replay and a target network and uses the Adam optimizer (Kingma and Ba, 2014) to train the MLP on mini-batches with experiences. We especially examine the effects of different learning rates on the resulting overestimation bias and final performance. We also propose the use of decaying the learning rate instead of using a static learning rate as commonly used in connectionist RL. Furthermore, we implemented a Double Q-learning algorithm in which two MLPs are used to learn two different state-action value functions similar to the original Double Q-learning algorithm (Van Hasselt, 2010). For this DQMLP method no target networks are used.

We created four tasks consisting of 2D or 3D maze environments and deterministic or stochastic transition functions. Besides, two classic control problems from the Open-AI gym environments were used to further examine the effect of learning rates on the overestimation bias and performance. The results showed that when lower learning rates are used for QMLP, no significant overestimation bias occurs. However, with higher learning rates, the overestimation bias does occur with QMLP. By using a decaying learning rate, the overestimation bias can be subdued and the value estimation can be improved. When Double Q-learning is combined with MLPs, the resulting DQMLP algorithm sometimes still leads to an overestimation bias when very low learning rates are used and becomes less stable when higher learning rates are used. Again, the value estimation can be improved by using a decaying learning rate. The results show that higher learning rates lead to worse final performances for both algorithms on the maze problems, although low learning rates lead to lower average performance during a whole simulation, which is caused by the initial slow learning behavior. Finally, the results demonstrate that in almost all experiments the use of a decaying learning rate can considerably improve the final and average performances.

Paper Outline. In Section 2, the used RL algorithms and the learning rate decay technique are described. Section 3 explains the experimental setup. In Section 4, the experimental results are presented and discussed. Section 5 concludes the paper and describes some directions for future work.

2 REINFORCEMENT LEARNING

In order to formalize most RL problems, the Markov decision process (MDP) framework can be used (Howard, 1960). A finite stationary MDP is a tuple $< S, A, R, P, \gamma >$, where:

- S represents the set of possible states in the environment and s_t denotes the state at time step t.

- A is the set of valid actions the agent can take and a_t denotes the selected action at time step t.
- $R(s,a,s')$ is the reward function which emits a reward r_{t+1} after the agent took action a in state s and moved to state s'.
- $P(s,a,s')$ is the state transition probability function and gives the probability of transitioning to state s' if the agent takes action a in state s.
- γ is a discount factor, $0 \leq \gamma \leq 1$, and determines the importance of rewards obtained in the future compared to immediate rewards.

The goal of an agent is to learn the optimal policy to obtain the highest possible cumulative discounted reward. In many RL algorithms, the agent uses value functions that estimate the goodness of states or state-action pairs. The value of a state, $v^{\pi}(s)$, denotes the expected discounted cumulative reward that an agent obtains when starting from state s and following policy π:

$$v^{\pi}(s) = \mathbb{E}[\sum_{k=0}^{\infty} \gamma^k r_{t+k+1} | s_t = s] \tag{1}$$

Where the expectancy operator is used to average over stochastic transitions and possibly over a stochastic policy and a stochastic reward function. Similarly, the state-action values $q^{\pi}(s,a)$ are defined by:

$$q^{\pi}(s,a) = \mathbb{E}[\sum_{k=0}^{\infty} \gamma^k r_{t+k+1} | s_t = s, a_t = a] \tag{2}$$

An optimal policy π^* is associated to the optimal Q-function $q^*(s,a)$, which is the maximum Q-function over all policies. When the agent correctly estimated $q^*(s,a)$, it can select optimal actions using $\pi^*(s) = \arg\max_a q^*(s,a)$.

2.1 Q-learning with a Multilayer Perceptron

Q-learning (Watkins, 1989; Watkins and Dayan, 1992) is the most often used algorithm for value-function based RL and can learn the optimal policy for finite MDPs even when actions are always taken completely randomly. This is because of its off-policy nature and this makes it also suitable for learning policies from expert demonstrations or for simultaneously learning multiple policies that solve different tasks. Given an experience tuple $(s_t, a_t, r_{t+1}, s_{t+1})$, Q-learning updates the Q-value $Q(s_t, a_t)$ according to:

$$\begin{aligned} Q(s_t,a_t) := & Q(s_t,a_t) + \\ & \eta(r_{t+1} + \gamma \max_{a'} Q(s_{t+1},a') - Q(s_t,a_t)) \end{aligned} \tag{3}$$

where η denotes the learning rate. As the agent explores the effects of different actions in the environment, the Q-function is continuously updated. When lookup tables are used to store the Q-function, Q-learning with a proper learning rate annealing schedule will converge after all actions have been tried an infinite number of times in all states (Jaakkola et al., 1994; Tsitsiklis, 1994)

Although the most straightforward way of applying Q-learning is using a look-up table to store the Q-values of all state-action pairs, this is infeasible if the state-action space is very large or continuous. Furthermore, a tabular representation does not have generalization power, which makes it impossible for the agent to select an informed action in a state that has not been visited before. Therefore, using function approximation is necessary when using Q-learning for solving complex sequential decision-making problems. The most often used function approximation techniques in RL are different kinds of neural networks. In 1993, Lin combined multi-layer perceptrons with Q-learning for several challenging navigation tasks (Lin, 1993), for which he also proposed the currently popular experience replay method. Tesauro also used an MLP when training a backgammon playing program that used self-play and temporal-difference learning (Tesauro, 1994). The resulting TD-Gammon program reached a human-expert level playing performance and was one of the first successes of RL.

More recently, approximating Q-functions by using convolutional neural networks (CNNs) has been shown to work very well for solving problems involving huge state spaces. In (Mnih et al., 2013; Mnih et al., 2015), the authors proposed a novel RL method, the deep Q-network (DQN), that combines Q-learning, a CNN, experience replay and a separate target Q-network. This DQN agent achieved state-of-the-art performances on 43 Atari games by only using images containing raw pixel values as input. This led to the active research field of deep reinforcement learning (DRL), in which deep neural networks are combined with RL algorithms.

When training neural networks with RL algorithms, an objective or loss function can be specified. When Q-learning and a target network are used, the loss function is:

$$L_{\theta} = \mathbb{E}[(r_{t+1} + \gamma \max_a Q(s_{t+1}, a; \theta^-) - Q(s_t, a_t; \theta))^2] \tag{4}$$

where θ consists of all trainable parameters of the current Q-network, and θ^- represents the parameters of the target Q-network which is a backup version of a previous Q-network. The expectancy operator is used because training occurs using stochastic sampling of mini-batches with experiences.

In this paper, we also use this objective function but instead of a CNN, we use a multilayer perceptron and train it with Q-learning. We will call this approach QMLP and similar to DQN, experience replay is used so that the collected experiences can be used multiple times for training the MLP (Lin, 1993). Furthermore, a separate target Q-network which is updated periodically is used, which is considered state-of-the-art practice in DRL to make the method more stable.

2.2 Double Q-learning with Two Multilayer Perceptrons

As described in the introduction, Q-learning can suffer from overestimation errors when learning the Q-function. Furthermore, the combination of off-policy RL, bootstrapping and function approximation form the deadly triad (Sutton and Barto, 2018), which can lead to poor learning performances (Van Hasselt et al., 2018). There have been different attempts to alleviate the overestimation bias in Q-learning. In (Van Hasselt, 2010), van Hasselt proposed an off-policy algorithm, Double Q-learning, which uses a double estimator approach for updating Q-values. Let Q^A and Q^B denote the two estimators, then for selecting an action one of them is selected randomly or the average action value of both Q-functions can be used. Then, one Q-function (e.g. Q^A) is randomly selected to be updated using:

$$Q^A(s_t,a_t) := Q^A(s_t,a_t) + \eta(r_{t+1} + \gamma Q^B(s_{t+1},a^\star) - Q^A(s_t,a_t)) \tag{5}$$

where $a^\star = \arg\max_a Q^A(s_{t+1},a)$ denotes the optimal action to take in the next state according to function Q^A. The updating rule for Q^B is the same with Q^A and Q^B swapped.

Subsequently, Double DQN was developed by adapting the DQN algorithm with respect to the idea of Double Q-learning (Van Hasselt et al., 2016). The update for the Q-network of Double DQN is similar to the one of DQN, except for the target value of an update:

$$y_t^{DDQN} \equiv r_{t+1} + \gamma Q(s_{t+1}, \arg\max_a Q(s_{t+1},a;\theta_t), \theta_t^-) \tag{6}$$

where θ and θ^- are again the parameters of the Q-network and the target Q-network. The basic difference is that the Q-network is used to compute the best action in the next state instead of letting the target Q-network determine this best action. It should be noted that this update is somewhat different from the original Double Q-learning algorithm, because only one network is used for selecting actions and updated on the experiences.

In our experiments, we developed a DQMLP algorithm that is more similar to the original Double Q-learning algorithm, which has also been proposed in (Schilperoort et al., 2018). Our DQMLP algorithm uses two MLPs to approximate the two estimators of the Double Q-learning algorithm. For selecting an action, the average Q-value of both MLPs is used. For each training step, one MLP is randomly picked to be updated, and the other MLP is used to compute the target value. The loss function for updating the Q^A network is:

$$L_{\theta^A} = \mathbb{E}[(r_{t+1} + \gamma Q^B(s_{t+1},a^*;\theta^B) - Q^A(s_t,a_t;\theta^A))^2] \tag{7}$$

where θ^A and θ^B represent the parameters of the two MLPs. The action a^* is again determined using network Q^A. The update for the Q^B network is again the same with indices swapped.

2.3 Learning Rate Decay

Learning rate decay is a commonly used technique in deep learning because of its simplicity and effectiveness (You et al., 2019). In the beginning of training, a reasonably high learning rate is important to learn fast, but once a good approximation has been learned, using a low learning rate helps with fine-tuning the model. It has been shown that when using stochastic gradient descent with mini-batches of training data, annealing the learning rate is necessary for the model to converge.

In RL, almost all researches use fixed learning rates and the effect of learning rate decay has hardly been studied. In (Van Hasselt, 2010), the author used a linear learning rate and a polynomial learning rate decay in the experiments, but the effect of using decaying learning rates compared to fixed learning rates was not researched. The relationship between learning rates and convergence rates in the Q-learning algorithm was explored in (Even-Dar and Mansour, 2003), but the algorithm studied was not combined with function approximators.

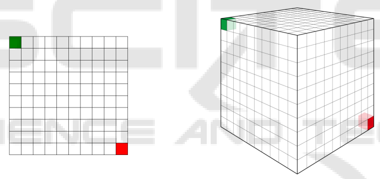
Therefore, besides the impact of learning rates on the overestimation bias, this paper investigates the effect of learning rate decay on the value estimation and performances of approximate RL algorithms.

3 EXPERIMENTAL SETUP

The experiments were performed in different maze environments and two Open-AI gym environments: CartPole-v0 and Acrobot-v1. In this section, we will first describe details of the maze environments. Then the structure of the MLPs is presented together with the used hyper-parameters. At the end of this section we explain how we measured the performances of the RL algorithms.

3.1 Maze Environments

To investigate when the overestimation occurs in the QMLP and DQMLP algorithms, an empty 2D maze and an empty 3D maze were constructed as illustrated in Figure 1. In each episode of the 2D maze, the agent starts from the upper left corner (green cell) and has to navigate to the lower right corner (red cell). The 2D maze consists of 100 states and four actions: north, south, west, and east. Similarly, in every episode of the 3D maze, the agent starts in the green cell and needs to find the red cell. The 3D maze consists of 1000 states and there are six actions: up, down, north, south, west, and east.



(a) 2D maze (b) 3D maze

Figure 1: Illustration of the 2D and 3D environments. The following states can be viewed: the starting state (green), the terminal state (red), and other states (white). Note that there is a wall around each entire maze, which is not shown in the figures.

State Transition Functions. Two types of state transition functions were created for both mazes: (1) deterministic transition functions, and (2) stochastic transition functions. Therefore, four kinds of environments were constructed in total: a deterministic 2D maze, a stochastic 2D maze, a deterministic 3D maze, and a stochastic 3D maze. In the maze with a deterministic transition function, the agent executes the selected action deterministically. For the stochastic transition function, the agent has a 20% chance of randomly moving to one of the neighboring states instead of executing the selected action.

Reward Function. The reward function is kept the same for all maze environments. The agent gets a reward of -5 for hitting the borders of the maze and $+20$ for reaching the terminal goal state. For every other step, a punishment of -1 is given to encourage the agent to find the exit using the least number of steps. The optimal cumulative reward intake in one episode is $+3$ for the deterministic 2D maze, and -6 for the deterministic 3D maze. Note that the optimal Q-value of the best action in the starting state (which we will also analyse) is different for these environments due to the discount factor.

3.2 Structure of Multilayer Perceptrons and Used Hyper-parameters

3.2.1 Maze Environments

In the maze-navigation problems, the MLPs in QMLP and DQMLP are implemented with the same structure, but DQMLP uses two MLPs. Each MLP is composed of an input layer, a single hidden layer, and an output layer.

Input Layer. The input of the MLP uses a continuous representation of the state of the agent. The current coordinates of the agent are normalized to values between 0 and 1. Therefore, the input layer of the 2D maze is composed of 2 nodes, which represent the row and column index of the agent. The starting state of the 2D maze is represented as [0.0, 0.0], and the terminal state is [1.0, 1.0]. Likewise, three inputs are used for the 3D maze.

Hidden Layer. The number of hidden units is 2048, which was enough to reliably learn to solve the tasks in preliminary experiments. As activation function, the hidden units use the rectified linear unit (ReLU) function (Glorot et al., 2011).

Output Layer. The outputs of the MLP represent the Q-values of the actions. Therefore, the number of output units is the same as the number of actions (4 or 6). A linear activation function is used in the output layer.

Other Hyper-parameters. The QMLP and DQMLP agent are both trained using the Adam optimizer (Kingma and Ba, 2014). We use the standard values $\beta_1 = 0.9$ and $\beta_2 = 0.999$ and varied the values of the learning rate. All other parameters were tuned using preliminary experiments. The epsilon-greedy policy is used as exploration strategy. For each simulation, each agent is trained for 5,000 episodes with the value of epsilon decaying linearly from 1.0 to 0.0 in the first 4,000 episodes and staying at 0.0 in the last 1,000 episodes. The maximum number of steps in one episode is set to 2,000. The discount factor is 0.95. In every updating step, a batch size of 128 experiences is sampled from the experience replay buffer for the agent to learn from. The capacity of the experience replay memory is 20,000. The best results were obtained by copying the current Q-network to the target network after each step and therefore this means that essentially no target network is used for QMLP.

3.2.2 Open-AI Gym Environments

For solving the Open-AI gym problems, the hyper-parameters were slightly re-tuned. Both QMLP and DQMLP agents are trained for 500 episodes. The epsilon value decreases linearly from 1.0 to 0.0 in the first 400 episodes and stays 0.0 for the remaining episodes. For both agents, the MLP used two hidden layers, the first with 48 and the second with 96 hidden units. The capacity of the memory buffer is 1,000,000. The batch size is 512 for both algorithms. The target network of QMLP is updated every 64 steps. The other hyper-parameters are kept the same as for the maze-solving agents.

3.3 Performance Measures

In the results, we report three different performance measures. First, the cumulative reward intake during the last episode is recorded for each simulation. Second, the average cumulative reward per episode during an entire simulation is computed. Third, to analyze possible overestimation or underestimation errors, we record the estimated maximal Q-value of the starting state. Because the Q-values of the initial state can change during the entire episode due to the use of experience replay after each time step, the average initial state value is computed for each episode. Equation (8) shows the computation of the initial state-value estimate per episode.

$$V_{start} = \frac{1}{T}\sum_{t=1}^{T}\max_{a} Q(S_{start}, a; \theta) \tag{8}$$

Where T denotes the number of steps in an episode ($T \leq 2000$ for maze-navigation problems), and θ represents the weight values of the Q-network. For DQMLP, two Q-functions are learned and we selected the highest Q-value for the initial state from both Q-functions to compute the initial state value. The true maximum state-action value of the starting state in each maze environment was calculated by using dynamic programming (Bellman, 1957).

4 EXPERIMENTAL RESULTS

The results are obtained by running 10 simulations with different random seeds. For each episode, the initial state value estimate V_{start} and the cumulative reward sum R are recorded. In each figure for the value estimates in the following subsections, the straight red line represents the true optimal value of the starting state in each corresponding environment. Each curve shows the average result of the 10 runs, and the shaded regions show the standard deviation. Some parts of the figures were zoomed in for clarity.

In 2D mazes, The learning curves of using the learning rates: $5e-3$, $1e-3$, $1e-4$, $1e-5$, $1e-6$, and a decaying learning rate are plotted. While in 3D mazes and the Open-AI gym control environments, the learning rates $1e-3$, $1e-4$, $1e-5$, $1e-6$, and a decaying learning rate are used. Specifically, the decaying learning rate is set as 10^x with x linearly annealing from -3 to -6 over all the episodes. The setting of the decaying learning rate is kept the same in the following experiments.

4.1 Results on Maze Environments

4.1.1 Deterministic 2D Maze

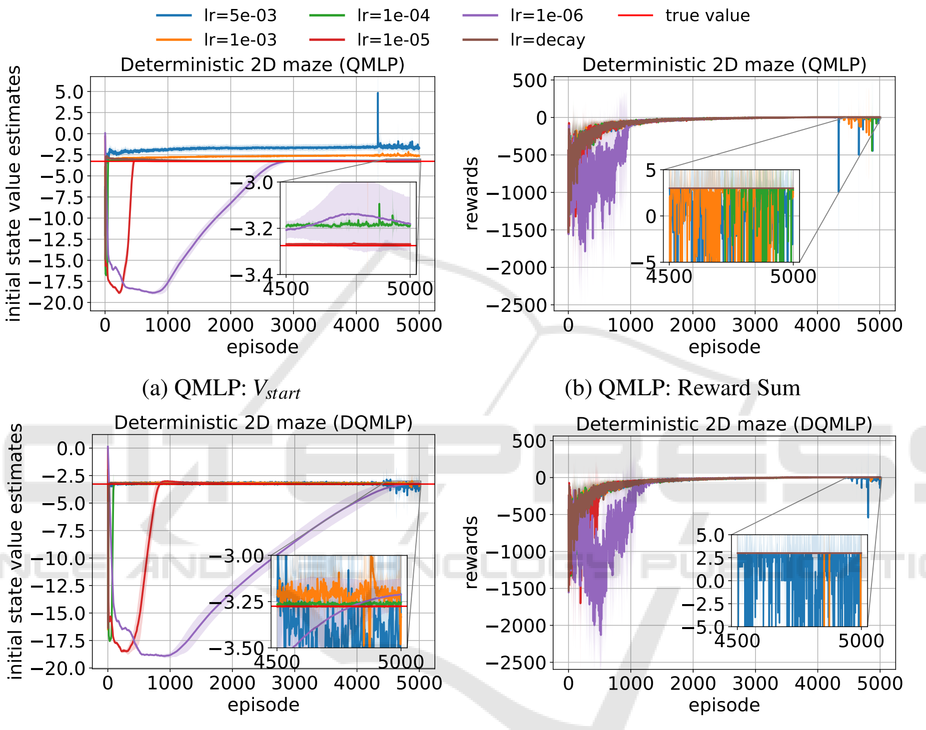
The first row in Figure 2 shows the estimated initial state value and the cumulative reward per episode of the QMLP agent in the deterministic 2D maze. We can see from Figure 2a that overestimation occurred with the learning rates $5e-3$ and $1e-3$. The corresponding cumulative reward curves became unstable after around 4300 episodes as shown in Figure 2b. The agent performed best with the decaying learning rate: the corresponding cumulative rewards smoothly converged and the average reward intake was highest.

When we examine the results for DQMLP in Figure 2c, we can observe that the initial state-value estimate curves become unstable for the learning rates $5e-3$ and $1e-3$, and their corresponding cumulative rewards were also deteriorating as shown in Figure 2d. The agent had a stable learning behavior and achieved the optimal cumulative rewards of $+3$ with a learning rate of $1e-4$, $1e-5$, $1e-6$ and the decaying learning rate. When we zoom in, we can observe that DQMLP with a learning rate of $1e-6$ surprisingly starts to overestimate the initial state value in the end.

Table 1 shows the results of both algorithms in the last episode for the initial state value estimate V and the cumulative reward R, where σ represents the standard deviation. The table also shows the average cumulative reward $\overline{R}$ over all episodes. The table clearly shows the overestimation errors for QMLP when high learning rates are used. The table also shows that DQMLP always obtains the highest cumulative reward in the last episode, but QMLP learns faster and obtains higher average rewards over all episodes. Decaying the learning rate leads to the best performances for both algorithms: the value function is accurate and

Table 1: Results of QMLP and DQMLP in a deterministic 2D maze. The true value for the initial state is -3.28 and the optimal cumulative reward is +3.0.

	QMLP			DQMLP		
l.r.	$V \pm \sigma$	$R \pm \sigma$	$\overline{R} \pm \sigma$	$V \pm \sigma$	$R \pm \sigma$	$\overline{R} \pm \sigma$
5e-3	-1.71 ± 0.37	2.5 ± 1.5	-57 ± 2	-3.33 ± 0.60	3.0 ± 0.0	-60 ± 1
1e-3	-2.63 ± 0.13	3.0 ± 0.0	-56 ± 1	-3.32 ± 0.33	3.0 ± 0.0	-59 ± 1
1e-4	-3.19 ± 0.01	2.8 ± 0.6	-56 ± 0	-3.26 ± 0.01	3.0 ± 0.0	-59 ± 1
1e-5	-3.27 ± 0.00	3.0 ± 0.0	-67 ± 2	-3.28 ± 0.00	3.0 ± 0.0	-78 ± 8
1e-6	-3.18 ± 0.12	3.0 ± 0.0	-151 ± 20	-3.21 ± 0.09	3.0 ± 0.0	-195 ± 26
decay	-3.28 ± 0.00	3.0 ± 0.0	-55 ± 1	-3.28 ± 0.00	3.0 ± 0.0	-58 ± 1



(a) QMLP: V_{start}

(b) QMLP: Reward Sum

(c) DQMLP: V_{start}

(d) DQMLP: Reward Sum

Figure 2: Performances of the QMLP agent and the DQMLP agent in a deterministic 2D maze with different learning rates. Each curve shows the average results of 10 runs. The shaded regions show the standard deviation.

the average reward intake over all episodes is highest. At the end of training this method is also much more stable than using static higher learning rates.

4.1.2 Deterministic 3D Maze

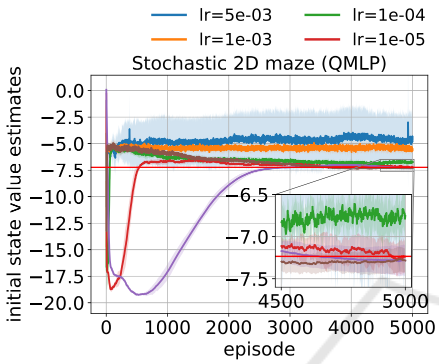
The results for the QMLP and DQMLP agent in a deterministic 3D maze are shown in Table 2. The table shows clear overestimation errors for QMLP when the learning rate is $1e-3$. For DQMLP, significant overestimation errors occur for the lowest learning rate. The use of a decaying learning rate is again very useful for both algorithms: the accuracy of the value function, the final performance and the average performance are all excellent using the decay function. We also noticed that using higher learning rates again led to instabilities in the learning process.

4.1.3 Stochastic 2D Maze

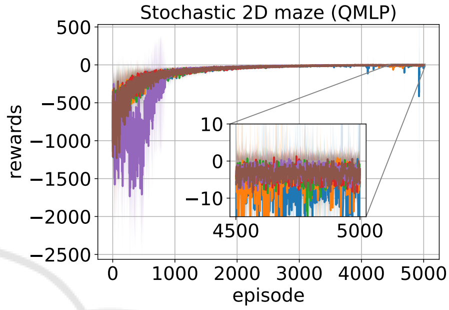
The first row of Figure 3 shows the training results of the QMLP agent in a stochastic 2D maze. As reported in Figure 3a, with a learning rate of $5e-3$, $1e-3$ and $1e-4$, the agent significantly overestimated the value of the starting state. Large standard deviations (shaded areas) can also be seen in the curve with a learning rate of $5e-3$. Figure 3b shows that QMLP with a learning rate of $1e-5$, $1e-6$ and the decaying

Table 2: Results of QMLP and DQMLP in a deterministic 3D maze. The true value for the initial state is -9.46 and the optimal cumulative reward is -6.0.

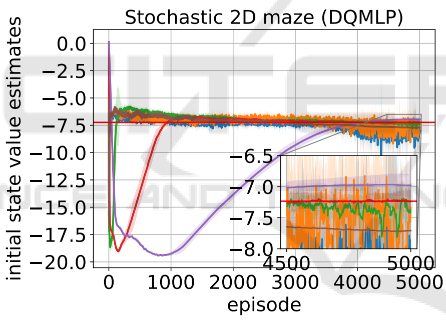
	QMLP			DQMLP		
l.r.	$V \pm \sigma$	$R \pm \sigma$	$\overline{R} \pm \sigma$	$V \pm \sigma$	$R \pm \sigma$	$\overline{R} \pm \sigma$
1e-3	-7.39 ± 1.52	-6.4 ± 0.8	-195 ± 80	-9.95 ± 1.73	-6.2 ± 0.6	-171 ± 2
1e-4	-9.18 ± 0.04	-6.0 ± 0.0	-156 ± 2	-9.40 ± 0.10	-6.0 ± 0.0	-163 ± 2
1e-5	-9.45 ± 0.00	-6.0 ± 0.0	-170 ± 11	-9.47 ± 0.01	-6.0 ± 0.0	-254 ± 21
1e-6	-9.35 ± 0.07	-6.0 ± 0.0	-734 ± 117	-8.67 ± 0.57	-6.0 ± 0.0	-1151 ± 41
decay	-9.44 ± 0.04	-6.0 ± 0.0	-157 ± 4	-9.47 ± 0.01	-6.0 ± 0.0	-117 ± 2



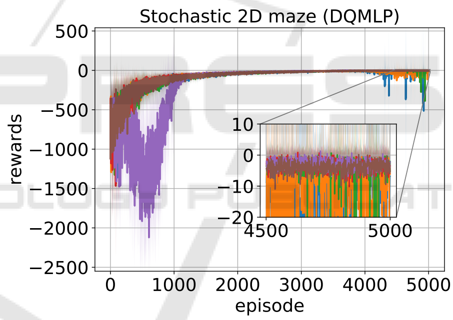
(a) QMLP: V_{start}



(b) QMLP: Reward Sum



(c) DQMLP: V_{start}



(d) DQMLP: Reward Sum

Figure 3: Performances of the QMLP and DQMLP agent in a stochastic 2D maze. Each curve shows the average results of 10 runs.

learning rate performed the most stable.

As can be seen from Figure 3c, for DQMLP with a learning rate of $5e-3$, $1e-3$ or $1e-4$, the value estimate became unstable. Their corresponding cumulative reward curves also became oscillating as shown in Figure 3d. The agent finally performed best with a learning rate of $1e-5$, $1e-6$ or a decaying learning rate. In the end, DQMLP again overestimates the initial state value with a learning rate of $1e-6$.

The results on the stochastic 2D maze are also shown in Table 3. The table shows large overestimation errors for QMLP with the highest learning rates. DQMLP significantly underestimates the initial state value when a learning rate of $1e-3$ is used and overestimates the value with a learning rate of $1e-6$. Due to the stochasticity of the environment, the cumulative rewards of the last episode have large standard deviations, but we can observe that the highest learning rate leads to worse performances for both algorithms. Decaying the learning rate was again beneficial for QMLP, but DQMLP had a few bad episodes with it at the end, which explains the high standard deviation.

4.1.4 Stochastic 3D Maze

The results for the QMLP and DQMLP agent in the stochastic 3D maze are shown in Table 4. The table shows significant overestimation errors for QMLP with the highest used learning rate. Furthermore, with all learning rates DQMLP learns to underestimate the

Table 3: Results of QMLP and DQMLP in a stochastic 2D maze. The true value for the initial state is -7.23 and the optimal cumulative reward is -2.4.

	QMLP			DQMLP		
l.r.	$V \pm \sigma$	$R \pm \sigma$	$\overline{R} \pm \sigma$	$V \pm \sigma$	$R \pm \sigma$	$\overline{R} \pm \sigma$
5e-3	-4.60 ± 2.65	-5.7 ± 5.2	-76 ± 3	-8.12 ± 1.02	-6.8 ± 6.7	-80 ± 3
1e-3	-5.54 ± 0.46	-2.2 ± 4.4	-74 ± 1	-7.34 ± 0.42	-6.1 ± 6.2	-76 ± 1
1e-4	-6.76 ± 0.29	-3.6 ± 5.9	-77 ± 2	-7.42 ± 0.35	-2.1 ± 5.0	-78 ± 2
1e-5	-7.24 ± 0.17	-2.8 ± 3.9	-74 ± 1	-7.23 ± 0.20	-3.4 ± 6.0	-76 ± 2
1e-6	-7.28 ± 0.19	-2.3 ± 3.9	-142 ± 27	-6.98 ± 0.19	-2.2 ± 5.3	-200 ± 15
decay	-7.28 ± 0.15	-2.1 ± 4.3	-74 ± 1	-7.71 ± 0.33	-5.3 ± 7.4	-76 ± 1

Table 4: Results of QMLP and DQMLP in a stochastic 3D maze. The true value for the initial state is -13.36 and the optimal cumulative reward is -14.2.

	QMLP			DQMLP		
l.r.	$V \pm \sigma$	$R \pm \sigma$	$\overline{R} \pm \sigma$	$V \pm \sigma$	$R \pm \sigma$	$\overline{R} \pm \sigma$
1e-3	-9.56 ± 0.48	-20.2 ± 10.5	-208 ± 2	-14.93 ± 1.18	-39.0 ± 60.8	-222 ± 3
1e-4	-13.27 ± 0.26	-18.1 ± 7.0	-381 ± 63	-14.78 ± 0.29	-18.9 ± 8.9	-317 ± 32
1e-5	-15.18 ± 0.83	-20.1 ± 7.5	-276 ± 49	-15.20 ± 0.35	-19.0 ± 8.4	-329 ± 12
1e-6	-15.45 ± 0.96	-20.1 ± 6.0	-1110 ± 280	-18.16 ± 3.44	-119 ± 116.2	-928 ± 77
decay	-14.89 ± 0.49	-17.1 ± 5.6	-209 ± 3	-15.54 ± 0.34	-19.9 ± 6.6	-225 ± 2

value of the initial state.

For QMLP, the value estimate converged when the learning rate is $1e-4$. With a learning rate of $1e-6$, the standard deviation of the value estimate curve became large and it seems the algorithm needs more episodes to converge. The best used learning rate is the decaying learning rate, which leads to the best final cumulative reward and average reward.

For DQMLP, the agent with a decaying learning rate performed best: it obtained the highest average reward and the final cumulative reward was among the best. When the learning rate is $1e-6$, the agent performed the worst. Also with DQMLP, the algorithm needs more episodes to converge with this low learning rate.

4.2 Results on Open-AI Gym Problems

As we have seen from the previous results, the agent using a decaying learning rate performed the best in the experiments with the empty maze environments. To further examine the effect of annealing the learning rate, we additionally conducted experiments on two classic control problems provided by Open-AI Gym: CartPole-v0 and Acrobot-v1.

4.2.1 Results on CartPole-V0

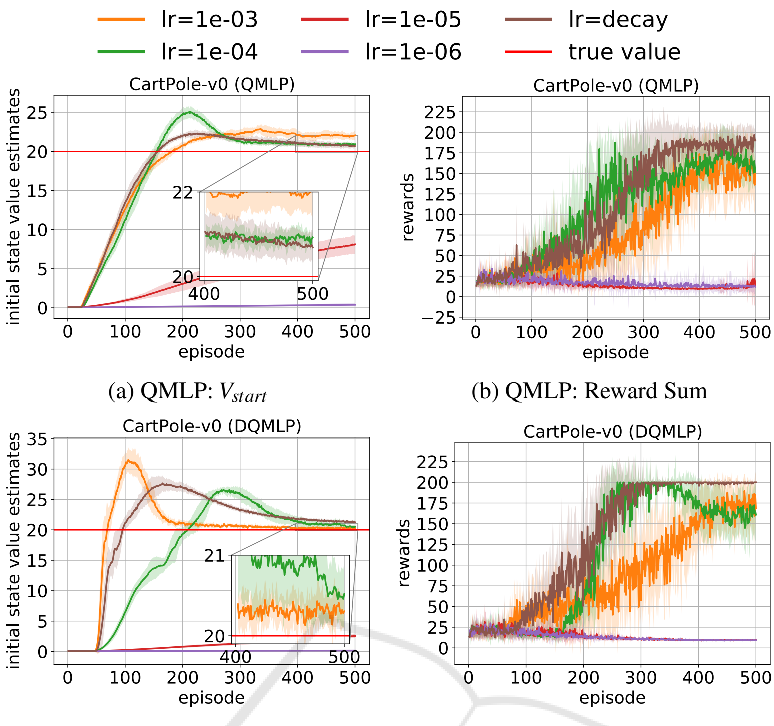
The CartPole-v0 problem requires the agent to push a cart left or right in order to keep a pole standing on the cart from falling, while keeping the cart on the track. There are four inputs: cart position and velocity and the pole's angle and the angular velocity. There are two actions: push the cart to the left or right. It is a well-known control problem that has been researched extensively.

Optimal Values. In the CartPole-v0 environment, the agent gets a reward of $+1$ for every step that the pole does not fall over. The episode terminates when either the pole angle or the cart position is out of range or the episode length reaches the maximum of 200 steps. The value of the discount factor γ is 0.95. This means that the maximal cumulative reward is 200 and the optimal Q-value is $\sum_{t=0}^{199} 1 * \gamma^t \approx 19.999$.

Results. The learning curves are presented in Figure 4. For this problem, low learning rates do not lead to good performances. The best performance for QMLP and DQMLP is obtained with the decaying learning rate. Table 5 shows some more details about the value estimation and performances of both algorithms in CartPole. It shows that DQMLP with the decaying learning rate is always able to obtain the optimal performance. Note that figure 4(c) shows that a large overestimation bias occurs in DQMLP, which was also shown in a similar algorithm (Fujimoto et al., 2018). Double DQN, which is a variant of DQMLP, can also overestimate the true value (Van Hasselt et al., 2016) in some cases.

4.2.2 Results on Acrobot-V1

The goal of the Acrobot-v1 problem is to swing up a robot consisting of 2 links and 2 joints. The problem consists of 6 inputs and there are 3 actions



(a) QMLP: V_{start} (b) QMLP: Reward Sum

(c) DQMLP: V_{start} (d) DQMLP: Reward Sum

Figure 4: Performances of the QMLP and DQMLP agent in CartPole-v0. Each curve shows the average results of 10 runs.

Table 5: Results of QMLP and DQMLP in the CartPole-v0 environment. The true value for the initial state of CartPole-v0 is 19.999 and the optimal cumulative reward is 200.

	QMLP (CartPole-v0)			DQMLP (CartPole-v0)		
l.r.	$V \pm \sigma$	$R \pm \sigma$	$\bar{R} \pm \sigma$	$V \pm \sigma$	$R \pm \sigma$	$\bar{R} \pm \sigma$
1e-3	22.06 ± 0.65	155.6 ± 43.8	81 ± 5	20.30 ± 0.19	168.7 ± 40.2	88 ± 7
1e-4	20.93 ± 0.18	151.9 ± 29.6	107 ± 7	20.52 ± 0.40	160.9 ± 32.5	110 ± 4
1e-5	8.12 ± 1.23	13.3 ± 8.6	14 ± 1	2.55 ± 0.30	9.8 ± 0.6	14 ± 1
1e-6	0.38 ± 0.21	11.9 ± 6.5	17 ± 7	0.15 ± 0.04	9.3 ± 0.6	14 ± 1
decay	20.69 ± 0.35	190.3 ± 18.7	110 ± 10	21.26 ± 0.32	200.0 ± 0.0	130 ± 3

left/right/do-nothing which is the torque applied on the second joint.

Optimal Values. For the Acrobot-v1 environment, there is not a clear optimal cumulative reward, so we approximate it. The agent gets a punishment of -1 for every time step until it reaches the goal. The problem is considered solved when the agent gets an average reward of -100.0 over 100 consecutive trials. The value of the discount factor γ is 0.95, and therefore we calculated the approximate optimal Q-value as follows: $\sum_{t=0}^{99} -1 * \gamma^t \approx -19.882$.

Results. Table 6 shows the value estimation and performances of both algorithms in the Acrobot environment. The learning curves are presented in Figure 5. As is shown in the results, QMLP with a decaying learning rate performed very well. The final performance is very good and the average performance is the best of all tested learning rates.

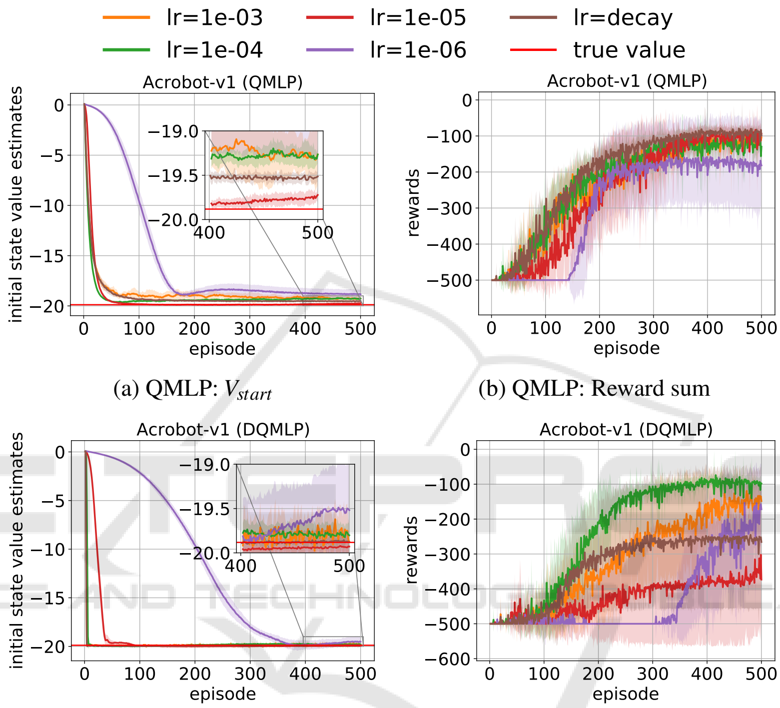
For the DQMLP agent, a learning rate of $1e-4$ works the best in the Acrobot environment. The decaying learning rate combined with DQMLP leads again to several bad runs.

4.3 Discussion

From the results, we observe that overestimation errors occur in QMLP with a high learning rate in the maze problems, no matter whether there is noise in the environment. For example, in both the deterministic 2D maze (first row of Figure 2) and the stochastic 2D maze (first row of Figure 3), obvious overestimation errors appeared for high learning rates. This means that noise in the environment is not the primary cause of overestimation in our cases. There-

Table 6: Results of QMLP and DQMLP in the Acrobot-v1 environment. The approximate true value for the initial state of Acrobot-v1 is -19.882 and the approximate optimal cumulative reward is -100.

	QMLP (Acrobot-v1)			DQMLP (Acrobot-v1)		
l.r.	$V \pm \sigma$	$R \pm \sigma$	$\overline{R} \pm \sigma$	$V \pm \sigma$	$R \pm \sigma$	$\overline{R} \pm \sigma$
1e-3	-19.26 ± 0.34	-104.0 ± 17.9	-236 ± 13	-19.87 ± 0.21	-148.4 ± 26.7	-315 ± 73
1e-4	-19.27 ± 0.10	-129.7 ± 40.8	-228 ± 27	-19.80 ± 0.13	-98.0 ± 16.9	-237 ± 28
1e-5	-19.72 ± 0.09	-93.7 ± 16.6	-255 ± 47	-19.93 ± 0.13	-373.4 ± 176.2	-428 ± 102
1e-6	-18.85 ± 0.46	-181.6 ± 123.1	-295 ± 73	-19.52 ± 0.53	-171.8 ± 126.5	-433 ± 30
decay	-19.51 ± 0.06	-98.7 ± 34.5	-207 ± 10	-19.88 ± 0.21	-265.7 ± 194.0	-335 ± 132



(a) QMLP: V_{start} (b) QMLP: Reward sum

(c) DQMLP: V_{start} (d) DQMLP: Reward sum

Figure 5: Performances of the QMLP and DQMLP agent in Acrobot-v1. Each curve shows the average results of 10 runs.

fore, we conclude that the problem occurs because of function approximation errors that lead to estimating inconsistent subsequent state-action values. For the problems we studied, using a decaying learning rate worked well to handle overestimation errors. Lower learning rates can also lead to good performances, but for some problems they lead to very slow learning.

When examining the used combination of Double Q-learning with MLPs, where two MLPs learn two different Q-functions, we observed that with low learning rates, the DQMLP algorithm can suffer from the overestimation bias. This could be caused when both Q-function approximations become almost identical, in which case the algorithm is similar to QMLP. In the DQMLP algorithm, actions were selected by combining both Q-functions. It might be possible that overestimation bias is less likely when one Q-function is randomly selected to select an action. The DQMLP algorithm did not use target networks and although for the deterministic environments, this did not seem to be a problem, the learning behavior in the stochastic environments was less stable than the one of QMLP.

5 CONCLUSION

This paper described a novel analysis of when and why the overestimation bias may occur when Q-learning is combined with multilayer perceptrons. The focus of our study was on the effect of the learning rate, and we proposed the use of decaying learning rates in reinforcement learning. The results on four different grid-worlds demonstrated that the overestimation problem always occurred when higher learning rates were used and not with lower learning rates or decaying learning rates.

We also analyzed the performance of Double Q-learning with multilayer perceptrons under the same conditions. This algorithm in general suffers from more underestimation when higher learning rates are used and surprisingly can also suffer from the overestimation bias when very low learning rates are used.

We also examined the performances of both algorithms on two Open-AI gym control problems. The results obtained on all six environments suggest that in general the best performances are achieved by using the decaying learning rates.

Our future work includes studying the connection between the learning rate and overestimation bias for more complex environments. We also want to perform more research on different methods for annealing the learning rate over time.

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