Application of Rodrigues Matrix in High Accuracy Geo-location for ZY-3 Panchromatic Imagery

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Abstract: In this paper, Rodrigues matrix is proposed to establish constant angular error calibration model, and interior orientation errors are compensated by additional parameters model. Bundle block adjustment model is established by these two models on the basis of the rigorous geometric model for ZY-3 panchromatic imagery. Once the constant angular errors and interior orientation errors are eliminated using a few GCPs, the geo-location accuracy will be significantly improved.

1 INTRODUCTION

The Ziyuan-3 (ZY-3) surveying and mapping satellite is the first civilian high-resolution satellite in China. Its ground resolution of nadir-view camera is better than 2.1 m, forward- and backward-view cameras are better than 3.5 m, and multi-spectral camera is better than 5.8 m. The main task of the ZY-3 satellite is performing stereo mapping at a scale of 1: 50,000, producing digital images, and updating topographic maps at scales of 1 : 25,000 and larger, as well as playing an important role in the fields of land resources surveying and monitoring, agriculture, disaster control, resources and environment, public safety, etc. (Sun and Tang, 2009).

Multiple studies have been conducted on the ZY-3 mapping satellite. For example, Deren Li constructed a geometric model for the imaging of the ZY-3 satellite and proposed an imaging technology based on a virtual CCD linear array, that was used to calibration (Li, 2012). Chubin Liu established a strict geometric model for the stereoscopic location of the panchromatic camera of the ZY-3 mapping satellite (Liu, 2012). Dazhao Fan adopted a linearized euler angle model to calibrate the constant error of the attitude (Fan et al., 2013). Chubin Liu utilized a self-calibrating method for the regional area adjustment of the three-line array images (Liu et al., 2014). Yonghua Jiang derived an internal orientation calibration model for the CCD, and achieved a high accuracy (Jiang et al., 2013). Finally, by analyzing the direction angle of each CCD joint in the star sensor coordinate system, Cao Jinshan proposed a direction angle calibration method (Cao et al., 2014).

In this study, based on the strict imaging geometric model of the ZY-3 satellite, we propose a calibration model for the constant error of the attitude angle using the Rodrigues rotation matrix. In addition, by introducing an additional parameter model considering the aberration to compensate for the internal orientation distortion, we constructed a block adjustment model using the self-calibrating bundle method, that could derive the correction numbers of both the internal and external orientation elements based on a small number of ground control points, then significantly improving the imaging geo-location accuracy of the system.

2 STRICT GEOMETRIC MODEL

The strict geometric model utilized in this study for the imaging of the ZY-3 mapping satellite can be expressed by formula (1):

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{CTS} = \begin{bmatrix} X_S \\ Y_S \\ Z_S \end{bmatrix}_{CTS} + mR_{CIS}^{CTS} R_{star}^{CIS} \begin{bmatrix} tan(\varphi_y)f \\ -tan(\varphi_x)f \\ -f \end{bmatrix} \quad (1)$$

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where $[X_S \quad Y_S \quad Z_S]^T$ is the external orientation line element; m is the scale factor; R_{CIS}^{CTS} is the rotation matrix from the earth inertial coordinate system to the earth-fixed reference coordinate system; and R_{star}^{CIS} is the rotation matrix from the body coordinate system to the earth inertial coordinate system. φ_x and φ_y denote the direction angles.

Formula (1) shows that the external orientation element in any row of images from the ZY-3 satellite, which can be compensated in adjustment processing.

3 ATTITUDE ANGLE CONSTANT ERROR CALIBRATION

As device installation, launching vibration, environment change and other factors, there would bring errors into the attitude and orbit data. If unprocessed attitude, orbit data and the original installation matrix of the satellite are used to restore the imaging bundle based on the strict geometric model, would resulting in substantial reduction in geometric location accuracy. Therefore, correction of the attitude constant angle error is very important for satellite stereo geo-location accuracy, and block adjustment (Yuan and Cao, 2012).

When the influence of atmospheric refraction is excluded, the spatial geometric position of the perspective center of the sensor and ground control points can be utilized to accurately restore the imaging bundle direction of the image point (Yuan and Yu, 2008), as shown in Figure. 1.

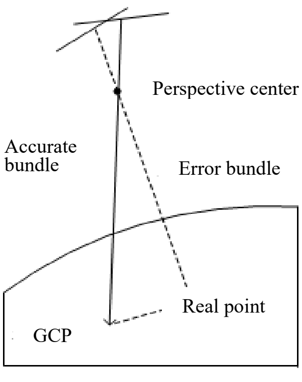


Figure 1: Target geo-location error due to attitude angle error.

3.1 Rodrigues Rotation Matrix

Suppose R is an orthogonal rotation matrix with three degrees of freedom, and S is the anti-symmetric matrix, then $S = \begin{bmatrix} 0 & -c & -b \\ c & 0 & -a \\ b & a & 0 \end{bmatrix}$, where a, b, and c are three independent unknowns. R can be seen as a Rodrigues rotation matrix composed of S, which has the following correlation:

$$R = (I - S)^{-1}(I + S) = \begin{bmatrix} 1 & c & -b \\ -c & 1 & a \\ b & -a & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & -c & b \\ c & 1 & -a \\ -b & a & 1 \end{bmatrix} \quad (2)$$

where I is a 3rd order unit matrix.

3.2 Model Construction and Solution

As shown in Figure. 1, the constant error in the attitude angle can be considered as an offset matrix R between the actual imaging bundle and the ideal imaging bundle. The strict geometric model is corrected as follow:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{CTS} = \begin{bmatrix} X_S \\ Y_S \\ Z_S \end{bmatrix}_{CTS} + mR_{CIS}^{CTS} R_{star}^{CIS} R \begin{bmatrix} tan(\varphi_y)f \\ -tan(\varphi_x)f \\ -f \end{bmatrix} \quad (3)$$

Suppose

$$u_1 = (R_{CIS}^{CTS} R_{star}^{CIS})^{-1} \begin{bmatrix} X - X_S \\ Y - Y_S \\ Z - Z_S \end{bmatrix}_{CTS} \quad (4)$$

$$u_2 = R \begin{bmatrix} tan(\varphi_y)f \\ -tan(\varphi_x)f \\ -f \end{bmatrix} \quad (5)$$

$$u_c = \frac{u_1}{\|u_1\|} \quad (6)$$

$$u_s = \frac{u_2}{\|u_2\|} \quad (7)$$

By substituting R into formula (3):

$$\begin{bmatrix} u_{cX} \\ u_{cY} \\ u_{cZ} \end{bmatrix} = \begin{bmatrix} 1 & c & -b \\ -c & 1 & a \\ b & -a & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & -c & b \\ c & 1 & -a \\ -b & a & 1 \end{bmatrix} \begin{bmatrix} u_{sX} \\ u_{sY} \\ u_{sZ} \end{bmatrix} \quad (8)$$

$$\begin{bmatrix} u_{cX} - u_{sX} \\ u_{cY} - u_{sY} \\ u_{cZ} - u_{sZ} \end{bmatrix} = \begin{bmatrix} 0 & u_{cZ} + u_{sZ} & -u_{cY} - u_{sY} \\ -u_{cZ} - u_{sZ} & 0 & u_{cX} + u_{sX} \\ u_{cY} + u_{sY} & -u_{cX} - u_{sX} & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \tag{9}$$

$$A = \begin{bmatrix} 0 & u_{cZ} + u_{sZ} & -u_{cY} - u_{sY} \\ -u_{cZ} - u_{sZ} & 0 & u_{cX} + u_{sX} \\ u_{cY} + u_{sY} & -u_{cX} - u_{sX} & 0 \end{bmatrix} \tag{10}$$

$$L = \begin{bmatrix} u_{cX} - u_{sX} \\ u_{cY} - u_{sY} \\ u_{cZ} - u_{sZ} \end{bmatrix} \tag{11}$$

$$X = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \tag{12}$$

$$V = AX - L \tag{13}$$

As u_c and u_s are unit vectors, and only two out of the three components are independent, only two independent formulas can be listed for one ground control point. Therefore, at least two control points are required to obtain a solution using the least squares method (Jia et al., 2012), so that the values of a, b, and c can be derived to construct the offset matrix R.

This model makes the offset matrix R equal to the Rodrigues parameters a, b, and c. They are resolved linearly, and the non-linear constraint is solved by a true linear method without compromising the accuracy.

4 SELF-CALIBRATING BUNDLE ADJUSTMENT

The geo-location accuracy is significantly improved after compensation of the attitude angle by the Rodrigues rotation matrix. However owing to an internal orientation distortion, it still does not meet the requirement for high-accuracy geo-location. Therefore, the internal orientation element must be calibrated. On the basis of the attitude angle constant error calibration model using Rodrigues rotation matrix, we introduced an additional parameter model taking into account the aberration to compensate for the internal orientation distortion. Based on the strict geometric model, a self-calibrating bundle adjustment model was constructed, which could correct both the internal and external orientation parameters.

4.1 Additional Parameter Model Considering Aberration

The systematic error of a linear-array camera can be roughly divided into an optical lens error and a CCD linear array error. Similar to the perspective camera, this systematic error is mainly composed of primary point offsets, CCD rotation changes, and pixel size variations (Lei, 2011). An additional parameter model is constructed accordingly, as shown below:

$$\begin{cases} \Delta x_s = \Delta x_p + (k_1 r^2 + k_2 r^4)\bar{x} + \\ \qquad p_1(r^2 + 2\bar{x}^2) + 2p_2\bar{x}\bar{y} + S\bar{x} \\ \Delta y_s = \Delta y_p + (k_1 r^2 + k_2 r^4)\bar{y} + \\ \qquad p_1\bar{x}\bar{y} + 2p_2(r^2 + 2\bar{y}^2) + R\bar{x} \end{cases} \tag{14}$$

where Δx_s and Δy_s are the systematic correction; Δx_p and Δy_p are the offsets of the primary imaging point; k_1 and k_2 are the radial distortion coefficients; p_1 and p_2 are the offset distortion coefficients; r is the radiant distance from the imaging point to the primary point; $(\bar{x}, \bar{y})$ is the difference between the imaging point coordinates and the primary point coordinates; S is the scale factor; and R is the rotation factor.

The additional parameter model taking into account the aberration comprehensively considered factors optical lens distortion, pixel size change and CCD rotation variation. However, in actual adjustments, different blocks present different geometrical characteristics; therefore, the type and number of additional parameters will affect the stability of the model in obtaining a solution (Tang et al., 2010; Gan and Yan, 2007). To prevent the occurrence of a strong correlation between parameters, the parameters of the additional parameter model must be selected carefully during adjustment.

4.2 Self-calibrating Bundle Adjustment Model

Based on the strict geometric model, an improved imaging geometric model that lays the foundation for the bundle adjustment model is constructed by comprehensively considering external orientation compensation and internal orientation distortion as shown

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{CTS} = \begin{bmatrix} X_S \\ Y_S \\ Z_S \end{bmatrix}_{CTS} + mR_{CIS}^{CTS} R_{star}^{CIS} R \begin{bmatrix} tan(\varphi_y)f + \Delta x \\ -tan(\varphi_x)f + \Delta y \\ -f \end{bmatrix} \quad (15)$$

In other words,

$$(R_{CIS}^{CTS} R_{star}^{CIS})^{-1} \begin{bmatrix} X - X_S \\ Y - Y_S \\ Z - Z_S \end{bmatrix}_{CTS} = mR \begin{bmatrix} tan(\varphi_y)f + \Delta x \\ -tan(\varphi_x)f + \Delta y \\ -f \end{bmatrix} \quad (16)$$

Suppose

$$(R_{CIS}^{CTS} R_{star}^{CIS})^{-1} \begin{bmatrix} X - X_S \\ Y - Y_S \\ Z - Z_S \end{bmatrix}_{CTS} = \begin{bmatrix} \bar{X} \\ \bar{Y} \\ \bar{Z} \end{bmatrix}_{CTS} \quad (17)$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R \begin{bmatrix} tan(\varphi_y)f + \Delta x \\ -tan(\varphi_x)f + \Delta y \\ -f \end{bmatrix} \quad (18)$$

We can therefore derive the collinear condition formula:

$$\begin{cases} \bar{X} = \bar{Z}\dfrac{y}{z} \\ \bar{Y} = Z\dfrac{y}{z} \end{cases} \quad (19)$$

By calculating the derivatives of a, b, c, Δx_p, Δy_p, k_1, k_2, p_1, p_2, S, and R, the above formula is linearized, as shown below:

$$\begin{cases} V_x = \dfrac{\partial \bar{X}}{\partial a}\mathrm{d}a + \dfrac{\partial \bar{X}}{\partial b}\mathrm{d}b + \dfrac{\partial \bar{X}}{\partial c}\mathrm{d}c + \dfrac{\partial \bar{X}}{\partial \Delta x_p}\mathrm{d}\Delta x_p + \\ \quad \dfrac{\partial \bar{X}}{\partial \Delta y_p}\mathrm{d}\Delta y_p + \dfrac{\partial \bar{X}}{\partial k_1}\mathrm{d}k_1 + \dfrac{\partial \bar{X}}{\partial k_2}\mathrm{d}k_2 + \dfrac{\partial \bar{X}}{\partial p_1}\mathrm{d}p \\ \quad + \dfrac{\partial \bar{X}}{\partial p_2}\mathrm{d}p_2 + \dfrac{\partial \bar{X}}{\partial S}\mathrm{d}S + \dfrac{\partial \bar{X}}{\partial R}\mathrm{d}R - l_x \\ V_y = \dfrac{\partial \bar{Y}}{\partial a}\mathrm{d}a + \dfrac{\partial \bar{Y}}{\partial b}\mathrm{d}b + \dfrac{\partial \bar{Y}}{\partial c}\mathrm{d}c + \dfrac{\partial \bar{Y}}{\partial \Delta x_p}\mathrm{d}\Delta x_p + \\ \quad \dfrac{\partial \bar{Y}}{\partial \Delta y_p}\mathrm{d}\Delta y_p + \dfrac{\partial \bar{Y}}{\partial k_1}\mathrm{d}k_1 + \dfrac{\partial \bar{Y}}{\partial k_2}\mathrm{d}k_2 + \dfrac{\partial \bar{Y}}{\partial p_1}\mathrm{d}p \\ \quad + \dfrac{\partial \bar{Y}}{\partial p_2}\mathrm{d}p_2 + \dfrac{\partial Y}{\partial S}\mathrm{d}S + \dfrac{\partial \bar{Y}}{\partial R}\mathrm{d}R - l_y \end{cases} \quad (20)$$

It can be expressed by matrix vectors as

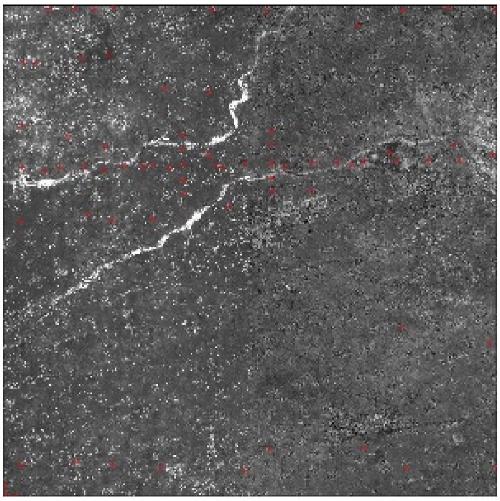
$$V = AX + BY - L \quad (21)$$

where X is the Rodrigues rotation matrix parameter vector, the corresponding coefficient matrix of which is A. Y is the parameter vector of the additional model that takes into account the aberration, the corresponding coefficient matrix of which is B. V is the correction number vector; and L is the observed value vector.

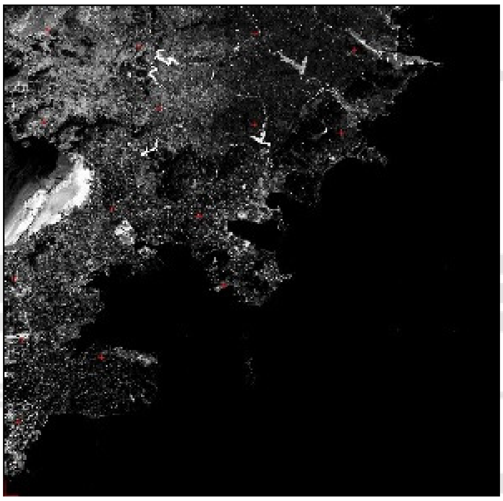
5 EXPERIMENTS AND ANALYSIS

5.1 Experimental Data

The experimental data used in this study were the images, attached files, and ground control point data of the ZY-3 mapping satellite acquired in the Hebei Anping area on 2012-02-18 and the Liaoning Dalian area on January 11, 2012. The Anping area had 70 ground control points, including 29 target ground control points and 41 normal ground object control points. The Dalian area had 19 ground control points, all of which were normal ground object control points. Most of the normal ground object control points were located at road intersections for farmland corners.



(a)



(b)

Figure 2: Control point distribution map, (a) Anping area control point distribution map, (b) Dalian area control point distribution map.

5.2 Rodrigues Rotation Matrix Imaging Bundle Correction Experiment

The geo-location accuracy statistics for the Anping and Dalian areas, when the strict geometric model was directly utilized for geo-location using the auxiliary data, are listed in Tables 1 and 2.

Table 1: Unprocessed geo-location accuracy of the Anping area.

Statistics Item	X/m	Y/m	Z/m
Average error	220.136	437.579	824.830
Maximum residual	225.107	515.886	845.053
Minimum residual	211.679	337.704	795.005
MSE	220.150	440.617	824.944

Table 2: Unprocessed geo-location accuracy of the Dalian area.

Statistics Item	X/m	Y/m	Z/m
Average error	77.199	623.378	603.520
Maximum residual	84.967	659.395	648.268
Minimum residual	73.497	563.374	530.545
MSE	77.253	624.164	604.618

It can be seen from Tables 1 and 2 that the geo-location accuracy of the images in the Dalian area, which was acquired earlier, was different from that of the images in the Anping area. This is because, in the early stage, the orbit of the satellite was not completely stable, and the camera was still performing a series of adjustments, which resulted in changes in the external orientation attitude angle and installation of the camera. Therefore, if unprocessed auxiliary data acquired at different times are used directly for geo-location, the accuracy of the geo-location will be different.

Subsequently, a small number of uniformly distributed ground control points were selected in the Anping images, and the Rodrigues rotation matrix was adopted to correct the imaging bundle. The corrected imaging bundle was then used for geo-location, with the resultant accuracy statistics presented in Table 3.

Table 3: Geo-location accuracy of the Anping area after imaging bundle correction.

Statistics Item	X/m	Y/m	Z/m
Average error	3.971	1.761	4.108
Maximum residual	12.126	4.999	12.394
Minimum residual	0.124	0.233	0.120
Medium error	5.037	2.102	4.888

The Rodrigues rotation matrix parameters of the Anping area were also used to perform extrapolative geo-location of the Dalian area. The accuracy statistics are listed in Table 4.

Table 4: Geo-location accuracy of the Dalian area by the Rodrigues rotation matrix extrapolation.

Statistics Item	X/m	Y/m	Z/m
Average error	13.057	34.493	27.700
Maximum residual	22.682	39.656	36.176
Minimum residual	7.273	29.100	18.066
Medium error	13.586	34.634	28.182

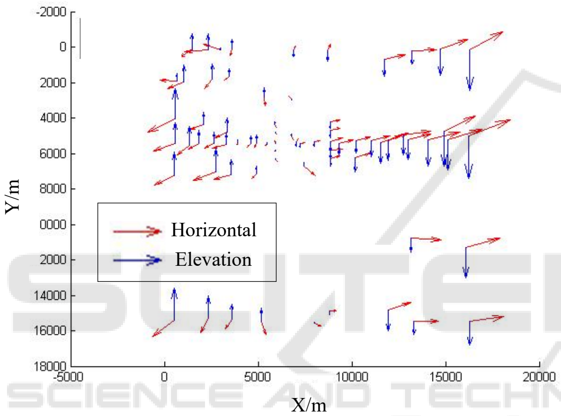


Figure 3: Geo-location residual map of the Anping area with the control points.

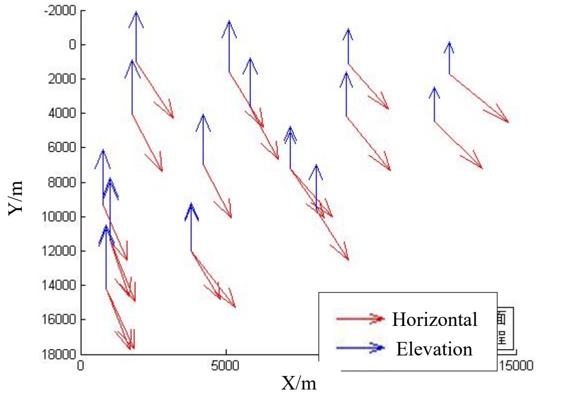


Figure 4: Geo-location residual map of the Dalian area after the use of extrapolation.

It is observed from Tables 1 and 3 that, by using a small number of ground control points for imaging bundle correction, the geo-location accuracy of the Anping area is substantially increased. Additionally, the geo-location residuals in Figure 3 are no longer systematic. Similarly, it is seen from Tables 2 and 4 that, by applying the Rodrigues rotation matrix parameters of the Anping area to the extrapolative geo-location of the Dalian area, the geo-location accuracy is also significantly improved, although not by as much as that of the Anping area. Furthermore, it is noted from Figure. 4 that after extrapolative geo-location, the geo-location residuals of the Dalian area remain systematic to a certain extent, indicating that the Rodrigues rotation matrix parameters of the Anping area can enhance its geo-location accuracy, but cannot fully eliminate the systematic error in the external orientation of the Dalian area. This is because of slight changes in the outer attitude angle and installation matrix of the camera over time.

5.3 Self-calibrating Bundle Adjustment Experiment

On the basis of the Rodrigues rotation matrix, considering the internal orientation distortion, an additional parameter model was adopted to perform block adjustment using the self-calibrating bundle method. The adjustment results are presented in Table 5.

Table 5: Geo-location accuracy of the Anping block after adjustment (RMS/m).

Control point	X	1.148
	Y	1.063
	Plane	1.564
	Elevation	1.121
Check point	X	1.196
	Y	1.400
	Plane	1.841
	Elevation	1.794

By using the Rodrigues rotation matrix parameters [illegible]d through an adjustment of the Anping area, as well as the Δx_s, Δy_s, k_1, k_2, p_1, p_2, S, and R, parameters, extrapolative geo-location of the Dalian area was performed, the accuracy of which is presented in Table 6.

Table 6: Geo-location accuracy of the Dalian area by the block adjustment parameter extrapolation (RMS/m).

Statistics Item	X/m	Y/m	Z/m
Average error	13.433	32.787	25.668
Maximum residual	19.073	35.828	29.003
Minimum residual	8.173	28.771	22.223
RMS	13.683	32.860	25.742

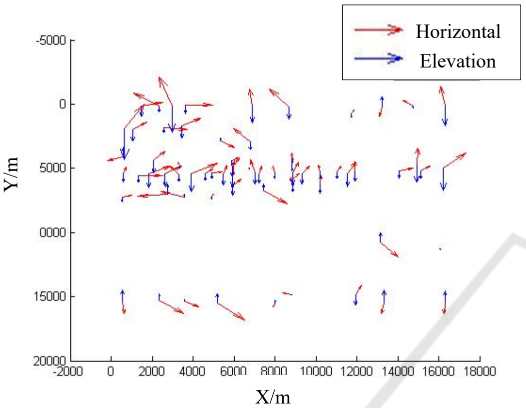


Figure 5: Geo-location residual map of the Anping block after adjustment.

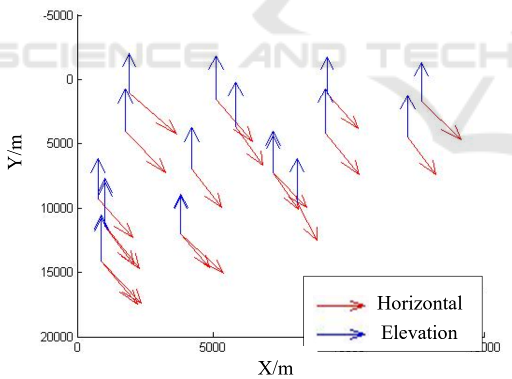


Figure 6: Geo-location residual map of the Dalian area after the use of the adjustment parameters of the Anping block for extrapolation.

It can be seen from Tables 5 and 3 that, through the self-calibrating block adjustment, the internal orientation distortion is compensated, thereby further escalating the geo-location accuracy of the Anping area. This result also confirms the presence of internal orientation distortion. In addition, from Tables 6 and 4, it is observed that, by applying the self-calibration block adjustment parameters of the Anping area to the extrapolative geo-location of the Dalian area, the geo-location accuracy of the Dalian area is also enhanced, although to a smaller extent. A comparison of Figs. 6 and 4 reveals a small change, indicating that there are still systematic errors in the geo-location residual map of the Dalian area. This result suggests that the block adjustment parameters of the Anping area cannot fully eliminate the systematic errors in the external orientation attitude angle and installation matrix or the internal orientation distortion in the Dalian area.

6 CONCLUSIONS

In this study, based on the strict geometric model of the ZY-3 satellite, we propose the Rodrigues rotation matrix to establish an attitude constant error calibration model. In addition, by introducing an additional parameter model considering the aberration to compensate for the internal orientation distortion, we constructed a block adjustment model using the self-calibrating bundle method. This model can derive the correction numbers of the internal and external orientation elements using only a small number of ground control points, thereby substantially improving the imaging geo-location accuracy. By introducing an additional parameter model considering the aberration to compensate for the internal orientation distortion, a block adjustment model using the self-calibrating bundle method was constructed. The model could derive the correction numbers of both the internal and the external orientation elements spontaneously, thereby significantly enhancing the geo-location accuracy. In addition, by applying the derived parameters to the extrapolative geo-location of other areas, the geo-location accuracy of these areas was also substantially escalated. Finally, we analyzed why the accuracy of the extrapolative geo-location areas could not reach that of the original area. The methods and findings of this study can serve as technical references for accurate geo-location solutions of aerospace three-line array imaging.

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