3D Spatial Dependencies Study in the Hawk and Dove Model

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Abstract: The aim of the research was to check spatial dependencies in evolutionary games in 3D grids and compare them with simulation results (2D) and theoretical or analytical considerations obtained from the replicator dynamics equations. In order to compare the results, the classic Hawk and Dove model was used and a series of simulations for both $v < c$ and $v > c$ cases was performed using our own software. The results are almost the same as the theoretical analysis of this model, but some small differences were observed and discussed. It seems, however, that the 3D model better reflects the behaviour of the population than 2D simulations.

1 INTRODUCTION

The theory of evolutionary game theory (EGT) initiated by JM. Smith and G. Price (Sigmund and Nowak, 1999; Smith, 1982) allow expressing the idea of Darwin's matching of genres and their evolution with the elegant mathematical apparatus of game theory. This field of knowledge has allowed the creation of methods for simulation and analysis of the dynamics of the population of individuals observed in the biological world. The players in those games are characterised by strategies or phenotypes that can compete or cooperate to achieve their evolutionary goals. Unlike standard game theory, players are instinct-based rather than particularly rational, and the expected outcome of the game is a better fit for the environment, thereby gaining food, partner, or living space. In the results of individual interactions, the population may be stable, monomorphic or multi-morphic. This state is called evolutionary stable, and the phenotype recognised as an evolutionary stable strategy (ESS) cannot be replaced by another (Smith and Price, 1973). The EGT methodology allows us to predict the behaviour of the population, e.g. whether one of the strategies will be dominant. Additional information about dynamics can be obtained from the so-called replicator dynamics equations (Hofbauer et al., 1979). Still, all this concerns the whole population and does not allow for the analysis of its spatial structure. The use of spatial evolutionary game theory (SEGT) (Bach et al., 2003) allows us to supplement the missing knowledge. Each new state of the population is obtained by performing the following steps: updating payoffs, removing cells, reproducing. Such models have been used e.g. in modelling cancer development (Bach et al., 2003), modelling inter-cellular interactions including avoidance of apoptosis and production of angiogenic factors (Tomlinson and Bodmer, 1997), modelling the production of the cytotoxic substances (Tomlinson, 1997), modelling production of growth factors (Bach et al., 2001), invasion and metastasis (Mansury et al., 2006), tumor-environment interactions (Gatenby and Vincent, 2003), interaction between osteoclasts and osteoblasts (Dingli et al., 2009), tumor-stroma interaction (Gerstung et al., 2011), neighbourhood effect modelling (Krzeslak and Swierniak, 2011), resistance to chemotherapy (Basanta et al., 2012a) or interaction of different types of cancer (Basanta et al., 2012b). An overview of the models can be found in the works (Basanta et al., 2008; Swierniak and Krzeslak, 2013). Until now, spatial analysis has been presented and analysed only in two dimensions (Krzeslak and Swierniak, 2016; Swierniak and Krzeslak, 2016; Swierniak and Krzeslak, 2013; Krzeslak and Swierniak, 2011) or the third dimension meant additional resources (Swierniak et al., 2018). To relate the models to any real, biological population, it seems necessary to examine spatial relationships in three-dimensional space, analysing whether the behaviour of the studied population will show significant differences. In this paper, we examine the spatial model of evolutionary games for the three-dimensional gam-

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ing space on the example of the Hawk-Dove model known in the literature. The results of 3D simulations were compared compare them with 2D simulation results and theoretical or analytical considerations obtained from the replicator dynamics equations. Both cases of parameter settings (for $v < c$ and $v > c$) were taken into account in the research. The influence of the choice of different settings in a spatial game was investigated and simulations for ordinary and mixed games (MSEG - multidimensional spatial evolutionary game), proposed in (Swierniak and Krzeslak, 2016; Krzeslak et al., 2016; Krzeslak and Swierniak, 2016; Swierniak et al., 2016), in which each cell contains information about the composition of different phenotypes, were carried out. Thus, a heterogeneous subpopulation within individual cells is represented.

2 MATERIALS AND METHODS

The game Hawk and Dove is one of the first evolutionary models proposed by John Maynard Smith (Smith, 1982). It includes two types of phenotypes or behaviour: combat (Hawks) or avoidance (Doves), within the population of a single species. This population is a symbolic representation of the ritual conflicts between two different strategies that evolved in the process of evolution.

This game has two players, and each player has his own set of decisions, which we call strategy here. Each pair of strategies for players will result in some game result for each player, which we call a payoff. These values, saved in the matrix form, can be treated as a profit or the cost of choosing a particular strategy. These values can also be used to model for e.g. Darwinians fitness. The payoff matrix (presented in general form in Table 1) has two parameters: v - the benefit of the competition and c - the cost of escalation.

In this model the replicator dynamics equation is as follows:

$$\dot{x} = cx(x-1)(x-\frac{v}{c}). \qquad (1)$$

For Hawk and Dove game stable polymorphism (coexistence between all phenotypes) is defined by the following evolutionarily stable strategy (ESS): $(v/c, 1-v/c)$ if $v < c$. This conditions can be found directly from the definition of ESS or from the Bishop-Canning theorem (Bishop and Cannings, 1978). To achieve a stable polymorphic result v must be less than c, otherwise the population is dominated by Hawks. The results for this model are independent of the initial frequencies of occurrence and the plot of

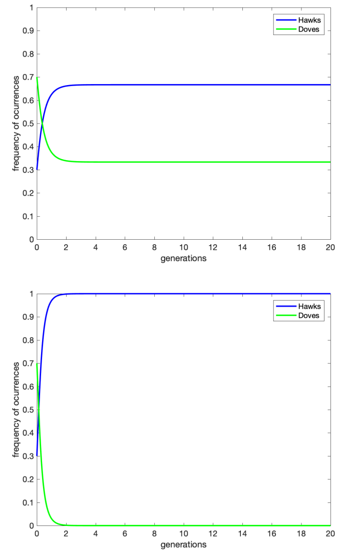

Figure 1: Result of mean-field (replication equations) dynamics for v=6, c=9 ($v < c$) in the top and v=9, c=6 ($v > c$) in the bottom.

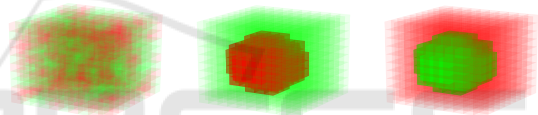

Figure 2: Initial conditions tested for Hawk and Dove model: random (*init0*), Hawks in the middle (*init1*), Doves in the middle (*init2*).

both cases ($v < c$ and $v > c$) are presented on Figure 1). Those plots are the result of solving the Equation 1. Any further plots in this work are the averaged result of simulation performed in 2D or 3D grid.

Table 1: The payoff matrix for original Hawk and Dove model.

Phenotypes	**Hawk**	**Dove**
Hawk	v-c	2v
Dove	0	v

In this paper, we have focused on the comparison of 2D and 3D game results, examining the impact of the initial condition and referring these simulations to the plots obtained from the replicator dynamic equations. The simulations were performed for three different initial conditions, which also gave a different share of particular phenotypes. These were: random distribution of individuals in the participation of 50/50 (called by us *init0*), Hawks concentration in the center of the area (as *init1*) and Doves concentration (*init2*). All those initial conditions are presented in Figure 2.

The in-house software was created to perform

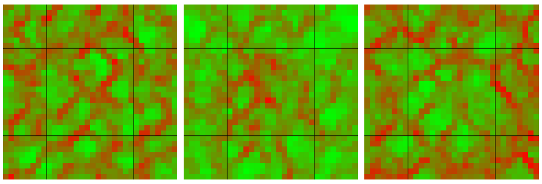

Figure 3: Result of mean 2D spatial game presented for v=6, c=9 ($v < c$) and appropriately for *init0*, *init1* and *init2* conditions.

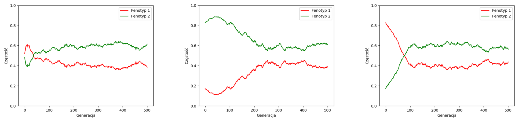

Figure 4: Averaged 2D spatial game dynamics presented in mean-field like plot for v=6, c=9 ($v < c$) and appropriately for *init0*, *init1* and *init2* conditions.

simulations in 2D and 3D cases. Exact steps of simulations were explained first by Bach et al. (Bach et al., 2003) and later in our previous works (Swierniak and Krzeslak, 2013; Swierniak and Krzeslak, 2016; Krzeslak et al., 2016; Krzeslak and Swierniak, 2016; Swierniak et al., 2018). All simulations were performed for 2D or 3D torus of size 32x32 or 10x10x10 cells. Results were analysed on spatial maps, average mean-field like plots and averaged spatial maps. Those averaged spatial images were analysed to show the areas occupied by particular phenotype during the simulation. Green colour represents Doves, and red represents Hawks in all plots that presents the simulation results.

3 RESULTS

In this section results for v=6, c=9 ($v < c$) and v=9, c=6 ($v > c$) will be presented in Figures 3-6 for different initial conditions and for 2D games. Averaged map as one in Figure 3 tells us where are areas of an increased occurrence of a specific phenotype. For example, pixels with almost pure red colour represents areas where almost all the time were observed Hawks phenotype, and vice versa - the greener the more often Doves was in that specific location: the more mixed colour, the more random occurrence of a specific phenotype.

In next section results for v=6, c=9 ($v < c$) are presented in Figures 7-9 for different initial conditions and summary mean image in Figure 10.

In the last section, results for v=9, c=6 ($v > c$) are presented in Figures 12-14 for different initial conditions and summary mean image in Figure 15.

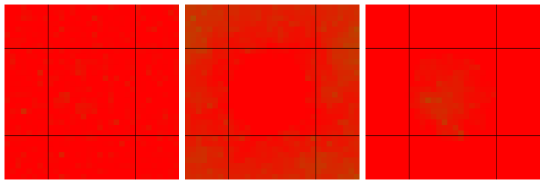

Figure 5: Result of mean 2D spatial game presented for v=9, c=6 ($v > c$) and appropriately for *init0*, *init1* and *init2* conditions.

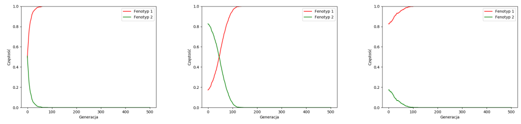

Figure 6: Averaged 2D spatial game dynamics presented in mean-field like plot for v=9, c=6 ($v > c$) and appropriately for *init0*, *init1* and *init2* conditions.

4 DISCUSSION AND CONCLUSIONS

Observation of average graphs (for example Fig. 4, Fig. 6 etc.) allows us to state that the results of spatial games on average overlap with theoretical considerations and replicator dynamics (Fig. 1). At this level, there were no significant differences between 2D and 3D simulations. Only it can be noticed that for the case $v < c$, where the coexistence of both populations occurs, we expected (referring to Figure 1 and the solution of ESS) a slightly better adaptation of the Hawks for the given parameters. However, for 2D spatial games, the Doves phenotype showed a slightly better adaptation. This may suggest that games simulated on 3D grids better reflect the behaviour of the population. Otherwise, in the opposite case ($v > c$), in each situation the results coincided with the theoreti-

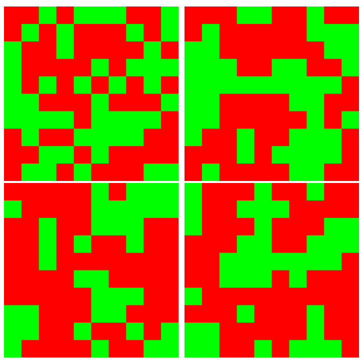

Figure 7: Result of 3D spatial game presented for the middle layer for v=6, c=9 ($v < c$), *init0* and appropriately in generations : 0 (left-top), 50 (right-top), 200 (left-bottom) and 500(right-bottom).

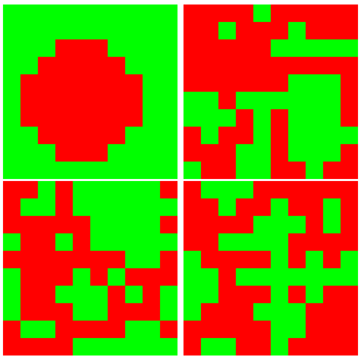

Figure 8: Result of 3D spatial game presented for the middle layer for v=6, c=9 ($v < c$), *init1* and appropriately in generations : 0 (left-top), 50 (right-top), 200 (left-bottom) and 500(right-bottom).

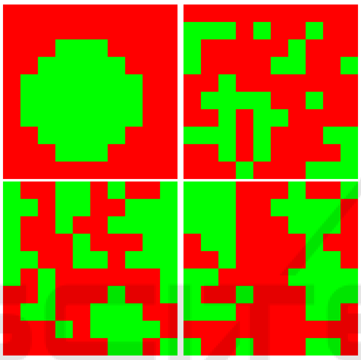

Figure 9: Result of 3D spatial game presented for the middle layer for v=6, c=9 ($v < c$), *init2* and appropriately in generations : 0 (left-top), 50 (right-top), 200 (left-bottom) and 500(right-bottom).

cal, quickly following the dominance of Hawks. The analysis of spatial results showed that even if, in the initial stage, the populations show some structure, the spatial distribution quickly starts to resemble a random one. On the average graphs in 2D, one can see a delicate structure, where particular phenotypes were grouped. This is not so clear for the 3D case (and $v < c$), although it is possible that this is due to the small size of the grid. An interesting structure was observed in the results of the 3D simulations, which seemed not to show much. After all, we expect a

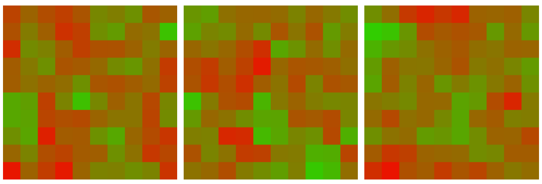

Figure 10: Result of mean 3D spatial game presented for the middle layer for v=6, c=9 ($v < c$) and appropriately for *init0*, *init1* and *init2* conditions.

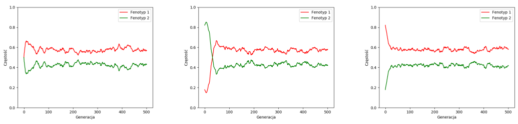

Figure 11: Averaged 3D spatial game dynamics presented in mean-field like plot for v=6, c=9 ($v < c$) and appropriately for *init0*, *init1* and *init2* conditions.

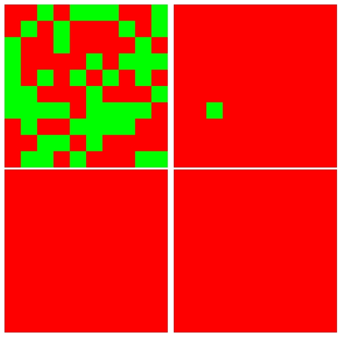

Figure 12: Result of 3D spatial game presented for the middle layer for v=9, c=6 ($v > c$), *init0* and appropriately in generations : 0 (left-top), 50 (right-top), 200 (left-bottom) and 500(right-bottom).

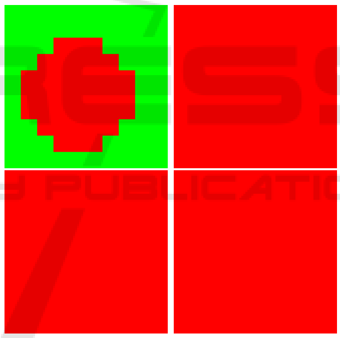

Figure 13: Result of 3D spatial game presented for the middle layer for v=9, c=6 ($v > c$), *init1* and appropriately in generations : 0 (left-top), 50 (right-top), 200 (left-bottom) and 500(right-bottom).

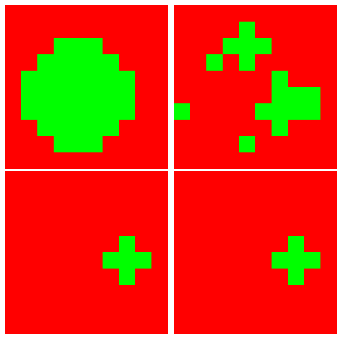

Figure 14: Result of 3D spatial game presented for the middle layer for v=9, c=6 ($v > c$), *init2* and appropriately in generations : 0 (left-top), 50 (right-top), 200 (left-bottom) and 500(right-bottom).

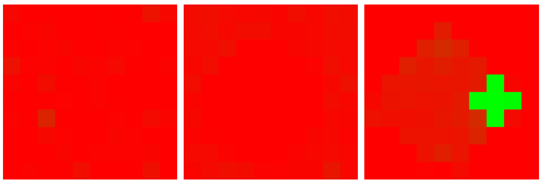

Figure 15: Result of mean 3D spatial game presented for the middle layer for v=9, c=6 ($v > c$) and appropriately for *init0*, *init1* and *init2* conditions.

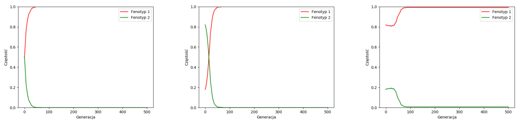

Figure 16: Averaged 3D spatial game dynamics presented in mean-field like plot for v=9, c=6 ($v > c$) and appropriately for *init0*, *init1* and *init2* conditions.

quick elimination of Doves. In spatial games, it turns out that the full elimination of Doves does not take place, and there is always a small number of representatives of this phenotype. What is more, on the Figure 14 one can see a formed structure in the form of a cross (corresponding to the settings of the simulations, i.e. Moore's neighbourhood), where Doves remain all the time. This trend is also clearly shown in Figure 15 (the mean of 3D spatial game result). On the basis of the carried out calculations, we can suggest that 3D simulations seem to reflect the population dynamics better, although the results are slightly more demanding for analysis. So concluding this study we suggest to perform spatial simulations of any game theoretical model using 3D grids.

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